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SuperIso v4.1: A program for calculating flavour physics observables in SM, 2HDM and supersymmetry

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Abstract

We describe `SuperIso v4.1` which is a public program for evaluation of flavour physics observables in the Standard Model (SM), general two-Higgs-doublet model (2HDM), minimal supersymmetric Standard Model (MSSM) and next to minimal supersymmetric Standard Model (NMSSM). `SuperIso v3.4`, in addition to the branching ratio of $\bar{B} \to X_s \gamma$ and the isospin asymmetry of $B \to K^* \gamma$, incorporates other flavour observables such as the branching ratios of $B_{s,d} \to \mu^+ \mu^-$, the branching ratio of $B \to \tau \nu_\tau$, the branching ratio of $B \to D \tau \nu_\tau$, the branching ratio of $K \to \mu \nu_\mu$, the branching ratio of $D \to \mu \nu_\mu$, and the branching ratios of $D_s \to \tau \nu_\tau$ and $D_s \to \mu \nu_\mu$, and several observables from the $B \to X_s \ell^+ \ell^-$ and $\bar{B} \to K^* \mu^+ \mu^-$ decays. The program also computes the muon anomalous magnetic moment $(g_\mu - 2)$. Several sample models are included in the package, namely SM, 2HDM, and CMSSM, NUHM, AMSB, HCAMSB, MMAMSB and GMSB for the MSSM, and CNMSSM, NGMSB and NNUHM for the NMSSM. `SuperIso` uses a SUSY Les Houches Accord file (SLHA1 or SLHA2) as input, which can be either generated automatically by the program via a call to external spectrum calculators, or provided by the user. The program can generate also outputs in the Flavour Les Houches Accord (FLHA) format. The calculation of the observables is detailed in the Appendices, where a suggestion for the allowed intervals for each observable is also provided.

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Contents

1 Introduction **8**

2 Content of the SuperIso v4.1 package **9**

2.1 Parameter structure 9

2.2 Main physics routines 12

2.3 Interpreter routines 20

2.4 Main statistical routines 23

3 Compilation and installation instructions **30**

4 Input and output description **32**

4.1 Standard Model main program 32

4.2 SLHA input file 34

4.3 CMSSM inputs 36

4.4 AMSB inputs 38

4.5 HCAMSB inputs 38

4.6 MMAMSB inputs 39

4.7 GMSB inputs 39

4.8 NUHM inputs 40

4.9 CNMSSM inputs 40

4.10 NGMSB inputs 41

4.11 NNUHM inputs 41

4.12 2HDM inputs 42

5 Results **42**

6 Conclusion **44**

Appendix A: QCD coupling **45**

Appendix B: Evolution of quark masses **45**

Appendix C: Wilson coefficients at matching scale **47**

C.1 Wilson coefficients $C_1 - C_8$ . . . 47

C.1.1 Standard Model contributions . . . 48

C.1.2 Charged Higgs contributions . . . 53

C.1.3 Supersymmetric contributions . . . 60

C.2 Wilson coefficients $C_9 - C_{10}$ . . . 66

C.2.1 Standard Model contributions . . . 66

C.2.2 Charged Higgs contributions . . . 70

C.2.3 Supersymmetric contributions . . . 71

C.3 Wilson coefficients $C_{Q_1} - C_{Q_2}$ . . . 74

C.3.1 2HDM contributions . . . 74

C.3.2 MSSM contributions . . . 75

C.3.3 NMSSM contributions . . . 78

C.4 Prime Wilson coefficients . . . 81

C.4.1 Prime Wilson coefficients $C'_{7,8}$ . . . 81

C.4.2 Prime Wilson coefficients $C'_{9,10}$ . . . 82

C.4.3 Prime Wilson coefficients C'_{Q_1,Q_2} . . . 83

C.5 Wilson coefficients C_L, C_R . . . 84

C.5.1 Standard Model contributions . . . 85

C.5.2 MSSM contributions . . . 85

C.6 Auxiliary functions . . . 89

Appendix D: Renormalization group equations **98**

D.1 RGE for the $C_{1\cdots10}$ Wilson coefficients in the standard operator basis . . . 98

D.2 RGE the $C_{1\cdots8}$ Wilson coefficients in the traditional operator basis . . . 104

D.3 Renormalization group equations for C_{Q_1,Q_2} . . . 106

D.4 Renormalization group equations for the prime Wilson coefficients . . . 106

Appendix E: Calculation of flavour observables **110**

E.1 Branching ratio of $\bar{B} \to X_s\gamma$ . . . 110

E.2 Isospin asymmetry of $B \to K^*\gamma$ . . . 122

E.3 $B \to X_s\ell^+\ell^-$ . . . 125

E.3.1 Main formulas . . . 126

E.3.2 Λ^2_{QCD}/m_b^2 and Λ^3_{QCD}/m_b^3 corrections . . . 130
E.3.3 Λ^2_{QCD}/m_c^2 correction . . . 131
E.3.4 Bremsstrahlung contributions . . . 132
E.3.5 Electromagnetic contributions . . . 136
E.3.6 Long distance contributions . . . 138
E.4 $B \to \bar{K}^*\ell^+\ell^-$ . . . 139
E.4.1 Angular coefficients in terms of transversity amplitudes . . . 140
E.4.2 Angular coefficients in terms of helicity amplitudes . . . 147
E.4.3 Translation between transversity and helicity amplitudes . . . 149
E.4.4 Calculation of $\mathcal{T}_a^\pm$ . . . 150
E.4.5 Observables . . . 157
E.5 $B \to K^*\gamma$ . . . 159
E.6 $B_s \to \phi\ell^+\ell^-$ . . . 160
E.6.1 Observables . . . 160
E.7 $B \to K\ell^+\ell^-$ . . . 161
E.7.1 Calculation of $\mathcal{T}_{\mathbf{P}}$ . . . 164
E.7.2 Observables . . . 165
E.8 Branching ratio of $B_{s,d} \to \mu^+\mu^-$ . . . 165
E.8.1 CP-averaged branching ratio . . . 165
E.8.2 Untagged Branching ratio . . . 166
E.9 Branching ratio of $K \to \pi\nu\nu$ . . . 167
E.10 Branching ratio of $K_{L,S} \to \mu^+\mu^-$ . . . 169
E.11 Branching ratio of $B_u \to \tau\nu_\tau$ . . . 172
E.12 Branching ratio of $B \to D\tau\nu_\tau$ . . . 172
E.13 Branching ratio of $K \to \mu\nu_\mu$ . . . 173
E.14 Branching ratio of $D_s \to \ell\nu_\ell$ . . . 174
E.15 Branching ratio of $D \to \mu\nu_\mu$ . . . 175

Appendix F: Muon anomalous magnetic moment **176**
F.1 Supersymmetry . . . 176
F.2 2HDM . . . 180

Appendix G: Useful parameters **181**

Appendix H: Suggested limits **181**

Appendix I: LHA file format for 2HDM **181**

Appendix J: Sample FLHA output file **186**

References **188**

List of Tables

1 $B \to K^* \mu^+ \mu^-$ observables . . . 18

2 Possible types of observables . . . 21

3 Possible decays . . . 22

4 Yukawa couplings for the four types of 2HDM . . . 53

5 Values of the RGE a_i numbers. . . . 98

6 $m_{kli}^{(00)}$ values . . . 99

7 $m_{kli}^{(1j)}$ values . . . 100

8 $m_{kli}^{(2j)}$ values . . . 101

9 $m_{9li}^{(00)}$ and $m_{9li}^{(1j)}$ values . . . 101

10 $m_{9li}^{(2j)}$ values . . . 102

11 a'_i, b_i, $d_i^{(j)}$ and $e_i^{(j)}$ values for C_{10} electroweak corrections . . . 104

12 $l_{kli}^{(00)}$ values . . . 107

13 $l_{kli}^{(10)}$ values . . . 108

14 $l_{kli}^{(11)}$ values . . . 109

15 Useful numbers for $K_c^{(0)}$ . . . 122

16 Meson masses and decay properties . . . 139

17 Input parameters . . . 182

18 Allowed intervals for the implemented observables . . . 183

19 Experimental results and theoretical predictions for the $b \to s\ell\ell$ observables [*to be updated*]. . . . 184

20 LEP and Tevatron Higgs and MSSM particle mass limits . . . 185

1 Introduction

Along with the direct searches for new physics effects and particles, indirect searches appear as very important and complementary tools to explore physics beyond the Standard Model (SM). We investigate here the most popular extensions of the SM, such as the two-Higgs-doublet model (2HDM) and supersymmetry (SUSY). The presence of new particles as virtual states in processes involving only ordinary external particles provides us with the opportunities to study the indirect effects of new physics. This is the case for example of the rare B decays or the anomalous magnetic moment of the charged leptons.

Phenomenological interests of indirect searches are numerous. This includes tests of the SM predictions and the possibility to reveal indirect effects of new physics. The later can be complementary to direct searches or even used as guideline. The results of indirect searches can also be employed in order to check consistencies with the direct search results.

The main purpose of the `SuperIso` program is to offer the possibility to evaluate the most important indirect observables and constraints.

`SuperIso` [1] was in its first versions devoted to the calculation of the isospin symmetry breaking in $B \to K^* \gamma$ decays in the minimal supersymmetric extension of the Standard Model (MSSM) with minimal flavour violation. This observable imposes stringent constraints on supersymmetric models [2], which justifies a dedicated program. The calculation of the $b \to s\gamma$ branching ratio was also included in the first version and has been improved by adding NNLO contributions since version 2. Also, a broader set of flavour physics observables has been implemented. This includes the branching ratios of $B_{s,d} \to \mu^+\mu^-$, the branching ratio of $B_u \to \tau\nu_\tau$, the branching ratios of $B \to D^0 \tau\nu_\tau$ and $B \to D^0 e\nu_e$, the branching ratio of $K \to \mu\nu_\mu$, the branching ratio of $D \to \mu\nu_\mu$, and the branching ratios of $D_s \to \tau\nu_\tau$ and $D_s \to \mu\nu_\mu$, and several observables from the $B \to X_s \ell^+\ell^-$ and $\bar{B} \to X_s \mu^+\mu^-$ decays. The calculation of the anomalous magnetic moment of the muon is also implemented in the program.

`SuperIso` has been extended to the general two-Higgs-doublet model since version 2.6 and to the next to minimal supersymmetric extension of the Standard Model (NMSSM) since version 3.0 [3]. Also, since version 2.8 an interface with the code `HiggsBounds` [4] is available[1] in order to obtain direct Higgs search constraints automatically in the output of `SuperIso`.

`SuperIso` uses a SUSY Les Houches Accord file (SLHA) [6, 7] as input, which can be either generated automatically by the program via a call to `SOFTSUSY` [8], `ISAJET` [9], `SPheno` [10], `SuSpect` [11] and `NMSSMTools` [12], or provided by the user. `SuperIso` can also use the LHA inspired format for the 2HDM generated by `2HDMC` [13], which will be described in Appendix I. The program is able to perform the calculations automatically for different types of 2HDM (I–IV), for different supersymmetric scenarios, such as the Constrained MSSM (CMSSM), the Non-Universal Higgs Mass model (NUHM), the Anomaly Mediated Supersymmetry Breaking scenario (AMSB), the Hypercharge Anomaly Mediated Supersymmetry Breaking scenario (HCAMSB), the Mixed Modulus Anomaly Mediated Supersymmetry Breaking sce-

[1] `FeynHiggs` [5] is necessary to use `HiggsBounds` with `SuperIso`.

nario (MMAMSB) and the Gauge Mediated Supersymmetry Breaking scenario (GMSB), and for the NMSSM scenarios namely CNMSSM, NGMSB and NNUHM. `SuperIso` is able to generate output files in the Flavour Les Houches Accord (FLHA) format [14].

In the following, we first discuss the content of the `SuperIso` package and give the list of the main routines. Then we describe the procedure to use `SuperIso`, and introduce the inputs and outputs of the program. Finally, we present some examples of results obtained with `SuperIso`. In the Appendices, a complete description of the formulas used to calculate the different observables in `SuperIso` is given for reference, as well as suggestions for the allowed intervals.

2 Content of the SuperIso v4.1 package

`SuperIso` is a C program respecting the C99 standard, devoted to the calculation of the most constraining flavour physics observables. Several main programs are provided in the package, but the users are also invited to write their own main programs. `slha.c` can scan files written following the SUSY Les Houches Accord formats, and calculates the corresponding observables. `sm.c` provides the values of the observables in the Standard Model while `thdm.c` computes the observables in 2HDM types I–IV and requires `2HDMC` [13] for the generation of the input file containing the Higgs masses and couplings. The main programs `cmssm.c`, `amsb.c`, `hcamsb.c`, `mmamsb.c`, `gmsb.c`, and `nuhm.c` have to be linked to at least one of the `SOFTSUSY` [8], `ISASUGRA/ISAJET` [9], `SPheno` [10] and/or `SuSpect` [11] packages, in order to compute supersymmetric mass spectra and couplings within respectively the CMSSM, AMSB, HCAMSB, MMAMSB, GMSB or NUHM scenarios. The programs `cnmssm.c`, `ngmsb.c`, and `nnuhm.c` have to be linked to the `NMSSMTools` [12] program to compute supersymmetric mass spectra and couplings within respectively the CNMSSM, NGMSB or NNUHM scenarios. For the general MSSM (or other supersymmetric scenarios) the user has to provide SLHA files containing all the needed masses and couplings.

The computation of the different observables in `SuperIso` proceeds following three main steps:

- Generation of the SLHA file with `ISAJET`, `SOFTSUSY`, `SPheno`, `SuSpect` or `NMSSMTools` (or supply of the SLHA file by the user),
- Scan of the SLHA file,
- Calculation of the observables.

The last point incorporates a complex procedure: to compute the inclusive branching ratio of $b \to s\gamma$ for example, `SuperIso` needs first to compute the Wilson coefficients at matching scale, and then to evolve them using the Renormalization Group Equations (RGE) to a lower scale, before using them to compute the branching ratio.

2.1 Parameter structure

The `SuperIso` package relies on the definition of a structure in `src/include.h`, supporting the SLHA1 and SLHA2 formats. This structure is defined as follows:

```
typedef struct parameters
/* structure containing all the scanned parameters from the SLHA file */
{
int SM;
int model; /* CMSSM=1, GMSB=2, AMSB=3 */
int generator; /* ISAJET=1, SOFTSUSY=3, SPHENO=4, SUSPECT=5, NMSSMTOOLS=6 */
double Q; /* Qmax ; default = M_EWSB = sqrt(m_stop1*mstop2) */
double m0,m12,tan_beta,sign_mu,A0; /* CMSSM parameters */
double Lambda,Mmess,N5,cgrav,m32; /* AMSB, GMSB parameters */
double mass_Z,mass_W,mass_b,mass_top_pole,mass_tau; /* SM parameters */
double inv_alpha_em,alphas_MZ,Gfermi,GAUGE_Q; /* SM parameters */
double charg_Umix[3][3],charg_Vmix[3][3],stop_mix[3][3],sbot_mix[3][3],
stau_mix[3][3],neut_mix[6][6],mass_neut[6],alpha; /* mass mixing matrices */
double Min,M1_Min,M2_Min,M3_Min,At_Min,Ab_Min,Atau_Min,M2H1_Min,M2H2_Min,
mu_Min,M2A_Min,tb_Min,mA_Min; /* optional input parameters at scale Min */
double MeL_Min,MmuL_Min,MtauL_Min,MeR_Min,MmuR_Min,MtauR_Min; /* optional
input parameters at scale Min */
double MqL1_Min,MqL2_Min,MqL3_Min,MuR_Min,McR_Min,MtR_Min,MdR_Min,MsR_Min,
MbR_Min; /* optional input parameters at scale Min */
double N51,N52,N53,M2H1_Q,M2H2_Q; /* optional input parameters (N51...3: GMSB) */
double mass_d,mass_u,mass_s,mass_c_pole,mass_b_pole,mass_e,mass_nue,mass_mu,
mass_num,mass_nut; /* SM masses */
double mass_gluon,mass_photon,mass_Z0; /* SM masses */
double mass_h0,mass_H0,mass_A0,mass_H,mass_dnl,mass_upl,mass_stl,mass_chl,
mass_b1,mass_t1; /* Higgs & superparticle masses */
double mass_el,mass_nuel,mass_mul,mass_numl,mass_tau1,mass_nutl,mass_gluino,
mass_cha1,mass_cha2; /* superparticle masses */
double mass_dnr,mass_upr,mass_str,mass_chr,mass_b2,mass_t2,mass_er,mass_mur,
mass_tau2; /* superparticle masses */
double mass_nuer,mass_numr,mass_nutr,mass_graviton,mass_gravitino; /* masses */
double gp,g2,gp_Q,g2_Q,g3_Q,YU_Q,yut[4],YD_Q,yub[4],YE_Q,yutau[4]; /* couplings */
double HMIX_Q,mu_Q,tanb_GUT,Higgs_VEV,mA2_Q,MSOFT_Q,M1_Q,M2_Q,M3_Q; /* parameters
at scale Q */
double MeL_Q,MmuL_Q,MtauL_Q,MeR_Q,MmuR_Q,MtauR_Q,MqL1_Q,MqL2_Q,MqL3_Q,MuR_Q,
McR_Q,MtR_Q,MdR_Q,MsR_Q,MbR_Q; /* masses at scale Q */
double AU_Q,A_u,A_c,A_t,AD_Q,A_d,A_s,A_b,AE_Q,A_e,A_mu,A_tau; /* trilinear
couplings */

/* SLHA2 */
int NMSSM,RV,CPV,FV;
double mass_nutau2,mass_e2,mass_nue2,mass_mu2,mass_numu2,mass_d2,mass_u2,
mass_s2,mass_c2;
double CKM_lambda,CKM_A,CKM_rhobar,CKM_etabar;
double PMNS_theta12,PMNS_theta23,PMNS_theta13,PMNS_delta13,PMNS_alpha1,
PMNS_alpha2;
double lambdaNMSSM_Min,kappaNMSSM_Min,AlambdaNMSSM_Min,AkappaNMSSM_Min,
```

```
lambdaSNMSSM_Min,xiFNMSSM_Min,xiSNMSSM_Min,mupNMSSM_Min,mSp2NMSSM_Min,
mS2NMSSM_Min,mass_H03,mass_A02,NMSSMRUN_Q,lambdaNMSSM,kappaNMSSM,
AlambdaNMSSM,AkappaNMSSM,lambdaSNMSSM,xiFNMSSM,xiSNMSSM,mupNMSSM,
mSp2NMSSM,mS2NMSSM; /* NMSSM parameters */
double PMNSU_Q,CKM_Q,IMCKM_Q,MSE2_Q,MSU2_Q,MSD2_Q,MSL2_Q,MSQ2_Q,
TU_Q,TD_Q,TE_Q;
double CKM[4][4],IMCKM[4][4]; /* CKM matrix */
double H0_mix[4][4],A0_mix[4][4]; /* Higgs mixing matrices */
double sU_mix[7][7],sD_mix[7][7],sE_mix[7][7], sNU_mix[4][4]; /* mixing
matrices */
double sCKM_msq2[4][4],sCKM_msl2[4][4],sCKM_msd2[4][4],sCKM_msu2[4][4],
sCKM_mse2[4][4]; /* super CKM matrices */
double PMNS_U[4][4]; /* PMNS mixing matrices */
double TU[4][4],TD[4][4],TE[4][4]; /* trilinear couplings */

/* non-SLHA*/
double mass_b_1S,mass_c,mass_top;
double Lambda5; /* Lambda QCD */

/* Flavour parameters */
double f_B,f_Bs,f_Ds,f_D,fK_fpi;
double f_K_par,f_K_perp;
double m_B,m_Bs,m_Bd,m_pi,m_Ds,m_K,m_Kstar,m_D0,m_D;
double life_pi,life_K,life_B,life_Bs,life_Bd,life_D,life_Ds;
double a1par,a2par,a1perp,a2perp;
double zeta3A,zeta3V,wA10,deltatp,deltatm;
double lambda_Bp,rho1,lambda2;
double BR_BXclnu_exp;

/* CKM matrix */
double complex Vud,Vus,Vub,Vcd,Vcs,Vcb,Vtd,Vts,Vtb;

/* 2HDM */
int THDM_model;
double lambda_u[4][4],lambda_d[4][4],lambda_l[4][4];

/* NMSSMTools */
int NMSSMcoll,NMSSMtheory,NMSSMups1S,NMSSMetab1S;

/* Decay widths */
double width_Z,width_W;
}
parameters;
```

This structure contains all the important parameters and is called by most of the main functions in the program.

2.2 Main physics routines

We now review the main routines of the code. The complete list of implemented routines can be found in `src/include.h`.

- `void Init_param(struct parameters* param)`

  This function initializes the `param` structure, setting all the parameters to 0, apart from the SM masses and couplings, which receive the values given in the PDG [15].

- `int Les_Houches_Reader(char name[], struct parameters* param)`

  This routine reads the SLHA file whose name is contained in `name`, and put all the read parameters in the structure `param`. This function has been updated to the SLHA2 format. This routine can also read the LHA inspired format for the 2HDM described in Appendix I. A negative value for `param->model` indicates a problem in reading the SLHA file, or a model not yet included in `SuperIso` (such as R-parity breaking models). In this case, `Les_Houches_Reader` returns 0, otherwise 1.

- `int test_slha(char name[])`

  This routine checks if the SLHA file whose name is contained in `name` is valid, and if so return 1. If not, -1 means that in the SLHA generator the computation did not succeed (*e.g.* because of tachyonic particles), -2 means that the considered model is not currently implemented in `SuperIso`, and -3 that the file provided is either not in the SLHA format, or some important elements are missing.

- `int isajet_cmssm(double m0, double m12, double tanb, double A0, double sgnmu, double mtop, char name[])`

- `int isajet_gmsb(double Lambda, double Mmess, double tanb, int N5, double cGrav, double sgnmu, double mtop, char name[])`

- `int isajet_amsb(double m0, double m32, double tanb, double sgnmu, double mtop, char name[])`

- `int isajet_mmamsb(double alpha, double m32, double tanb, double sgnmu, double mtop, char name[])`

- `int isajet_hcamsb(double alpha, double m32, double tanb, double sgnmu, double mtop, char name[])`

- `int isajet_nuhm(double m0, double m12, double tanb, double A0, double mu, double mA, double mtop, char name[])`

  The above routines call `ISAJET` to compute the mass spectrum corresponding to the input parameters (more details are given in the next sections), and return an SLHA file whose name has to be specified in the string `name`. It should however be noted `isajet_gmsb`, `isajet_amsb`, `isajet_mmamsb`, `isajet_hcamsb` and `isajet_nuhm` only work with `ISAJET v7.80` or later versions.

- `int softsusy_cmssm(double m0, double m12, double tanb, double A0, double sgnmu, double mtop, double mbot, double alphas_mz, char name[])`

- `int softsusy_gmsb(double Lambda, double Mmess, double tanb, int N5, double cGrav, double sgnmu, double mtop, double mbot, double alphas_mz, char name[])`
- `int softsusy_amsb(double m0, double m32, double tanb, double sgnmu, double mtop, double mbot, double alphas_mz, char name[])`
- `int softsusy_nuhm(double m0, double m12, double tanb, double A0, double mu, double mA, double mtop, double mbot, double alphas_mz, char name[])`
- `int softsusy_mssm(double m1, double m2, double m3, double tanb, double mA, double at, double ab, double atau, double mu, double mer, double mel, double mstaul, double mstaur, double mql, double mq3l, double mqur, double mqtr, double mqdr, double mqbr, double Q, double mtop, double mbot, double alphas_mz, char name[])`

  The above routines call `SOFTSUSY` to compute the mass spectrum corresponding to the input parameters (more details are given in the next sections), and return an SLHA file whose name has to be specified in the string `name`.

- `int spheno_cmssm(double m0, double m12, double tanb, double A0, double sgnmu, double mtop, double mbot, double alphas_mz, char name[])`
- `int spheno_gmsb(double Lambda, double Mmess, double tanb, int N5, double sgnmu, double mtop, double mbot, double alphas_mz, char name[])`
- `int spheno_amsb(double m0, double m32, double tanb, double sgnmu, double mtop, double mbot, double alphas_mz, char name[])`

  The above routines call `SPheno` to compute the mass spectrum corresponding to the input parameters (more details are given in the next sections), and return an SLHA file whose name has to be specified in the string `name`.

- `int suspect_cmssm(double m0, double m12, double tanb, double A0, double sgnmu, double mtop, double mbot, double alphas_mz, char name[])`
- `int suspect_gmsb(double Lambda, double Mmess, double tanb, int N5, double sgnmu, double mtop, double mbot, double alphas_mz, char name[])`
- `int suspect_amsb(double m0, double m32, double tanb, double sgnmu, double mtop, double mbot, double alphas_mz, char name[])`
- `int suspect_nuhm(double m0, double m12, double tanb, double A0, double mu, double mA, double mtop, double mbot, double alphas_mz, char name[])`
- `int suspect_mssm(double m1, double m2, double m3, double tanb, double mA, double at, double ab, double atau, double mu, double mer, double mel, double mstaul, double mstaur, double mql, double mq3l, double mqur, double mqtr, double mqdr, double mqbr, double Q, double mtop, double mbot, double alphas_mz, char name[])`

  The above routines call `SuSpect` to compute the mass spectrum corresponding to the input parameters (more details are given in the next sections), and return an SLHA file whose name has to be specified in the string `name`.

- `int thdmc_types(double l1, double l2, double l3, double l4, double l5, double l6, double l7, double m12_2, double tanb, int type, char name[])`

  This routine calls `2HDMC` to compute the masses and couplings corresponding to the 2HDM input parameters, and returns a LHA inspired file whose name has to be specified in the string `name`.

- `int nmssmtools_cnmssm(double m0, double m12, double tanb, double A0, double lambda, double AK, double sgnmu, double mtop, double mbot, double alphas_mz, char name[])`

- `int nmssmtools_nnuhm(double m0, double m12, double tanb, double A0, double MHDGUT, double MHUGUT, double lambda, double AK, double sgnmu, double mtop, double mbot, double alphas_mz, char name[])`

- `int nmssmtools_ngmsb(double Lambda, double Mmess, double tanb, int N5, double lambda, double AK, double Del_h, double sgnmu, double mtop, double mbot, double alphas_mz, char name[])`

  The above routines call `NMSSMTools` to compute the mass spectrum corresponding to the input parameters (more details are given in the next sections) and return the SLHA2 file `name`.

- `alphas_running(double Q, double mtop, double mbot, struct parameters* param)`

  This function computes the strong coupling constant at the energy scale `Q` using the parameters in `param`, provided the top quark mass `mtop` and bottom quark mass `mbot` used for the matching between the scales corresponding to different flavour numbers are specified. The main formula for calculating α_s is given in Appendix A.

- `double running_mass(double quark_mass, double Qinit, double Qfin, double mtop, double mbot, struct parameters* param)`

  This function calculates the running quark mass at the scale `Qfin`, for a quark of mass `quark_mass` at the scale `Qinit` using the structure `param`, knowing the matching scales `mtop` and `mbot`. A description of the quark mass calculations can be found in Appendix B.

- `void CW_calculator(double C0w[], double C1w[], double C2w[], double mu_W, struct parameters* param)`

- `void C_calculator_base1(double C0w[], double C1w[], double C2w[], double mu_W, double C0b[], double C1b[], double C2b[], double mu, struct parameters* param)`

- `void C_calculator_base2(double C0w[], double C1w[], double mu_W, double C0b[], double C1b[], double mu, struct parameters* param)`

  These three routines compute the Wilson coefficients $C_1 \cdots C_{10}$.
  The procedure `CW_calculator` computes the LO contributions to the Wilson coefficients `C0w[]`, the NLO contributions `C1w[]` and the NNLO contributions `C2w[]` at the matching scale `mu_W`, using the parameters of `param`, as described in Appendix C.

C_calculator_base1 evolves the LO, NLO and NNLO Wilson coefficients C0w[], C1w[], C2w[] initially at scale mu_W to C0b[], C1b[], C2b[] at scale mu, in the standard operator basis described in Appendix D.1.

C_calculator_base2 evolves the LO and NLO Wilson coefficients C0w[], C1w[] initially at scale mu_W to C0b[], C1b[] at scale mu, in the traditional operator basis described in Appendix D.2.

- `void CQ_calculator(double complex CQ0b[], double complex CQ1b[], double mu_W, double mu, struct parameters* param)`

  This routine computes the Wilson coefficients corresponding to the scalar operators Q_1 and Q_2 as described in Appendix C.3.

- `void Cprime_calculator(double Cpb[], double complex CQpb[], double mu_W, double mu, struct parameters* param)`

  This routine computes the primed Wilson coefficients (with flipped chirality) as described in Appendix C.4.

- `int excluded_mass_calculator(char name[])`

  This routine, with the name of the SLHA file in the argument, checks whether the parameter space point is excluded by the LEP and Tevatron constraints on the particle masses and if so returns 1. The implemented mass limits are given in Appendix H, and can be updated by the users in src/excluded_masses.c . These limits are valid only in the MSSM.

- `int NMSSM_collider_excluded(char name[])`

- `int NMSSM_theory_excluded(char name[])`

  These two routines only apply to the SLHA file name generated by NMSSMTools, as they need NMSSMTools specific outputs. They respectively check if a parameter space point is excluded by collider constraints [16] or by theoretical constraints (such as unphysical global minimum). The output 1 means that the point is excluded.

- `double higgsbounds_calculator(char name[])`

  The higgsbounds_calculator routine, with the name of the SLHA file in the argument, calls HiggsBounds to check the direct search constraints on the Higgs masses. If for a given point the result is larger than 1, the point is excluded.

- `int charged_LSP_calculator(char name[])`

  This routine, with the name of the SLHA file in the argument, checks whether the LSP is charged or not. It returns 0 if the LSP is a neutralino, 1 if it is charged, 2 if the LSP is a sneutrino, and 3 if it is a gluino.

- `void flha_generator(char name[], char name_output[])`

  This routine generates an output FLHA file using the input SLHA file. The first argument is the name of the SLHA file and the second of the FLHA file. An example of FLHA output is given in Appendix J.

The following routines encode the implemented observables:

- `double bsgamma(double C0[], double C1[], double C2[], double Cp[], double mu, double mu_W, struct parameters* param)`

  This function has replaced the calculation of $b \to s\gamma$ in the first version, which was performed at NLO accuracy. Here, knowing the LO, NLO, NNLO and primed Wilson coefficients `C0[]`, `C1[]`, `C2[]`, `Cp[]` at scale `mu`, and given the matching scale `mu_W` this procedure computes the inclusive branching ratio of $b \to s\gamma$ at NNLO, as described in Appendix E.1.
  The container routine `bsgamma_calculator`, in which `name` contains the name of the SLHA file, automatizes the whole calculation, as it first calls `Init_param` and `Les_Houches_Reader`, then `CW_calculator`, `C_calculator_base1`, `Cprime_calculator`, and finally `bsgamma`.

- `double delta0(double C0[],double C0_spec[],double C1[],double C1_spec[], double Cp[], struct parameters* param,double mu_b,double mu_f)`

  This function computes the isospin asymmetry in $B \to K^*\gamma$ as described in Appendix E.2, using the LO, NLO and primed Wilson coefficients at scale `mu_b` (`C0[]`, `C1[]` and `Cp[]`), and at the spectator scale `mu_f` (`C0_spec[]` and `C1_spec[]`). Compared to the first version, the calculation has been improved, and all the involved integrals have been coded in separate routines. Again, an automatic container routine which only needs the name of the SLHA file is provided: `delta0_calculator`.

- `double Bsmumu(double C0b[], double C1b[], double C2b[], double complex CQ0b[], double complex CQ1b[], double Cpb[], double complex CQpb[], struct parameters* param, double mu_b)`

  `double Bdmumu(double C0b[], double C1b[], double C2b[], double complex CQ0b[], double complex CQ1b[], struct parameters* param, double mu_b)`

  These functions compute the CP-averaged branching ratios of the rare decays $B_s \to \mu^+\mu^-$ and $B_d \to \mu^+\mu^-$ using two loop electroweak and three loop QCD corrections as described in Appendix E.8. The container routines `Bsmumu_calculator(char name[])` and `Bdmumu_calculator (char name[])`, in which `name` contains the name of the SLHA file, automatize the whole calculation, and first call `Init_param` and `Les_Houches_Reader`, then `CW_calculator`, `C_calculator_base1`, `Cprime_calculator`, `CQ_calculator`, and finally `Bsmumu` and `Bdmumu`.

- `double Bsmumu_untag(double C0b[], double C1b[], double C2b[], double complex CQ0b[], double complex CQ1b[], double Cpb[], double complex CQpb[], struct parameters* param, double mu_b)`

  This functions compute the untagged branching ratio of the rare decay $B_s \to \mu^+\mu^-$ as described in Appendix E.8.2. The container routine `Bsmumu_untag_calculator` automatize the calculation. The resulting value can be directly compared to the experimental limits.

- `dBR_BXsmumu_dshat(double shat, double C0b[], double C1b[], double C2b[], double complex CQ0b[], double complex CQ1b[], double Cpb[], double complex CQpb[], struct parameters* param, double mu_b)`

  `double A_BXsmumu(double shat, double C0b[], double C1b[], double C2b[], double complex CQ0b[], double complex CQ1b[], double Cpb[], double complex CQpb[], struct parameters* param, double mu_b)`

  These functions compute the differential branching fraction and forward-backward asymmetry of $B \to X_s\mu^+\mu^-$ for $\hat{s}$ =`shat` using the LO, NLO, NNLO and primed Wilson coefficients `C0b/CQ0b`, `C1b/CQ1b`, `C2b` and `Cpb/CQpb`, at scale `mu_b`, as described in Appendix E.3. They are called for the calculation of all the $B \to X_s\mu^+\mu^-$ observables.

- `BRBXsmumu_lowq2(double C0b[], double C1b[], double C2b[], double complex CQ0b[], double complex CQ1b[], double Cpb[], double complex CQpb[], struct parameters* param, double mu_b)`

  `BRBXsmumu_highq2(double C0b[], double C1b[], double C2b[], double complex CQ0b[], double complex CQ1b[], double Cpb[], double complex CQpb[], struct parameters* param, double mu_b)`

  `double A_BXsmumu_zero(double C0b[], double C1b[], double C2b[], double complex CQ0b[], double complex CQ1b[], double Cpb[], double complex CQpb[], struct parameters* param, double mu_b)`

  These functions compute the branching fractions in the low q^2 region ($1 < q^2 < 6$ GeV^2), in the high q^2 region ($q^2 > 14.4$ GeV^2), and the zero-crossing of the forward-backward asymmetry of $B \to X_s\mu^+\mu^-$ using the LO, NLO, NNLO and primed Wilson coefficients `C0b/CQ0b`, `C1b/CQ1b`, `C2b` and `Cpb/CQpb`, at scale `mu_b`, as described in Appendix E.3. Automatic container routines which only need the name of the SLHA file are provided: `BRBXsmumu_lowq2_calculator`, `BRBXsmumu_highq2_calculator` and `A_BXsmumu_zero_calculator`.

- `BRBXstautau_highq2(double C0b[], double C1b[], double C2b[], double complex CQ0b[], double complex CQ1b[], double Cpb[], double complex CQpb[], struct parameters* param, double mu_b)`

  This function computes the branching fraction in the high q^2 region ($q^2 > 14.4$ GeV^2) of $B \to X_s\tau^+\tau^-$ using the LO, NLO, NNLO and primed Wilson coefficients `C0b/CQ0b`, `C1b/CQ1b`, `C2b` and `Cpb/CQpb` at scale `mu_b`, as described in Appendix E.3. An automatic container routine which only needs the name of the SLHA file is provided: `BRBXstautau_highq2_calculator`.

- `double dGamma_BKstarmumu_dq2(double q2, double obs[][3], double C0b[], double C1b[], double C2b[], double complex CQ0b[], double complex CQ1b[], double Cpb[], double complex CQpb[], struct parameters* param, double mu_b)`

  `double dAI_BKstarmumu_dq2(double q2, double C0b[], double C1b[], double C2b[], struct parameters* param, double mu_b)`

  These functions compute the differential decay rate and isospin asymmetry of $B \to$

	Observable		Observable
obs[0]	$q_0^2(A_{FB})$	obs[11]	$H_T^{(3)}$
obs[1]	A_{FB}	obs[12]	α_{K^*}
obs[2]	F_L	obs[13]	A_{Im}
obs[3]	F_T	obs[14]	P_2
obs[4]	$A_T^{(1)}$	obs[15]	P_3
obs[5]	$A_T^{(2)} = P_1$	obs[16]	P_6
obs[6]	$A_T^{(3)}$	obs[17]	P_4'
obs[7]	$A_T^{(4)}$	obs[18]	P_5'
obs[8]	$A_T^{(5)}$	obs[19]	P_6'
obs[9]	$H_T^{(1)} = P_4$	obs[20]	P_8
obs[10]	$H_T^{(2)} = P_5$	obs[21]	P_8'

Table 1: $B \to K^*\mu^+\mu^-$ observables contained in the array `obs[]`. The definitions are given in Appendix E.4.

$K^*\mu^+\mu^-$ for q^2 =`q2` using the LO, NLO and NNLO Wilson coefficients `C0b/CQ0b`, `C1b/CQ1b`, `C2b` and the primed Wilson coefficients `Cpb` and `CQpb` at scale `mu_b`, as described in Appendix E.4. They are called for the calculation of all the $B \to K^*\mu^+\mu^-$ observables. The array `obs` contains the values of the observables given in Table 1.

- `double BRBKstarmumu_lowq2(double obs[], double C0b[], double C1b[], double C2b[], double complex CQ0b[], double complex CQ1b[], double Cpb[], double complex CQpb[], struct parameters* param, double mu_b)`

  `double BRBKstarmumu_highq2(double obs[], double C0b[], double C1b[], double C2b[], double complex CQ0b[], double complex CQ1b[], double Cpb[], double complex CQpb[], struct parameters* param, double mu_b)`

  These functions compute branching fraction of $B \to K^*\mu^+\mu^-$ as well of all the observables given in Table 1, in the low q^2 ($1 < q^2 < 6$ GeV^2) and high q^2 ($14.18 < q^2 < 16$ GeV^2) regions respectively, using the LO, NLO and NNLO Wilson coefficients `C0b/CQ0b`, `C1b/CQ1b`, `C2b` and the primed Wilson coefficients `Cpb` and `CQpb` at scale `mu_b`. The array `obs` contains the values of the observables in the corresponding q^2 region.

  The minimum and maximum q^2 values for the averages can be modified using the function:
  `double BRBKstarmumu(double smin, double smax, double obs[], double C0b[], double C1b[], double C2b[], double complex CQ0b[], double complex CQ1b[], double Cpb[], double complex CQpb[], struct parameters* param, double mu_b)`

  Automatic container routines which need the name of the SLHA file and an array `obs[]` to get the other functions are provided:
  `BRobs_BKstarmumu_lowq2_calculator` and `BRobs_BKstarmumu_highq2_calculator`

  Specific functions for the observables of Table 1 can be found in `src/include.h`.

- `double AI_BKstarmumu_lowq2(double C0b[], double C1b[], double C2b[], struct parameters* param, double mu_b)`

  `double AI_BKstarmumu_highq2(double C0b[], double C1b[], double C2b[], struct parameters* param, double mu_b)`

  `double AI_BKstarmumu_zero(double C0b[], double C1b[], double C2b[], struct parameters* param, double mu_b)`

  These functions compute the averaged isospin asymmetries in the low q^2 ($1 < q^2 < 6$ GeV^2) and high q^2 ($14.18 < q^2 < 16$ GeV^2) regions and the isospin asymmetry zero-crossing of $B \to K^* \mu^+ \mu^-$ respectively, using the LO, NLO and NNLO Wilson coefficients `C0b`, `C1b`, `C2b` at scale `mu_b`, as described in Appendix E.4. Automatic container routines which need the name of the SLHA file are provided:
  `AI_BKstarmumu_lowq2_calculator`, `AI_BKstarmumu_highq2_calculator` and
  `AI_BKstarmumu_zero_calculator`

- `double Btaunu(struct parameters* param)`
  `double RBtaunu(struct parameters* param)`
  `double Btaunu_calculator(char name[])`
  `double RBtaunu_calculator(char name[])`

  These routines compute the branching ratio of the leptonic decay $B_u \to \tau\nu_\tau$ and the ratio $\mathrm{BR}(B_u \to \tau\nu_\tau)/\mathrm{BR}(B_u \to \tau\nu_\tau)^{\mathrm{SM}}$ as described in Appendix E.11. These leptonic decays occur at tree level, and we consider also higher order SUSY corrections to the Yukawa coupling.

- `double BDtaunu(struct parameters* param)`
  `double BDtaunu_BDenu(struct parameters* param)`
  `double BDtaunu_calculator(char name[])`
  `double BDtaunu_BDenu_calculator(char name[])`

  These routines compute the branching ratio of the semileptonic decay $B \to D^0 \tau\nu_\tau$ and the ratio $\mathrm{BR}(B \to D^0 \tau\nu_\tau)/\mathrm{BR}(B \to D^0 e\nu_e)$ as described in Appendix E.12. These semileptonic decays occur at tree level, and we consider also higher order SUSY corrections to the Yukawa coupling.

- `double Kmunu_pimunu(struct parameters* param)`
  `double Rmu23(struct parameters* param)`
  `double Kmunu_pimunu_calculator(char name[])`
  `double Rmu23_calculator(char name[])`

  These functions compute the ratio $\mathrm{BR}(K \to \mu\nu_\mu)/\mathrm{BR}(\pi \to \mu\nu_\mu)$ and the observable $R_{\mu 23}$ as described in Appendix E.13. These leptonic decays occur at tree level, and we consider also higher order SUSY corrections to the Yukawa coupling.

- `double Dstaunu(struct parameters* param)`
  `double Dsmunu(struct parameters* param)`
  `double Dstaunu_pimunu_calculator(char name[])`
  `double Dsmunu_calculator(char name[])`

  These routines compute the branching ratios of the leptonic decays $D_s \to \tau\nu_\tau$ and

$D_s \to \mu\nu_\mu$ as described in Appendix E.14. These leptonic decays occur at tree level, and we consider also higher order SUSY corrections to the Yukawa coupling.

- `double Dmunu(struct parameters* param)`
  `double Dmunu_calculator(char name[])`

  These routines compute the branching ratio of the leptonic decay $D \to \mu\nu_\mu$ as described in Appendix E.15.

- `double muon_gm2(struct parameters* param)`
  `double muon_gm2_calculator(char name[])`

  These routines compute the muon anomalous magnetic moment (δa_μ) at two loop, as described in Appendix F.

2.3 Interpreter routines

The interpreter code facilitates the use of `SuperIso` by linking the functions used to calculate the different observables to their names in plain text. In particular it allows the user to compute the values and theoretical uncertainties of a list of observables by giving as input the name of the observables. It relies on a specific type for the observable names:

```
typedef struct obsname
/* structure for observable names */
{
char type[20]; /* BR, AFB, ... */
char decay[20];

double low; /* low q2 bin value */
double high; /* high q2 bin value */

char other[50];

char name[100];
}
obsname;
```

The observable names are composed of one type and one decay, and optionally of low and high q^2 bin values and another describer, all separated by underscores. For example: `BR_BKstarmumu_1_6_LHCb`. The recognized types and decays are given in Tables 2 and 3.

The interpreter routines can be found in `src/interpreter.c`:

- `int read_nameobs(char name[], obsname* obs)`

  This routine transforms a name in string format into an `obsname`.

- `int check_nameobs(obsname* obs)`

  This routine verifies that `obs` is defined in `SuperIso`.

SuperIso name	Corresponding observable
BR	branching ratio
BRuntag	untagged branching ratio
dGamma/dq2	differential decay width
R-1	ratio minus one
AI	isospin asymmetry
ACP	CP asymmetry
AFB	forward-backward asymmetry
FL	longitudinal fraction
FT	transverse fraction
ATRe	
AT1...5	
HT1...3	
alpha	
AIm	
P1...3	angular observable
P6	angular observable
P4...6prime	angular observable
P8	angular observable
P8prime	angular observable
A3...9	CP-violating angular observable
S1...9	CP-conserving angular observable
FH	
AlFB	
AhFB	
AlhFB	
(Re)C1...10	Wilson coefficients
(Re)CQ1,2	scalar/pseudoscalar Wilson coefficients
(Re)Cprime1...10	prime Wilson coefficients
(Re)CprimeQ1,2	prime scalar/pseudoscalar Wilson coefficients
ImC1...10	imaginary part of Wilson coefficients
ImCQ1,2	imaginary part of scalar/pseudoscalar Wilson coefficients
ImCprime1...10	imaginary part of prime Wilson coefficients
ImCprimeQ1,2	imaginary part of prime scalar/pseudoscalar Wilson coefficients
(Re)Vud	(real part of) CKM matrix element
(Re)Vus	(real part of) CKM matrix element
(Re)Vub	(real part of) CKM matrix element
(Re)Vcd	(real part of) CKM matrix element
(Re)Vcs	(real part of) CKM matrix element
(Re)Vcb	(real part of) CKM matrix element
(Re)Vtd	(real part of) CKM matrix element
(Re)Vts	(real part of) CKM matrix element
(Re)Vtb	(real part of) CKM matrix element
ImVud	(imaginary part of) CKM matrix element
ImVus	(imaginary part of) CKM matrix element
ImVub	(imaginary part of) CKM matrix element
ImVcd	(imaginary part of) CKM matrix element
ImVcs	(imaginary part of) CKM matrix element
ImVcb	(imaginary part of) CKM matrix element
ImVtd	(imaginary part of) CKM matrix element
ImVts	(imaginary part of) CKM matrix element
ImVtb	(imaginary part of) CKM matrix element

Table 2: Possible types of observables

SuperIso name	Corresponding decay
BXsgamma	$B \to X_s\gamma$
BXdgamma	$B \to X_d\gamma$
BXsmumu	$B \to X_s\mu^+\mu^-$
BXsee	$B \to X_s e^+e^-$
BXstautau	$B \to X_s\tau^+\tau^-$
BXsll	$B \to X_s\ell^+\ell^-$
BKstargamma	$B \to K^*\gamma$
B0Kstar0gamma	$B^0 \to K^{*0}\gamma$
Bsmumu	$B_s \to \mu^+\mu^-$
Bsee	$B_s \to e^+e^-$
Bstautau	$B_s \to \tau^+\tau^-$
Bsll	$B_s \to \ell^+\ell^-$
Bdmumu	$B_d \to \mu^+\mu^-$
Bdee	$B_d \to e^+e^-$
Bdtautau	$B_d \to \tau^+\tau^-$
Bdll	$B_d \to \ell^+\ell^-$
BKstarmumu	$B \to K^*\mu^+\mu^-$
B0Kstar0mumu	$B^0 \to K^{*0}\mu^+\mu^-$
BKstaree	$B \to K^*e^+e^-$
B0Kstar0ee	$B^0 \to K^{*0}e^+e^-$
BKstartautau	$B \to K^*\tau^+\tau^-$
B0Kstar0tautau	$B^0 \to K^{*0}\tau^+\tau^-$
BKstarll	$B \to K^*\ell^+\ell^-$
B0Kstar0ll	$B^0 \to K^{*0}\ell^+\ell^-$
BKmumu	$B \to K\mu^+\mu^-$
B0K0mumu	$B^0 \to K^0\mu^+\mu^-$
BKee	$B \to Ke^+e^-$
B0K0ee	$B^0 \to K^0e^+e^-$
BKtautau	$B \to K\tau^+\tau^-$
B0K0tautau	$B^0 \to K^0\tau^+\tau^-$
BKll	$B \to K\ell^+\ell^-$
B0K0ll	$B^0 \to K^0\ell^+\ell^-$
Bsphimumu	$B_s \to \phi\mu^+\mu^-$
Bsphiee	$B_s \to \phi e^+e^-$
Bsphitautau	$B_s \to \phi\tau^+\tau^-$
Bsphill	$B_s \to \phi\ell^+\ell^-$
Bmunu	$B \to \mu\nu$
Benu	$B \to e\nu$
Btaunu	$B \to \tau\nu$
Dsmunu	$D_s \to \mu\nu$
Dsenu	$D_s \to e\nu$
Dstaunu	$D_s \to \tau\nu$
Dmunu	$D \to \mu\nu$
Denu	$D \to e\nu$
Kmunu/pimunu	$K \to \mu\nu/\pi \to \mu\nu$
BDmunu	$B \to D\mu\nu$
BDenu	$B \to De\nu$
BDtaunu	$B \to D\tau\nu$
BDstarmunu	$B \to D^*\mu\nu$
BDstarenu	$B \to D^*e\nu$
BDstartaunu	$B \to D^*\tau\nu$
LambdabLambdamumu	$\Lambda_b \to \Lambda\mu^+\mu^-$
LambdabLambdaee	$\Lambda_b \to \Lambda e^+e^-$
LambdabLambdatautau	$\Lambda_b \to \Lambda\tau^+\tau^-$
LambdabLambdall	$\Lambda_b \to \Lambda\ell^+\ell^-$
KLpi0nunu	$K_L \to \pi^0\nu\nu$
Kpinunu	$K \to \pi\nu\nu$
KLmumu	$K_L \to \mu^+\mu^-$
KSmumu	$K_S \to \mu^+\mu^-$
e	electron final state (Wilson coefficients)
mu	mu final state (Wilson coefficients)
tau	tau final state (Wilson coefficients)
CKM	CKM matrix

Table 3: Possible decays

- `void make_obslist(char names[][50], obsname obs[], int *nobs)`

  This routine converts a list of observables in string format into a list of `obsname`, keeping only the observables implemented in `SuperIso`. `nobs` is the number of valid observables.

- `int read_obs_list(char filename[], char names[][50])`

  This function converts a list of observables from a file `filename` into a string array `names` and returns the number of observables. No verification of validity is performed at this stage.

- `double compute_nameobs_ref(obsname* obs, struct parameters* param)`

  This function computes the value of the observable `obsname` using the parameters of `param`.

- `double compute_nameobs(obsname* obs, int ke, struct parameters* param)`

  This function computes the value of the observable `obsname` using the parameters of `param` but changing the value of the `ke`-th nuisance parameter by one standard deviation in order to compute the theoretical covariance matrix (see `get_th_covariance` below).

- `void get_predictions(char names[][50], int *nbobs, double** predictions, struct parameters* param)`

  This routine computes the values of the observables in the list `names` and puts them into `predictions`. *nobs* is the number of valid observables.

- `void get_th_covariance(double ***covariance_th, char names[][50], int *nbobs, struct parameters* param)`

  This routine computes the theoretical covariance matrix for the observables in the list `names` and puts them into `covariance_th`. It uses by default the nuisance parameters defined in `chi2_input/nuisance.in` together with their correlations in `chi2_input/nuisance_corr.in`. The names of these two files can be changed directly in `param.nuisance_values` and `param.nuisance_corr`.

2.4 Main statistical routines

`SuperIso` features an automatic calculation of the theoretical uncertainties, which is based on a numerical differentiation method.

In order to compute the theoretical uncertainties and covariance matrix, we use an approach similar to the one of [**?**]. To illustrate this method, we consider two observables Q and T, which are subject to nuisance parameters numbered $a = (1, \cdots, n)$. The variations of the nuisance parameters are denoted as δ_a, and can affect both observables Q and T. Under the assumption that the uncertainties are small enough to affect the observables linearly, the variation of observable Q reads:

$$Q = Q_0 \left(1 + \sum_{a=1}^{n} \delta_a \Delta_Q^a \right) , \tag{1}$$

where Δ_Q^a is the relative variance generated by the nuisance parameter a, and Q_0 the central value. Defining the covariance matrix of the nuisance parameters as

$$\rho_{ab} = \texttt{cov}[\delta_a, \delta_b]\,, \tag{2}$$

the total relative variance of observable Q can be written as

$$(\Delta_Q)^2 = \sum_{a,b} \rho_{ab} \Delta_Q^a \Delta_Q^b\,, \tag{3}$$

and the correlation coefficient between Q and T is

$$(\Delta_{QT})^2 = \sum_{a,b} \rho_{ab}\, \Delta_Q^a \Delta_T^b\,. \tag{4}$$

The covariance matrix of observables Q and T therefore writes:

$$\texttt{cov}[Q,T] = \begin{pmatrix} (\Delta_Q)^2 (Q_0)^2 & (\Delta_{QT})^2\, Q_0 T_0 \\ (\Delta_{QT})^2\, Q_0 T_0 & (\Delta_T)^2 (T_0)^2 \end{pmatrix}\,. \tag{5}$$

The uncertainties of the different observables are then obtained as the square root of the diagonal elements.

In general, the nuisance parameters are uncorrelated so that $\rho_{ab} = \delta_{ab}$, but on the other hand the form factors are strongly correlated and their correlations have to be taken into account.

The χ^2 for a set of n observables can be obtained after summing the theoretical and experimental covariance matrix into a total covariance matrix $\mathcal{C}$:

$$\chi^2 = \sum_{i,j=1}^{n} (O_i - E_i) \mathcal{C}_{ij}^{-1} (O_j - E_j)\,, \tag{6}$$

where $\mathcal{C}^{-1}$ is the inverse of the covariance matrix, O_i the central value of the theoretical prediction and E_i the central value of the experimental measurement.

Most of the statistics related routines can be found in `src/chi2.c`.

We define the `indnuis` type as

```
typedef struct indnuis
/* structure for individual nuisance parameters */
{
double cent,dev; /* central value, standard deviation */
int type; /* 1=gaussian distribution, 2= flat distribution */
char* name;
}
indnuis;
```

The nuisance parameters are gathered in the `nuisance` structure:

```
typedef struct nuisance
/* structure containing nuisance parameters for the statistical analysis */
{
/* SM parameters */
indnuis alphas_MZ,mass_b,mass_c,mass_s,mass_top_pole,mass_h0;
/* CKM parameters */
indnuis CKM_lambda,CKM_A,CKM_rhobar,CKM_etabar;
/* Scales of Wilson coefficients */
indnuis log_mu_W_mass_W,log_mu_b_mass_b;

/* inclusive b -> s */
indnuis BR_BXclnu_exp;
/* b -> s gamma */
indnuis mu_G2_bsg,rho_D3_bsg,rho_LS3_bsg,bsgamma_rand,mu_c_bsg;
/* b -> s mu mu */
indnuis BRBXsmumu_lowq2_rand,BRBXsmumu_highq2_rand,BRBXsmumu_full_rand;
/* b -> s e e */
indnuis BRBXsee_lowq2_rand,BRBXsee_highq2_rand,BRBXsee_full_rand;
/* b -> s tau tau */
indnuis BRBXstautau_lowq2_rand,BRBXstautau_highq2_rand,BRBXstautau_full_rand;

/* B */
indnuis f_B,lambda_Bp;
/* B -> K* */
indnuis f_Kstar_par,f_Kstar_perp,a1perp,a2perp,a1par,a2par;
/* B -> K* gamma */
indnuis T1_BKstar,log_mu_spec_lambda_h_mass_b;

/* low */
indnuis BtoKstarlow_ALperp_err_noq2,BtoKstarlow_ARperp_err_noq2,
BtoKstarlow_ALpar_err_noq2,BtoKstarlow_ARpar_err_noq2,BtoKstarlow_AL0_err_noq2,
BtoKstarlow_AR0_err_noq2,BtoKstarlow_At_err_noq2,BtoKstarlow_AS_err_noq2;
indnuis BtoKstarlow_ALperp_err_q2,BtoKstarlow_ARperp_err_q2,
BtoKstarlow_ALpar_err_q2,BtoKstarlow_ARpar_err_q2,BtoKstarlow_AL0_err_q2,
BtoKstarlow_AR0_err_q2,BtoKstarlow_At_err_q2,BtoKstarlow_AS_err_q2;

indnuis real_alpha_perp0,real_alpha_perp1,real_alpha_perp2,real_alpha_par0,
real_alpha_par1,real_alpha_par2,real_alpha_zero0,real_alpha_zero1,
imag_alpha_perp0,imag_alpha_perp1,imag_alpha_perp2,imag_alpha_par0,
imag_alpha_par1,imag_alpha_par2,imag_alpha_zero0,imag_alpha_zero1;

indnuis DeltaC9_M1_q2bar,r1_M1,r2_M1,DeltaC9_M2_q2bar,r1_M2,r2_M2,
DeltaC9_M3_q2bar,r1_M3,r2_M3;

/* high */
indnuis BtoKstarhigh_ALperp_err,BtoKstarhigh_ARperp_err,BtoKstarhigh_ALpar_err,
```

```
BtoKstarhigh_ARpar_err,BtoKstarhigh_AL0_err,BtoKstarhigh_AR0_err,
BtoKstarhigh_At_err,BtoKstarhigh_AS_err;

/* B -> K */
indnuis f_K,a1K,a2K;

/* Form factors B->K ll */
indnuis a00_BK,a10_BK,a20_BK,a30_BK;
indnuis a0p_BK,a1p_BK,a2p_BK;
indnuis a0T_BK,a1T_BK,a2T_BK;

/* low */
indnuis BtoKlow_FV_err_noq2,BtoKlow_FA_err_noq2,BtoKlow_FS_err_noq2,BtoKlow_FP_err_noq2;
indnuis BtoKlow_FV_err_q2,BtoKlow_FA_err_q2,BtoKlow_FS_err_q2,BtoKlow_FP_err_q2;

/* high */
indnuis BtoKhigh_FV_err,BtoKhigh_FA_err,BtoKhigh_FS_err,BtoKhigh_FP_err;

/* Form factors B->K* ll */
indnuis a0A0_BKstar,a1A0_BKstar,a2A0_BKstar;
indnuis a0A1_BKstar,a1A1_BKstar,a2A1_BKstar;
indnuis a0A12_BKstar,a1A12_BKstar,a2A12_BKstar;
indnuis a0V_BKstar,a1V_BKstar,a2V_BKstar;
indnuis a0T1_BKstar,a1T1_BKstar,a2T1_BKstar;
indnuis a0T2_BKstar,a1T2_BKstar,a2T2_BKstar;
indnuis a0T23_BKstar,a1T23_BKstar,a2T23_BKstar;

/* Bs */
indnuis life_Bs,f_Bs,lambda_Bsp;

/* Bs -> phi */
indnuis f_phi_par,f_phi_perp,a1phi_perp,a1phi_par,a2phi_perp,a2phi_par;

/* low */
indnuis Bstophilow_ALperp_err_noq2,Bstophilow_ARperp_err_noq2,
Bstophilow_ALpar_err_noq2,Bstophilow_ARpar_err_noq2,Bstophilow_AL0_err_noq2,
Bstophilow_AR0_err_noq2,Bstophilow_At_err_noq2,Bstophilow_AS_err_noq2;
indnuis Bstophilow_ALperp_err_q2,Bstophilow_ARperp_err_q2,Bstophilow_ALpar_err_q2,
Bstophilow_ARpar_err_q2,Bstophilow_AL0_err_q2,Bstophilow_AR0_err_q2,
Bstophilow_At_err_q2,Bstophilow_AS_err_q2;

/* high */
indnuis Bstophihigh_ALperp_err,Bstophihigh_ARperp_err,Bstophihigh_ALpar_err,
Bstophihigh_ARpar_err,Bstophihigh_AL0_err,Bstophihigh_AR0_err,Bstophihigh_At_err,
Bstophihigh_AS_err;
```

```
/* Form factors Bs->phi ll */
indnuis a0A0_Bsphi,a1A0_Bsphi,a2A0_Bsphi;
indnuis a0A1_Bsphi,a1A1_Bsphi,a2A1_Bsphi;
indnuis a0A12_Bsphi,a1A12_Bsphi,a2A12_Bsphi;
indnuis a0V_Bsphi,a1V_Bsphi,a2V_Bsphi;
indnuis a0T1_Bsphi,a1T1_Bsphi,a2T1_Bsphi;
indnuis a0T2_Bsphi,a1T2_Bsphi,a2T2_Bsphi;
indnuis a0T23_Bsphi,a1T23_Bsphi,a2T23_Bsphi;

/* Lambda_b -> Lambda l+l- */
indnuis life_Lb, alphaL_LbLll;
indnuis a0_HO_fplus_LbLll,a1_HO_fplus_LbLll,a2_HO_fplus_LbLll,a0_HO_fperp_LbLll,
a1_HO_fperp_LbLll,a2_HO_fperp_LbLll,a0_HO_gpp_LbLll,a1_HO_gplus_LbLll,
a2_HO_gplus_LbLll,a1_HO_gperp_LbLll,a2_HO_gperp_LbLll,a0_HO_hplus_LbLll,
a1_HO_hplus_LbLll,a2_HO_hplus_LbLll,a0_HO_hperp_LbLll,a1_HO_hperp_LbLll,
a2_HO_hperp_LbLll,a0_HO_htildepp_LbLll,a1_HO_htildeplus_LbLll,a2_HO_htildeplus_LbLll,
a1_HO_htildeperp_LbLll,a2_HO_htildeperp_LbLll;

/* kaons */
indnuis deltaPcu_Kppipnunu;
indnuis err_Pc_Xlambda_Kppipnunu;
indnuis BR_KLgammagamma_exp;
indnuis Aterm_mu_KLmumu;
indnuis BR_KSgammagamma_exp;
indnuis Iterm_mu_KSmumu;
}
nuisance;
```

The correlation between the different nuisance parameters are defined via the type:

```
typedef struct nuiscorr
/* structure containing nuisance correlations for the statistical analysis */
{
char obs1[50],obs2[50];
double value;
}
nuiscorr;
```

- `void set_nuisance(struct nuisance* nuisparam)`
  This routine sets the central values and errors of the nuisance parameters of `nuisparam` to their default values defined in the routine.

- `void set_nuisance_deviation_to_zero(struct nuisance* nuisparam)`
  This routine sets the errors of the nuisance parameters in `nuisparam` to zero.

- `void set_nuisance_value_from_param(struct nuisance* nuisparam, struct parameters* param)`

This routine sets the central values of the nuisance parameters in `nuisparam` to the values in `param`.

- `void read_nuisance(char name[], struct nuisance* nuisparam)`

  This routine reads nuisance parameters into `nuisparam` from file `name`.

- `void write_nuisance(struct nuisance* nuisparam, char name[])`

  This routine writes the nuisance parameters defined in `*nuisparam` into file `name` in a format readable by `SuperIso`.

- `void observables(int ke, obsname obs[], int nobs, double values[], double values_ref[], struct nuisance* nuisparam, char namenuisance[][50], struct parameters* param)`

  This routine computes the `values` of the `nobs` observables `obs` for the `ke`-th iteration of the theory error calculation, using the nuisance parameters `nuisparam` as named in `namenuisance`. `values_ref` contains the values of the observables obtained at the 0th reference iteration.

- `void write_correlation_nuisance(char name[], double **corr, char nameparam[][50], int n)`

  This routine writes the correlation matrix `corr` of dimension `n`×`n` into the file `name`, using the name of the corresponding parameters. Only the upper part of the matrix is written, and the diagonal and null elements are omitted.

- `void read_correlation(char name[], double **corr, char nameparam[][50], int n)`

  This routine reads file `name` into the correlation matrix `corr` of dimension `n`×`n`.

- `int read_experimental_covariance(char name_val_exp[], char name_corr_exp[], char namesin[][50], int nbobsin, double* central_exp, double* errors_exp, double **correlations)`

  This routine reads the experimental values, errors and correlations from files `name_val_exp` and `name_corr_exp`, and if they match the names given in the string array `namesin`, puts them into `central_exp`, `errors_exp` and `correlations`, respectively. `nbobsin` is the number of observables in `namesin`. The function returns the number of observables for which experimental values have been found. Example of input files can be found in `chi2_input/exp_values.in` and `chi2_input/exp_corr.in`.

- `void get_th_covariance_nuisance(double ***covariance_th, char names[][50], int *nbobs, struct parameters* param, struct nuisance* nuisparam, double **nuiscorr)`

  This routine computes the theory covariance matrix for the `nbobs` observables which are defined in `names`, using the nuisance parameters contained in the structure `nuisparam` and array `nuiscorr`, and put it in `covariance_th`.

- `void get_exp_covariance(double ***covariance_exp, double **central_exp, char names[][50], int *nbobs, struct parameters* param)`

  This routine reads the experimental central values, errors and correlation matrix of the observables contained in `names` from the files defined in `param.exp_values` and param.exp_corr and derives the experimental covariance matrix in covariance_exp for the nbobs observables for which experimental values are found.

- void get_exp_values(double **central_exp, char names[][50], int *nbobs, struct parameters* param)

  This routine reads the experimental central values of the observables contained in names from the file defined in param.exp_values for the nbobs observables for which experimental values are found.

- void get_covariance(double ***covariance_th, double ***covariance_exp, double **central_exp, char names[][50], int *nbobs, struct parameters* param)

  This routine is a wrapper which computes the theoretical covariance matrix and reads the experimental central values and covariance matrix from the files whose names are defined in the variables param.exp_values and param.exp_corr.

- void get_predictions_nuisance(char names[][50], int *nbobs, double** predictions, struct parameters* param, struct nuisance* nuisparam)

  This routine computes the reference values predictions of observables defined in the string array names, using the central values of the nuisance parameters given in nuisparam.

- double get_chi2(double **inv_cov_tot, double *predictions, double *central_exp, int nbobs)

  This function returns the χ^2 value, taking as input the inverse of the covariance matrix inv_cov_tot, as well as the predictions predictions and experimental central values central_exp of the observables. nbobs is the number of observables.

- double chi2(char names[][50], int nbobs, struct parameters* param)

  This function automatically computes the chi2 corresponding to the list of nbobs observables names, using experimental input from the files defined in param.exp_values and param.exp_corr, and computing the theoretical correlations based on the nuisance parameters and their correlations from the files defined in param.nuisance_values and param.nuisance_corr.

- int get_invcovtot(double ***covariance_tot, double ***inv_cov_tot, int nbobs)

  This function computes the inverse inv_cov_tot of the covariance matrix covariance_tot, and return 1 if it succeeds, and −1 otherwise. nbobs is the number of observables.

- void get_covtot(double ***covariance_th, double ***covariance_exp, double ***covariance_tot, int nbobs)

  This function sums the theoretical covariance matrix covariance_th and experimental covariance matrix covariance_exp into the total covariance matrix covariance_tot. nbobs is the number of observables.

- int reduce_covariance(double ***covariance_in, char* namesin[], int nbobsin, double ***covariance_out, char* namesout[], int nbobsout)

  This function allows the user to reduce the size of the covariance matrix to use a smaller sample of observables. It takes as input the original covariance matrix `covariance_in` corresponding to the `\nbobsin` observables of names `namesin`, as well as the list of observables expected in output `namesout` and their number `nbobsout`, and output the reduced covariance matrix `covariance_out`. It returns 1 in case of success, 0 otherwise (e.g. if `namesout` contains observables which are not in `namesin`).

- `int reduce_values(double **values_in, char* namesin[], int nbobsin, double **values_out, char* namesout[], int nbobsout)`

  Similarly to `reduce_covariance`, this function reduces the set of observables with values in `values_in` into `values_out`.

- `void read_covariance(char name[], double **cov, char nameparam[][50], int n)`

  This routine reads the covariance matrix `cov` of size `n`×`n` from the file `name`. This routine can be used for both observable covariance matrices and nuisance parameter covariance matrices (see e.g. `chi2_input/nuisance_corr.in` and `chi2_input/exp_corr.in`).

- `void write_covariance(char name[], double **cov, char nameparam[][50], int n)`

  This routine writes the covariance matrix `cov` of size `n`×`n` into the file `name` in a format recognized by `SuperIso`.

- `int read_experimental_values(char name_val_exp[], char namesin[][50], int nbobsin, double *central_exp)`

  This function reads experimental values from file `name_val_exp`, and if they match the names given in the string array `namesin`, puts them into `central_exp`. `nbobsin` is the number of observables in `namesin`. The function `read_experimental_values` returns the number of observables for which experimental values have been found. An example of input file can be found in `chi2_input/exp_values.in`.

3 Compilation and installation instructions

The main structure of the `SuperIso` package is unchanged since the first version, and the spirit of the program relies on the idea of simplicity of use.

The `SuperIso` package[2] can be downloaded from:

`http://superiso.in2p3.fr`

The following main directory is created after unpacking:

`superiso_vX.X`

It contains the `src/` directory, in which all the source files can be found. The main directory contains also a `Makefile`, a `README`, twelve sample main programs (`sm.c`, `thdm.c`,

[2] An alternative package including the calculation of the relic density, `SuperIso Relic` [17], is also available at: `http://superiso.in2p3.fr/relic` .

cmssm.c, amsb.c, hcamsb.c, mmamsb.c, gmsb.c, nuhm.c, cnmssm.c, ngmsb.c, nnuhm.c and slha.c) and one example of input file in the SLHA format (example.lha).

In the Makefile the user has to specify the following items:

- The compiler name and the compilation options.
- Optionally, the path to the external programs:
  - path to isasugra.x for ISAJET,
  - path to softpoint.x for SOFTSUSY,
  - path to the suspect2 executable file for SuSpect,
  - path to the SPheno executable file for SPheno,
  - path to the 2HDMC directory,
  - path to the NMSSMTools main directory,
  - path to the HBwithFH executable file for HiggsBounds.

To use the limits from HiggsBounds, HBwithFH is used which needs FeynHiggs to be installed and linked to HiggsBounds. More information about how to compile HBwithFH can be found in the HiggsBounds manual or in the README file of SuperIso.
If the above optional programs are not used, the corresponding lines have to be commented or removed from the main programs (*e.g.* "#define USE_ISAJET" in cmssm.c).

SuperIso is written for a C compiler respecting the C99 standard. In particular, it has been tested successfully with the GNU C Compiler and the Intel C Compiler on Linux and Mac 32-bits or 64-bits machines, and with the latest versions of ISAJET, SOFTSUSY, SPheno, SuSpect, NMSSMTools, 2HDMC, and HiggsBounds.
Additional information can be found in the README file.
To compile the library, type

```
make
```

This creates libisospin.a in src/. Then, to compile one of the provided main programs, type

```
make name     or     make name.c
```

where name can be sm, thdm, cmssm, amsb, hcamsb, mmamsb, gmsb, nuhm, cnmssm, ngmsb, nnuhm or slha. This generates an executable program with the .x extension. Note that sm and slha do not need any additional program, but cmssm, amsb, gmsb need ISAJET or SOFTSUSY or SPheno or SuSpect, nuhm needs either ISAJET or SOFTSUSY, hcamsb and mmamsb need ISAJET, cnmssm, ngmsb and nnuhm require NMSSMTools and 2HDMC is necessary for thdm.

The main programs are further detailed in the following:

- sm.x calculates the different observables described in the Appendices in the Standard Model, using the parameters of Table 17.
- slha.x calculates the different observables using the parameters contained in the SLHA file whose name has to be passed as input parameter.
- thdm.x calculates the observables in 2HDM (general model or types I–IV), starting first by calculating the mass spectrum and couplings thanks to 2HDMC.
- amsb.x, hcamsb.x, mmamsb.x, gmsb.x, cmssm.x and nuhm.x compute the observables, first by calculating the mass spectrum and couplings thanks to ISAJET (amsb.x, hcamsb.x, mmamsb.x, gmsb.x and nuhm.x only work with ISAJET v7.80 or later versions) and/or SOFTSUSY and/or SPheno and/or SuSpect, within respectively the AMSB, HCAMSB, MMAMSB, GMSB, CMSSM or NUHM parameter spaces.
- cnmssm.x, ngmsb.x or nnuhm.x compute the observables, after calculating the mass spectrum and couplings thanks to NMSSMTools within respectively the CNMSSM, NGMSB or NNUHM parameter spaces.

For all these programs (except sm.c), arguments referring to the usual input parameters have to be passed to the program. If not, a message will describe which parameters have to be specified.

4 Input and output description

The input and output of the main programs provided in SuperIso are detailed in the following. Using the main programs as examples, the user is encouraged to write his/her own programs in order to, for example, perform scans in a given scenario. The full output is reproduced for sm.x, slha.x and cmssm.x as examples.

4.1 Standard Model main program

The program sm.x is a standalone program which computes the different observables in the Standard Model. No argument is necessary for this program, and the input parameters are given in Appendix G. The command

```
./sm.x
```

returns

```
Observable                        Value

BR(b->s gamma)                    3.174e-04
delta0(B->K* gamma)               5.856e-02

BR(Bs->mu mu)                     3.227e-09
BR(Bs->mu mu)_untag               3.538e-09
BR(Bd->mu mu)                     1.067e-10
```

```
BR(B->K* mu mu)_low             2.487e-07
AFB(B->K* mu mu)_low            -4.054e-02
FL(B->K* mu mu)_low             7.073e-01
P1=AT1(B->K* mu mu)_low         9.966e-01
AT2(B->K* mu mu)_low            -5.690e-02
AT3(B->K* mu mu)_low            6.384e-01
AT4(B->K* mu mu)_low            8.585e-01
AT5(B->K* mu mu)_low            3.651e-01
P4=HT1(B->K* mu mu)_low         4.864e-01
P5=HT2(B->K* mu mu)_low         -3.631e-01
HT3(B->K* mu mu)_low            1.974e-01
P2(B->K* mu mu)_low             9.856e-02
P3(B->K* mu mu)_low             -3.903e-04
P6(B->K* mu mu)_low             -6.135e-02
P4'(B->K* mu mu)_low            5.000e-01
P5'(B->K* mu mu)_low            -3.534e-01
P6'(B->K* mu mu)_low            -6.013e-02
P8(B->K* mu mu)_low             4.936e-02
P8'(B->K* mu mu)_low            4.793e-02
AI(B->K* mu mu)_low             -2.823e-02

BR(B->K* mu mu)_high            1.373e-07
AFB(B->K* mu mu)_high           4.454e-01
FL(B->K* mu mu)_high            3.108e-01
AT1(B->K* mu mu)_high           8.723e-01
P1=AT2(B->K* mu mu)_high        -4.875e-01
AT3(B->K* mu mu)_high           1.704e+00
AT4(B->K* mu mu)_high           5.810e-01
AT5(B->K* mu mu)_high           6.188e-02
P4=HT1(B->K* mu mu)_high        9.997e-01
P5=HT2(B->K* mu mu)_high        -9.896e-01
HT3(B->K* mu mu)_high           -9.904e-01
P2(B->K* mu mu)_high            -4.324e-01
P3(B->K* mu mu)_high            0.000e+00
P6(B->K* mu mu)_high            0.000e+00
P4'(B->K* mu mu)_high           1.219e+00
P5'(B->K* mu mu)_high           -7.095e-01
P6'(B->K* mu mu)_high           0.000e+00
P8(B->K* mu mu)_high            0.000e+00
P8'(B->K* mu mu)_high           0.000e+00
AI(B->K* mu mu)_high            -4.707e-04

q0^2(AFB(B->K* mu mu))          4.024e+00
q0^2(AI(B->K* mu mu))           1.707e+00

BR(B->Xs mu mu)_low             1.684e-06
```

```
BR(B->Xs mu mu)_high            2.125e-07
q0^2(AFB(B->Xs mu mu)           3.407e+00
BR(B->Xs tau tau)_high          1.564e-07

BR(B->tau nu)                   8.093e-05
R(B->tau nu)                    1.000e+00
BR(B->D tau nu)                 6.859e-03
BR(B->D tau nu)/BR(B->D e nu)   2.975e-01
BR(Ds->tau nu)                  5.127e-02
BR(Ds->mu nu)                   5.261e-03
BR(D->mu nu)                    4.101e-04
BR(K->mu nu)/BR(pi->mu nu)      6.355e-01
Rmu23(K->mu nu)                 1.000e+00
```

where `BR(b->s gamma)` refers to the branching ratio of $\bar{B} \to X_s\gamma$, `delta0(B->K* gamma)` the isospin symmetry breaking in $B \to K^*\gamma$ decays, `BR(Bs->mu mu)` and `BR(Bs->mu mu)_untag` the CP-averaged and untagged branching ratios of $B_s \to \mu^+\mu^-$ respectively, `BR(Bd->mu mu)` the branching ratio of $B_d \to \mu^+\mu^-$. `BR(B->K* mu mu)_low`, `AFB(B->K* mu mu)_low`, `FL(B->K* mu mu)_low` and `AI(B->K* mu mu)_low` stand for the averaged branching fraction, forward-backward asymmetry, longitudinal fraction F_L and isospin asymmetry of $B \to K^*\mu^+\mu^-$ in the low q^2 region ($1 < q^2 < 6$ GeV^2) respectively, and `BR(B->K* mu mu)_high`, `AFB(B->K* mu mu)_high`, `FL(B->K* mu mu)_high` and `AI(B->K* mu mu)_high` for the averaged branching fraction, forward-backward asymmetry, longitudinal fraction F_L and isospin asymmetry of $B \to K^*\mu^+\mu^-$ in the high q^2 region ($14.18 < q^2 < 16$ GeV^2) respectively. The other $B \to K^*\mu^+\mu^-$ observables are described in section E.4. `q0^2(AFB(B->K* mu mu))` and `q0^2(AI(B->K* mu mu))` correspond to the zero-crossing of the forward-backward and isospin asymmetries of $B \to K^*\mu^+\mu^-$. Also, `BR(B->Xs mu mu)_low`, `BR(B->Xs mu mu)_high` and `q0^2(AFB(B->Xs mu mu)` are respectively the branching fractions in the low q^2 region ($1 < q^2 < 6$ GeV^2), in the high q^2 region ($q^2 > 14.4$ GeV^2), and the zero-crossing of the forward asymmetry of $B \to X_s\mu^+\mu^-$. `BR(B->Xs tau tau)_high` is the branching ratio of $B \to X_s\tau^+\tau^-$ in the high q^2 region ($q^2 > 14.4$ GeV^2). `BR(B->tau nu)` refers to the branching ratio of $B_u \to \tau\nu_\tau$, `R(B->tau nu)` the normalized ratio to the SM value, `BR(B->D tau nu)` the branching ratio of $B \to D^0\tau\nu_\tau$, `BR(B->D tau nu)/BR(B->D e nu)` the ratio $\mathrm{BR}(B \to D^0\tau\nu_\tau)/\mathrm{BR}(B \to D^0 e\nu_e)$, `BR(Ds->tau nu)` and `BR(Ds->mu nu)` the branching ratios of $D_s \to \tau\nu_\tau$ and $D_s \to \mu\nu_\mu$ respectively, `BR(D->mu nu)` the branching ratio of $D \to \mu\nu_\mu$, `BR(K->mu nu)/BR(pi->mu nu)` the ratio $\mathrm{BR}(K \to \mu\nu_\mu)/\mathrm{BR}(\pi \to \mu\nu_\mu)$, `Rmu23(K->mu nu)` the ratio $R_{\mu 23}$, `a_muon` the deviation in the anomalous magnetic moment of the muon. More details on the definitions and calculations of these observables are given in the appendices.

4.2 SLHA input file

The program `slha.x` reads the needed parameters in the input SLHA file and calculates the observables. For example, the command

```
./slha.x example.lha
```

returns

```
Observable                        Value

BR(b->s gamma)                    3.112e-04
delta0(B->K* gamma)               5.770e-02

BR(Bs->mu mu)                     3.156e-09
BR(Bs->mu mu)_untag               3.461e-09
BR(Bd->mu mu)                     1.043e-10

BR(B->K* mu mu)_low               2.445e-07
AFB(B->K* mu mu)_low              -3.544e-02
FL(B->K* mu mu)_low               7.108e-01
P1(B->K* mu mu)_low               -5.877e-02
P2(B->K* mu mu)_low               8.726e-02
P4'(B->K* mu mu)_low              5.041e-01
P5'(B->K* mu mu)_low              -3.707e-01
P6'(B->K* mu mu)_low              -6.003e-02
P8'(B->K* mu mu)_low              4.912e-02
AI(B->K* mu mu)_low               -2.941e-02

BR(B->K* mu mu)_high              1.361e-07
AFB(B->K* mu mu)_high             4.460e-01
FL(B->K* mu mu)_high              3.108e-01
P1(B->K* mu mu)_high              -4.875e-01
P2(B->K* mu mu)_high              -4.330e-01
P4'(B->K* mu mu)_high             1.219e+00
P5'(B->K* mu mu)_high             -7.106e-01
P6'(B->K* mu mu)_high             -4.355e-09
P8'(B->K* mu mu)_high             0.000e+00
AI(B->K* mu mu)_high              -4.570e-04

q0^2(AFB(B->K* mu mu))            3.966e+00
q0^2(AI(B->K* mu mu))             1.671e+00

BR(B->Xs mu mu)_low               1.665e-06
BR(B->Xs mu mu)_high              2.174e-07
q0^2(AFB(B->Xs mu mu)             3.361e+00
BR(B->Xs tau tau)_high            1.600e-07

BR(B->tau nu)                     8.058e-05
R(B->tau nu)                      9.966e-01
BR(B->D tau nu)                   6.845e-03
BR(B->D tau nu)/BR(B->D e nu)     2.972e-01
BR(Ds->tau nu)                    5.113e-02
BR(Ds->mu nu)                     5.254e-03
BR(D->mu nu)                      4.096e-04
```

```
BR(K->mu nu)/BR(pi->mu nu)      6.355e-01
Rmu23(K->mu nu)                 1.000e+00

a_muon                          2.029e-10

excluded_LEP/Tevatron_mass      0
charged_LSP                     0

output.flha generated
```

corresponding to the observables described in the previous section. In addition, `excluded_LEP/Tevatron_mass`, if equal to 1, indicates that the point is excluded by the mass limits from LEP and Tevatron, as given in Appendix H, otherwise 0. Finally, `charged_LSP`, if equal to 1, shows that the lightest supersymmetric particle (LSP) is charged, which is generally disfavoured by cosmological data, 0 otherwise. If `HiggsBounds` is available, this line is replaced by `excluded_Higgsbounds` which gives 1 if the point is excluded by the `HiggsBounds` constraints, 0 otherwise. The program also provides an FLHA output (reproduced in Appendix J) containing more information on the flavour observables.

If the SLHA file provided to `slha.x` is inconsistent, a message will be displayed:

- `Invalid point` means that the SLHA generator had not succeeded in generating the mass spectrum (*e.g.* due to the presence of tachyonic particles).
- `Model not yet implemented` means that the SLHA file is intended for a model not implemented in `SuperIso`, such as R-parity violating models.
- `Invalid SLHA file` means that the SLHA file is broken and important parameters are missing.

4.3 CMSSM inputs

The program `cmssm.x` computes the observables in the CMSSM parameter space, using `ISAJET` and/or `SOFTSUSY` and/or `SPheno` and/or `SuSpect`, to generate the mass spectra. If a generator is unavailable, the corresponding `#define` in `cmssm.c` has to be commented. The necessary arguments to this program are:

- m_0: universal scalar mass at GUT scale,
- $m_{1/2}$: universal gaugino mass at GUT scale,
- A_0: trilinear soft breaking parameter at GUT scale,
- $\tan\beta$: ratio of the two Higgs vacuum expectation values.

Optional arguments can also be given:

- $sign(\mu)$: sign of Higgsino mass term, positive by default,
- m_t^{pole}: top quark pole mass, by default 173.34 GeV,
- $\overline{m}_b(\overline{m}_b)$: scale independent b-quark mass, by default 4.19 GeV (option unavailable for `ISAJET`),

- $\alpha_s(M_Z)$: strong coupling constant at the Z-boson mass, by default 0.1184 (option unavailable for `ISAJET`).

If the arguments are not specified, a message will describe the needed parameters in a correct order.

With `SOFTSUSY 3.5.1`, running the program with:

```
./cmssm.x 500 500 -500 50
```

```
CMSSM - SLHA file generated by SOFTSUSY

Observable                         Value

BR(b->s gamma)                     2.217e-04
delta0(B->K* gamma)                6.742e-02

BR(Bs->mu mu)                      2.890e-08
BR(Bs->mu mu)_untag                3.042e-08
BR(Bd->mu mu)                      9.237e-10

BR(B->K* mu mu)_low                2.484e-07
AFB(B->K* mu mu)_low               2.993e-02
FL(B->K* mu mu)_low                7.345e-01
P1(B->K* mu mu)_low                -7.894e-02
P2(B->K* mu mu)_low                -8.153e-02
P4'(B->K* mu mu)_low               6.684e-01
P5'(B->K* mu mu)_low               -5.629e-01
P6'(B->K* mu mu)_low               -6.058e-02
P8'(B->K* mu mu)_low               5.037e-02
AI(B->K* mu mu)_low                -4.058e-02

BR(B->K* mu mu)_high               1.447e-07
AFB(B->K* mu mu)_high              4.472e-01
FL(B->K* mu mu)_high               3.102e-01
P1(B->K* mu mu)_high               -4.875e-01
P2(B->K* mu mu)_high               -4.350e-01
P4'(B->K* mu mu)_high              1.219e+00
P5'(B->K* mu mu)_high              -7.153e-01
P6'(B->K* mu mu)_high              5.148e-05
P8'(B->K* mu mu)_high              0.000e+00
AI(B->K* mu mu)_high               -4.695e-04

q0^2(AFB(B->K* mu mu))             3.251e+00
q0^2(AI(B->K* mu mu))              1.379e+00

BR(B->Xs mu mu)_low                1.717e-06
```

```
BR(B->Xs mu mu)_high          2.186e-07
q0^2(AFB(B->Xs mu mu)         2.762e+00
BR(B->Xs tau tau)_high        1.674e-07

BR(B->tau nu)                 5.424e-05
R(B->tau nu)                  6.701e-01
BR(B->D tau nu)               6.374e-03
BR(B->D tau nu)/BR(B->D e nu) 2.765e-01
BR(Ds->tau nu)                5.097e-02
BR(Ds->mu nu)                 5.239e-03
BR(D->mu nu)                  4.100e-04
BR(K->mu nu)/BR(pi->mu nu)    6.334e-01
Rmu23(K->mu nu)               9.984e-01

a_muon                        1.956e-09

excluded_LEP/Tevatron_mass    0
charged_LSP                   0

output1.flha generated
```

corresponding to the observables described in section 4.1.

4.4 AMSB inputs

The program `amsb.x` computes the observables using the corresponding parameters generated by `ISAJET` and/or `SOFTSUSY` and/or `SPheno` and/or `SuSpect`, in the AMSB scenario. The necessary arguments to this program are:

- m_0: universal scalar mass at GUT scale,
- $m_{3/2}$: gravitino mass at GUT scale,
- $\tan\beta$: ratio of the two Higgs vacuum expectation values.

Optional arguments are the same as for CMSSM. If the input parameters are absent, a message will ask for them.

Example:

```
./amsb.x 500 5000 5 -1
```

4.5 HCAMSB inputs

The program `hcamsb.x` computes the observables using the corresponding parameters generated by `ISAJET` in the HCAMSB scenario. The necessary arguments to this program are:

- α: hypercharge anomaly mixing parameter,
- $m_{3/2}$: gravitino mass at GUT scale,

- $\tan\beta$: ratio of the two Higgs vacuum expectation values.

Optional arguments are the same as for CMSSM. If the input parameters are absent, a message will ask for them.

Example:

```
./hcamsb.x 0.2 10000 20
```

4.6 MMAMSB inputs

The program `mmamsb.x` computes the observables using the corresponding parameters generated by `ISAJET` in the MMAMSB scenario. The necessary arguments to this program are:

- α: modulus anomaly mixing parameter,
- $m_{3/2}$: gravitino mass at GUT scale,
- $\tan\beta$: ratio of the two Higgs vacuum expectation values.

Optional arguments are the same as for CMSSM. If the input parameters are absent, a message will ask for them.

Example:

```
./mmamsb.x 5 10000 10
```

4.7 GMSB inputs

The program `gmsb.x` computes the observables using the GMSB parameters generated by `ISAJET` and/or `SOFTSUSY` and/or `SPheno` and/or `SuSpect`. The necessary arguments to this program are:

- Λ: scale of the SUSY breaking in GeV (usually 10000-100000 GeV),
- M_{mess}: messenger mass scale ($> \Lambda$),
- N_5: equivalent number of $5 + \bar{5}$ messenger fields,
- $\tan\beta$: ratio of the two Higgs vacuum expectation values.

Optional arguments are the same as for CMSSM, with an additional one:

- c_{Grav} (≥ 1): ratio of the gravitino mass to its value for a breaking scale Λ, 1 by default.

Again, in the case of lack of arguments, a message will be displayed.

Example:

```
./gmsb.x 1e5 1e8 1 10
```

4.8 NUHM inputs

The program `nuhm.x` computes the observables using the NUHM parameters generated by `ISAJET` and/or `SOFTSUSY` and/or `SuSpect`. The necessary arguments to this program are the same as for CMSSM, with two additional ones, the values of μ and m_A:

- m_0: universal scalar mass at GUT scale,
- $m_{1/2}$: universal gaugino mass at GUT scale,
- A_0: trilinear soft breaking parameter at GUT scale,
- $\tan\beta$: ratio of the two Higgs vacuum expectation values,
- μ: μ parameter,
- m_A: CP-odd Higgs mass.

Optional arguments can also be given:

- m_t^{pole}: top quark pole mass, by default 173.34 GeV,
- $\overline{m}_b(\overline{m}_b)$: scale independent b-quark mass, by default 4.19 GeV (option unavailable for `ISAJET`),
- $\alpha_s(M_Z)$: strong coupling constant at the Z-boson mass, by default 0.1184 (option unavailable for `ISAJET`).

In the absence of arguments, a message will be shown.

Example:

```
./nuhm.x 500 500 0 50 500 500
```

4.9 CNMSSM inputs

The program `cnmssm.x` computes the observables using the CNMSSM parameters generated by `NMSSMTools`[3]. The necessary arguments to this program are:

- m_0: universal scalar mass at GUT scale,
- $m_{1/2}$: universal gaugino mass at GUT scale,
- A_0: trilinear soft breaking parameter at GUT scale,
- $\tan\beta$: ratio of the two Higgs vacuum expectation values,
- λ: cubic Higgs coupling.

Optional arguments can also be given:

- $sign(\mu)$: sign of Higgsino mass term, positive by default,
- A_κ: trilinear soft breaking parameter at GUT scale, by default $A_\kappa = A_0$,
- m_t^{pole}: top quark pole mass, by default 173.34 GeV,

[3]As the soft singlet mass m_S^2 and the singlet self coupling κ are both determined in terms of the other parameters through the minimization equations of the Higgs potential in `NMSSMTools`, what we call CNMSSM here is a partially constrained NMSSM scenario.

- $\overline{m}_b(\overline{m}_b)$: scale independent b-quark mass, by default 4.19 GeV,
- $\alpha_s(M_Z)$: strong coupling constant at the Z-boson mass, by default 0.1184.

In the absence of arguments, a message will be shown.

Example:

```
./cnmssm.x 500 500 0 50 0.01
```

4.10 NGMSB inputs

The program `ngmsb.x` computes the observables using the NGMSB parameters generated by `NMSSMTools`. The necessary arguments to this program are:

- Λ: scale of the SUSY breaking in GeV (usually 10000-100000 GeV),
- M_{mess}: messenger mass scale ($> \Lambda$),
- N_5: equivalent number of $5 + \bar{5}$ messenger fields,
- $\tan\beta$: ratio of the two Higgs vacuum expectation values,
- λ: cubic Higgs coupling.

Optional arguments are the same as for CNMSSM, with an additional one:

- Δ_H: 0 by default.

`NMSSMTools` allows also for other optional parameters such as μ', B', ξ_S and ξ_F.

Example:

```
./ngmsb.x 1e5 2e10 5 20 0.1 1 -1000
```

4.11 NNUHM inputs

The program `nnuhm.x` computes the observables using the NNUHM parameters generated by `NMSSMTools`. The necessary arguments to this program are the same as for CNMSSM, with two additional ones:

- m_0: universal scalar mass at GUT scale,
- $m_{1/2}$: universal gaugino mass at GUT scale,
- A_0: trilinear soft breaking parameter at GUT scale,
- $\tan\beta$: ratio of the two Higgs vacuum expectation values,
- λ: cubic Higgs coupling,
- M_{H_D}: down Higgs mass parameter at GUT scale,
- M_{H_U}: up Higgs mass parameter at GUT scale.

Optional arguments are the same as for CNMSSM. In the absence of arguments, a message will be shown.

Example:

```
./nnuhm.x 500 500 0 50 0.1 500 500
```

4.12 2HDM inputs

The program `thdm.x` computes the observables using the 2HDM parameters generated by `2HDMC`. The necessary arguments to this program are:

- *type*: Yukawa type (1-4),
- $\tan\beta$: ratio of the two Higgs vacuum expectation values,
- m_A: CP-odd Higgs mass.

Optional arguments can also be given:

- $\lambda_1, \cdots, \lambda_7$: Higgs potential parameters,
- m_{12}^2: Higgs potential parameter alternative to m_A.

By specifying the Higgs potential parameters, it is possible to do the calculations in general 2HDM. If not specified, the optional arguments are set to the default tree level MSSM-like values:

$$\lambda_1 = \lambda_2 = \frac{g^2 + g'^2}{4} \;, \qquad \lambda_3 = \frac{g^2 - g'^2}{4} \;, \qquad \lambda_4 = -\frac{g^2}{2} \;, \qquad \lambda_5 = \lambda_6 = \lambda_7 = 0 \;,$$
$$m_{12}^2 = m_A^2 \cos\beta \sin\beta \;.$$

In the absence of the necessary arguments, a message will be displayed.

Example:

```
./thdm.x 4 10 300
```

5 Results

We illustrate in this section the constraints on the SUSY parameter space that can be obtained using observables calculated with `SuperIso`. For more extended discussions about the constraints obtained using `SuperIso`, see for example [18–24]. In Figures 1 and 2, two examples of the obtained constraints in the CMSSM and NUHM scenarios using `SuperIso v2.3` are displayed. The different areas in the figures correspond to the following observables:

- red region: excluded by the isospin asymmetry,
- blue region: excluded by the inclusive branching ratio of $b \to s\gamma$,
- black hatched region: excluded by the collider mass limits,
- violet region: excluded by the branching ratio of $B_s \to \mu^+\mu^-$,
- gray hatched region: **favoured** by the anomalous magnetic moment of the muon,
- yellow hatched region: the LSP is charged, therefore disfavoured by cosmology,
- green region: excluded by the branching ratio of $B_u \to \tau\nu_\tau$,
- orange region: excluded by the branching ratio of $B \to D^0\tau\nu_\tau$,
- cyan region: excluded by the branching ratio of $K \to \mu\nu_\mu$.

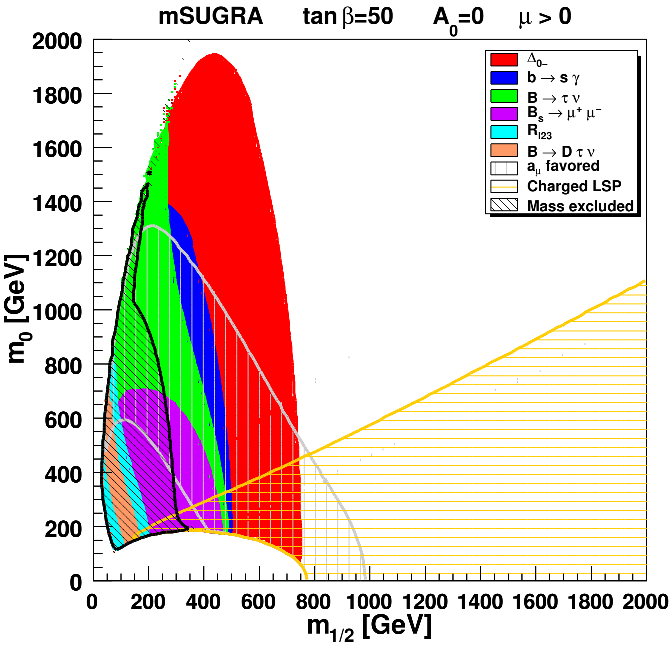

Figure 1: Constraints in CMSSM ($m_{1/2} - m_0$) parameter plane. For the description of the various coloured zones see the text. The contours are superimposed in the order given in the legend.

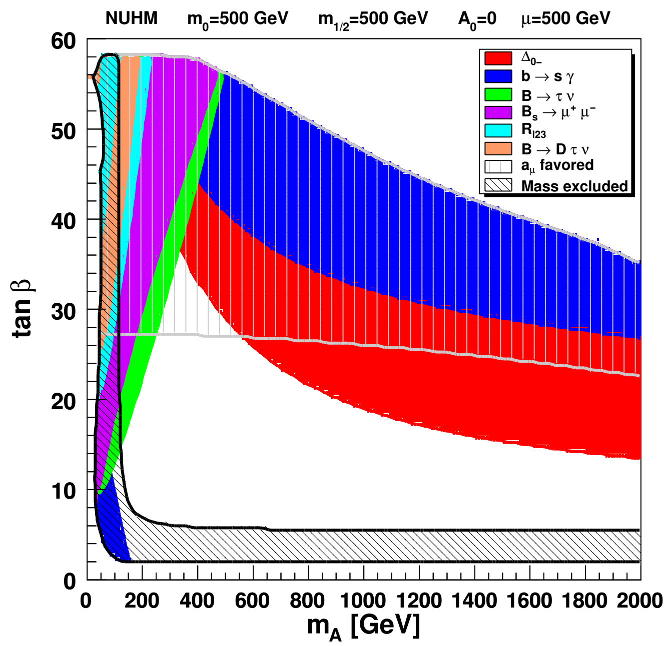

Figure 2: Constraints in NUHM ($m_A - \tan\beta$) parameter plane. For the description of the various coloured zones see the text. The contours are superimposed in the order given in the legend.

The allowed interval for each observable is given in Appendix H.

In Figure 1, the exclusion regions in the CMSSM parameter plane ($m_{1/2}$–m_0) for $\tan\beta = 50$, $A_0 = 0$ and $\mu > 0$ are displayed. One can notice that small values of m_0 and $m_{1/2}$ are disfavoured by the observables. The unfilled region in the bottom left corner corresponds to points with tachyonic particles.
In Figure 2, the exclusion zones are displayed in the NUHM parameter plane ($m_A - \tan\beta$) for $m_0 = 500$ GeV, $m_{1/2} = 500$ GeV, $A_0 = 0$ and $\mu = 500$ GeV. Most observables tend to disfavour the high $\tan\beta$ region in this plane. The white top right triangle corresponds to a region where tachyonic particles are encountered.

6 Conclusion

`SuperIso v4.1` features many new additions and improvements as compared to the first versions of the program. It is now able to compute numerous flavour physics observables – as well as the muon anomalous magnetic moment – which have already proved to be very useful in the exploration of the MSSM and NMSSM parameter spaces. Investigating the indirect constraints has many interesting phenomenological impacts, and can provide us with important information. They can also be used as guidelines for the LHC direct searches, and will be very valuable for the consistency checks.

In spite of the numerous changes, the spirit of the program is still based on the simplicity of use. The code will continue to incorporate other flavour physics observables. Also, the extension of the program to beyond minimal flavour violation is under development.

Appendix A QCD coupling

The α_s evolution is expressed as [15]:

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda_{n_f}^2)} \left\{ 1 - \frac{\beta_1}{\beta_0^2} \frac{\ln\big(\ln(\mu^2/\Lambda_{n_f}^2)\big)}{\ln(\mu^2/\Lambda_{n_f}^2)} + \frac{\beta_1^2}{\beta_0^4 \ln^2(\mu^2/\Lambda_{n_f}^2)} \times \left[\left(\ln\big(\ln(\mu^2/\Lambda_{n_f}^2)\big) - \frac{1}{2} \right)^2 + \frac{\beta_2 \beta_0}{2\beta_1^2} - \frac{5}{4} \right] \right\} , \tag{7}$$

with

$$\beta_0 = 11 - \frac{2}{3} n_f \ , \quad \beta_1 = 102 - \frac{38}{3} n_f \ , \quad \beta_2 = 2857 - \frac{5033}{9} n_f + \frac{325}{27} n_f^2 \ . \tag{8}$$

n_f denotes the number of active flavours (*i.e.* 4 for energies between the charm and the bottom masses, 5 between the bottom and the top masses, and 6 beyond the top mass). We compute the associated Λ_{n_f} by requiring continuity of α_s. In particular in `SuperIso`, Λ_5 is calculated so that $\alpha_s(M_Z)$ matches the value given in the input SLHA file, and then Λ_4 and Λ_6 are calculated if needed by imposing the continuity at the bottom and top mass scales respectively, which are also input parameters.

Appendix B Evolution of quark masses

We use the following two loop formula[4] to compute the pole mass of quarks [15]:

$$m_q^{\rm pole} = \overline{m}_q(\overline{m}_q) \left\{ 1 + \frac{4\alpha_s(\overline{m}_q)}{3\pi} + \left[-1.0414 \sum_{k=1}^{n_{f_l}} \left(1 - \frac{4}{3} \frac{\overline{m}_{q_k}}{\overline{m}_q} \right) + 13.4434 \right] \left(\frac{\alpha_s(\overline{m}_q)}{\pi} \right)^2 \right\} , \tag{9}$$

where n_{f_l} is the number of flavours q_k lighter than q.

For the $\overline{MS}$ top mass, we use [15]

$$\overline{m}_t(\overline{m}_t) = m_t^{\rm pole} \left(1 - \frac{4}{3} \frac{\alpha_s(m_t^{\rm pole})}{\pi} \right) \ . \tag{10}$$

The running mass of the quarks is given by [25]

$$\overline{m}_q(\mu_1) = \frac{R\big(\alpha_s(\mu_1)\big)}{R\big(\alpha_s(\mu_2)\big)} \overline{m}_q(\mu_2) \ , \tag{11}$$

[4]The two and three loop corrections are comparable in size and have the same sign as the one loop term. Since this is a signal of the asymptotic nature of the perturbation series, we leave the three loop corrections [15].

where

$$R(\alpha_s) = \left(\frac{\beta_0}{2}\frac{\alpha_s}{\pi}\right)^{2\gamma_0/\beta_0} \left\{1 + \left(2\frac{\gamma_1}{\beta_0} - \frac{\beta_1\gamma_0}{\beta_0^2}\right)\frac{\alpha_s}{\pi}\right. \tag{12}$$

$$\left. +\frac{1}{2}\left[\left(2\frac{\gamma_1}{\beta_0} - \frac{\beta_1\gamma_0}{\beta_0^2}\right)^2 + 2\frac{\gamma_2}{\beta_0} - \frac{\beta_1\gamma_1}{\beta_0^2} - \frac{\beta_2\gamma_0}{16\beta_0^2} + \frac{\beta_1^2\gamma_0}{2\beta_0^3}\right]\left(\frac{\alpha_s}{\pi}\right)^2 + O(\alpha_s^3)\right\},$$

with

$$\gamma_0 = 2\,, \tag{13}$$

$$\gamma_1 = \frac{101}{12} - \frac{5}{18}n_f\,, \tag{14}$$

$$\gamma_2 = \frac{1}{32}\left[1249 - \left(\frac{2216}{27} + \frac{160}{3}\zeta(3)\right)n_f - \frac{140}{81}n_f^2\right]\,, \tag{15}$$

with n_f the number of active flavours, and β's given in Eq. (8).

The $1S$ bottom quark mass is given by [26]:

$$m_b^{1S} = m_b^{\rm pole}\left[1 - \Delta^{\rm LO} - \Delta^{\rm NLO} - \Delta^{\rm NNLO}\right]\,, \tag{16}$$

where

$$\Delta^{\rm LO} = \frac{C_F^2\,\alpha_s^2(\mu_b)}{8}\,, \tag{17}$$

$$\Delta^{\rm NLO} = \frac{C_F^2\,\alpha_s^2(\mu_b)}{8}\left(\frac{\alpha_s(\mu_b)}{\pi}\right)\left[\beta_0'\,(L_0+1) + \frac{a_1}{2}\right]\,, \tag{18}$$

$$\Delta^{\rm NNLO} = \frac{C_F^2\,\alpha_s^2(\mu_b)}{8}\left(\frac{\alpha_s(\mu_b)}{\pi}\right)^2\left[\beta_0'^2\left(\frac{3}{4}L_0^2 + L_0 + \frac{\zeta(3)}{2} + \frac{\pi^2}{24} + \frac{1}{4}\right)\right. \tag{19}$$

$$\left. +\beta_0'\,\frac{a_1}{2}\left(\frac{3}{2}L_0 + 1\right) + \frac{\beta_1'}{4}\,(L_0+1) + \frac{a_1^2}{16} + \frac{a_2}{8} + \left(C_A - \frac{C_F}{48}\right)C_F\pi^2\right]\,,$$

with

$$L_0 \equiv \ln\left(\frac{\mu_b}{C_F\,\alpha_s(\mu_b)\,m_b^{\rm pole}}\right)\,, \tag{20}$$

$$\zeta(3) \approx 1.2020569\,, \tag{21}$$

and $\mu_b = O(m_b)$.

Also,

$$
\begin{aligned}
\beta_0' &= \frac{11}{3}\,C_A - \frac{4}{3}\,T\,n_{f_l}\ , \\
\beta_1' &= \frac{34}{3}\,C_A^2 - \frac{20}{3} C_A\,T\,n_{f_l} - 4\,C_F\,T\,n_{f_l}\ , \\
a_1 &= \frac{31}{9}\,C_A - \frac{20}{9}\,T\,n_{f_l}\ , \\
a_2 &= \left(\frac{4343}{162} + 4\,\pi^2 - \frac{\pi^4}{4} + \frac{22}{3}\,\zeta(3)\right) C_A^2 - \left(\frac{1798}{81} + \frac{56}{3}\,\zeta(3)\right) C_A\,T\,n_{f_l} \\
&\quad -\left(\frac{55}{3} - 16\,\zeta(3)\right) C_F\,T\,n_{f_l} + \left(\frac{20}{9}\,T\,n_{f_l}\right)^2 .
\end{aligned}
\tag{22}
$$

In the above equations $C_A = 3$, $T = 1/2$, $n_{f_l} = 4$ and $C_F = 4/3$.

Appendix C Wilson coefficients at matching scale

The Wilson coefficients at different orders are calculated separately and the following convention for the perturbative expansion is used:

$$
\begin{aligned}
C_i(\mu) &= C_i^{(0)}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_{i,s}^{(1)}(\mu) + \left(\frac{\alpha_s(\mu)}{4\pi}\right)^2 C_{i,s}^{(2)}(\mu) \\
&\quad + \frac{\alpha(\mu)}{4\pi} C_{i,e}^{(1)}(\mu) + \frac{\alpha(\mu)}{4\pi}\frac{\alpha_s(\mu)}{4\pi} C_{i,es}^{(2)}(\mu) + \cdots\ ,
\end{aligned}
\tag{23}
$$

where $C_{i,j}^{(k)}$ is the Wilson coefficient at order k in the perturbative expansion in $\alpha(\mu)$ $(j = e)$ or $\alpha_s(\mu)$ $(j = s)$. In the following, the indices s and e are omitted for simplicity.

C.1 Wilson coefficients $C_1 - C_8$

The effective Hamiltonian describing the $b \to s\gamma$ transitions has the following generic structure:

$$
\mathcal{H}_{eff} = \frac{4G_F}{\sqrt{2}} \sum_{p=u,c} V_{ps}^* V_{pb} \sum_{i=1}^{8} C_i(\mu)\, O_i\ , \tag{24}
$$

where G_F is the Fermi coupling constant, V_{ij} are elements of the CKM matrix, $O_i(\mu)$ are the relevant operators and $C_i(\mu)$ are the corresponding Wilson coefficients evaluated at the scale μ.

The Wilson coefficients are given in the standard operator basis [27]:

$$\begin{aligned}
O_1 &= (\bar{s}\gamma_\mu T^a P_L c)(\bar{c}\gamma^\mu T^a P_L b)\;, \\
O_2 &= (\bar{s}\gamma_\mu P_L c)(\bar{c}\gamma^\mu P_L b)\;, \\
O_3 &= (\bar{s}\gamma_\mu P_L b)\sum_q(\bar{q}\gamma^\mu q)\;, \\
O_4 &= (\bar{s}\gamma_\mu T^a P_L b)\sum_q(\bar{q}\gamma^\mu T^a q)\;, \\
O_5 &= (\bar{s}\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} P_L b)\sum_q(\bar{q}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} q)\;, \\
O_6 &= (\bar{s}\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} T^a P_L b)\sum_q(\bar{q}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} T^a q)\;, \\
O_7 &= \frac{e}{16\pi^2}\Big[\bar{s}\sigma^{\mu\nu}(m_s P_L + m_b P_R)b\Big]F_{\mu\nu}\;, \\
O_8 &= \frac{g}{16\pi^2}\Big[\bar{s}\sigma^{\mu\nu}(m_s P_L + m_b P_R)T^a b\Big]G^a_{\mu\nu}\;,
\end{aligned} \tag{25}$$

where $P_{L,R} = (1 \mp \gamma_5)/2$.

C.1.1 Standard Model contributions

We express here the SM contributions to the Wilson coefficients following [28, 29].

The LO coefficients are:

$$\begin{aligned}
C_2^{c(0)}(\mu_W) &= -1\;, & & \\
C_7^{c(0)}(\mu_W) &= \frac{23}{36}\;, & C_7^{t(0)}(\mu_W) &= -\frac{1}{2}\,A_0^t(x_{tW})\;, \\
C_8^{c(0)}(\mu_W) &= \frac{1}{3}\;, & C_8^{t(0)}(\mu_W) &= -\frac{1}{2}\,F_0^t(x_{tW})\;,
\end{aligned} \tag{26}$$

the NLO coefficients are:

$$\begin{aligned}
C_1^{c(1)}(\mu_W) &= -15 - 6\,L\;, & & \\
C_2^{c(1)}(\mu_W) &= 0\;, & & \\
C_3^{c(1)}(\mu_W) &= 0\;, & C_3^{t(1)}(\mu_W) &= 0\;, \\
C_4^{c(1)}(\mu_W) &= \frac{7}{9} - \frac{2}{3}\,L\;, & C_4^{t(1)}(\mu_W) &= E_0^t(x_{tW})\;, \\
C_5^{c(1)}(\mu_W) &= 0\;, & C_5^{t(1)}(\mu_W) &= 0\;, \\
C_6^{c(1)}(\mu_W) &= 0\;, & C_6^{t(1)}(\mu_W) &= 0\;, \\
C_7^{c(1)}(\mu_W) &= -\frac{713}{243} - \frac{4}{81}\,L\;, & C_7^{t(1)}(\mu_W) &= -\frac{1}{2}\,A_1^t(x_{tW})\;, \\
C_8^{c(1)}(\mu_W) &= -\frac{91}{324} + \frac{4}{27}\,L\;, & C_8^{t(1)}(\mu_W) &= -\frac{1}{2}\,F_1^t(x_{tW})\;,
\end{aligned} \tag{27}$$

and the NNLO coefficients are:

$$\begin{aligned}
C_1^{c(2)}(\mu_W) &= T(x_{tW}) - \frac{7987}{72} - \frac{17}{3}\pi^2 - \frac{475}{6}L - 17L^2 , \\
C_2^{c(2)}(\mu_W) &= -\frac{127}{18} - \frac{4}{3}\pi^2 - \frac{46}{3}L - 4L^2 , \\
C_3^{c(2)}(\mu_W) &= \frac{680}{243} + \frac{20}{81}\pi^2 + \frac{68}{81}L + \frac{20}{27}L^2 , \\
C_4^{c(2)}(\mu_W) &= -\frac{950}{243} - \frac{10}{81}\pi^2 - \frac{124}{27}L - \frac{10}{27}L^2 , \\
C_5^{c(2)}(\mu_W) &= -\frac{68}{243} - \frac{2}{81}\pi^2 - \frac{14}{81}L - \frac{2}{27}L^2 , \\
C_6^{c(2)}(\mu_W) &= -\frac{85}{162} - \frac{5}{108}\pi^2 - \frac{35}{108}L - \frac{5}{36}L^2 , \\
C_3^{t(2)}(\mu_W) &= G_1^t(x_{tW}) , \\
C_4^{t(2)}(\mu_W) &= E_1^t(x_{tW}) , \\
C_5^{t(2)}(\mu_W) &= -\frac{1}{10}G_1^t(x_{tW}) + \frac{2}{15}E_0^t(x_{tW}) , \\
C_6^{t(2)}(\mu_W) &= -\frac{3}{16}G_1^t(x_{tW}) + \frac{1}{4}E_0^t(x_{tW}) ,
\end{aligned} \tag{28}$$

where

$$x_{tW} = \left(\frac{\overline{m}_t(\mu_W)}{M_W}\right)^2 , \tag{29}$$

$$L = \ln\left(\frac{\mu_W^2}{M_W^2}\right) , \tag{30}$$

and $\mu_W = O(M_W)$. The necessary functions in Eqs. (26-28) are:

$$A_0^t(x) = \frac{-3x^3 + 2x^2}{2(1-x)^4}\ln x + \frac{22x^3 - 153x^2 + 159x - 46}{36(1-x)^3} , \tag{31}$$

$$E_0^t(x) = \frac{-9x^2 + 16x - 4}{6(1-x)^4}\ln x + \frac{-7x^3 - 21x^2 + 42x + 4}{36(1-x)^3} , \tag{32}$$

$$F_0^t(x) = \frac{3x^2}{2(1-x)^4}\ln x + \frac{5x^3 - 9x^2 + 30x - 8}{12(1-x)^3} , \tag{33}$$

$$
\begin{aligned}
A_1^t(x) &= \frac{32x^4+244x^3-160x^2+16x}{9(1-x)^4}\,\mathrm{Li}_2\left(1-\frac{1}{x}\right) \\
&\quad + \frac{-774x^4-2826x^3+1994x^2-130x+8}{81(1-x)^5}\ln x \\
&\quad + \frac{-94x^4-18665x^3+20682x^2-9113x+2006}{243(1-x)^4} \\
&\quad + \left[\frac{-12x^4-92x^3+56x^2}{3(1-x)^5}\ln x + \frac{-68x^4-202x^3-804x^2+794x-152}{27(1-x)^4}\right]\ln\left(\frac{\mu_W^2}{m_t^2}\right) \ , \qquad (34)
\end{aligned}
$$

$$
\begin{aligned}
E_1^t(x) &= \frac{515x^4-614x^3-81x^2-190x+40}{54(1-x)^4}\,\mathrm{Li}_2\left(1-\frac{1}{x}\right) \qquad (35) \\
&\quad + \frac{-1030x^4+435x^3+1373x^2+1950x-424}{108(1-x)^5}\ln x \\
&\quad + \frac{-29467x^4+45604x^3-30237x^2+66532x-10960}{1944(1-x)^4} \\
&\quad + \left[\frac{133x^4-2758x^3-2061x^2+11522x-1652}{324(1-x)^4}\right. \\
&\quad \left. + \frac{-1125x^3+1685x^2+380x-76}{54(1-x)^5}\ln x\right]\ln\left(\frac{\mu_W^2}{m_t^2}\right) \ ,
\end{aligned}
$$

as well as:

$$
\begin{aligned}
F_1^t(x) &= \frac{4x^4-40x^3-41x^2-x}{3(1-x)^4}\,\mathrm{Li}_2\left(1-\frac{1}{x}\right) \qquad (36) \\
&\quad + \frac{-144x^4+3177x^3+3661x^2+250x-32}{108(1-x)^5}\ln x \\
&\quad + \frac{-247x^4+11890x^3+31779x^2-2966x+1016}{648(1-x)^4} \\
&\quad + \left[\frac{17x^3+31x^2}{(1-x)^5}\ln x + \frac{-35x^4+170x^3+447x^2+338x-56}{18(1-x)^4}\right]\ln\left(\frac{\mu_W^2}{m_t^2}\right) \ , \\
G_1^t(x) &= \frac{10x^4-100x^3+30x^2+160x-40}{27(1-x)^4}\,\mathrm{Li}_2\left(1-\frac{1}{x}\right) \qquad (37) \\
&\quad + \frac{30x^3-42x^2-332x+68}{81(1-x)^4}\ln x + \frac{-6x^3-293x^2+161x+42}{81(1-x)^3} \\
&\quad + \left[\frac{90x^2-160x+40}{27(1-x)^4}\ln x + \frac{35x^3+105x^2-210x-20}{81(1-x)^3}\right]\ln\left(\frac{\mu_W^2}{m_t^2}\right) \ , \\
T(x) &= -(16x+8)\sqrt{4x-1}\,\mathrm{Cl}_2\left(2\arcsin\frac{1}{2\sqrt{x}}\right) + \left(16x+\frac{20}{3}\right)\ln x + 32x + \frac{112}{9} \ .
\end{aligned}
$$

The integral representations for the functions Li_2 and Cl_2 are as follows:

$$\mathrm{Li}_2(z) \;=\; -\int_0^z dt\,\frac{\ln(1-t)}{t}\;, \tag{38}$$

$$\mathrm{Cl}_2(x) \;=\; \mathrm{Im}\Big[\mathrm{Li}_2(e^{ix})\Big] = -\int_0^x d\theta\,\ln|2\sin(\theta/2)|\;. \tag{39}$$

The remaining NNLO coefficients take the form:

$$C_7^{c(2)}(\mu_W) \;=\; C_7^{c(2)}(\mu_W=M_W)+\frac{13763}{2187}\ln\left(\frac{\mu_W^2}{M_W^2}\right)+\frac{814}{729}\ln^2\left(\frac{\mu_W^2}{M_W^2}\right)\;, \tag{40}$$

$$C_8^{c(2)}(\mu_W) \;=\; C_8^{c(2)}(\mu_W=M_W)+\frac{16607}{5832}\ln\left(\frac{\mu_W^2}{M_W^2}\right)+\frac{397}{486}\ln^2\left(\frac{\mu_W^2}{M_W^2}\right)\;, \tag{41}$$

$$\begin{aligned}C_7^{t(2)}(\mu_W) \;=\;& C_7^{t(2)}(\mu_W=m_t)\\
&+\ln\left(\frac{\mu_W^2}{m_t^2}\right)\left[\frac{-592x^5-22x^4+12814x^3-6376x^2+512x}{27(x-1)^5}\,\mathrm{Li}_2\left(1-\frac{1}{x}\right)\right.\\
&+\frac{-26838x^5+25938x^4+627367x^3-331956x^2+16989x-460}{729(x-1)^6}\ln x\\
&\left.+\frac{34400x^5+276644x^4-2668324x^3+1694437x^2-323354x+53077}{2187(x-1)^5}\right]\\
&+\ln^2\left(\frac{\mu_W^2}{m_t^2}\right)\left[\frac{-63x^5+532x^4+2089x^3-1118x^2}{9(x-1)^6}\ln x\right.\\
&\left.+\frac{1186x^5-2705x^4-24791x^3-16099x^2+19229x-2740}{162(x-1)^5}\right]\;,\end{aligned} \tag{42}$$

$$\begin{aligned}C_8^{t(2)}(\mu_W) \;=\;& C_8^{t(2)}(\mu_W=m_t)\\
&+\ln\left(\frac{\mu_W^2}{m_t^2}\right)\left[\frac{-148x^5+1052x^4-4811x^3-3520x^2-61x}{18(x-1)^5}\,\mathrm{Li}_2\left(1-\frac{1}{x}\right)\right.\\
&+\frac{-15984x^5+152379x^4-1358060x^3-1201653x^2-74190x+9188}{1944(x-1)^6}\ln x\\
&\left.+\frac{109669x^5-1112675x^4+6239377x^3+8967623x^2+768722x-42796}{11664(x-1)^5}\right]\\
&+\ln^2\left(\frac{\mu_W^2}{m_t^2}\right)\left[\frac{-139x^4-2938x^3-2683x^2}{12(x-1)^6}\ln x\right.\\
&\left.+\frac{1295x^5-7009x^4+29495x^3+64513x^2+17458x-2072}{216(x-1)^5}\right]\;.\end{aligned} \tag{43}$$

As regard to the three-loop quantities $C_7^{c(2)}(\mu_W=M_W)$, $C_8^{c(2)}(\mu_W=M_W)$, $C_7^{t(2)}(\mu_W=m_t)$ and $C_8^{t(2)}(\mu_W=m_t)$, we have access to their expansions at $x\to 1$ and $x\to\infty$.

Denoting $z = 1/x$ and $w = 1 - z$, the coefficients become:

$$C_7^{c(2)}(\mu_W = M_W) \underset{x\to\infty}{\simeq} 1.525 - 0.1165z + 0.01975z\ln z + 0.06283z^2 + 0.005349z^2\ln z + 0.01005z^2\ln^2 z - 0.04202z^3 + 0.01535z^3\ln z - 0.00329z^3\ln^2 z + 0.002372z^4 - 0.0007910z^4\ln z + \mathcal{O}(z^5)\ , \tag{44}$$

$$C_7^{c(2)}(\mu_W = M_W) \underset{x\to 1}{\simeq} 1.432 + 0.06709w + 0.01257w^2 + 0.004710w^3 + 0.002373w^4 + 0.001406w^5 + 0.0009216w^6 + 0.00064730w^7 + 0.0004779w^8 + \mathcal{O}(w^9)\ , \tag{45}$$

$$C_8^{c(2)}(\mu_W = M_W) \underset{x\to\infty}{\simeq} -1.870 + 0.1010z - 0.1218z\ln z + 0.1045z^2 - 0.03748z^2\ln z + 0.01151z^2\ln^2 z - 0.01023z^3 + 0.004342z^3\ln z + 0.0003031z^3\ln^2 z - 0.001537z^4 + 0.0007532z^4\ln z + \mathcal{O}(z^5)\ , \tag{46}$$

$$C_8^{c(2)}(\mu_W = M_W) \underset{x\to 1}{\simeq} -1.676 - 0.1179w - 0.02926w^2 - 0.01297w^3 - 0.007296w^4 - 0.004672w^5 - 0.003248w^6 - 0.002389w^7 - 0.001831w^8 + \mathcal{O}(w^9)\ , \tag{47}$$

$$C_7^{t(2)}(\mu_W = m_t) \underset{x\to\infty}{\simeq} 12.06 + 12.93z + 3.013z\ln z + 96.71z^2 + 52.73z^2\ln z + 147.9z^3 + 187.7z^3\ln z - 144.9z^4 + 236.1z^4\ln z + \mathcal{O}(z^5)\ , \tag{48}$$

$$C_7^{t(2)}(\mu_W = m_t) \underset{x\to 1}{\simeq} 11.74 + 0.3642w + 0.1155w^2 - 0.003145w^3 - 0.03263w^4 - 0.03528w^5 - 0.03076w^6 - 0.02504w^7 - 0.01985w^8 + \mathcal{O}(w^9)\ , \tag{49}$$

$$C_8^{t(2)}(\mu_W = m_t) \underset{x\to\infty}{\simeq} -0.8954 - 7.043z - 98.34z^2 - 46.21z^2\ln z - 127.1z^3 - 181.6z^3\ln z + 535.8z^4 - 76.76z^4\ln z + \mathcal{O}(z^5)\ , \tag{50}$$

$$C_8^{t(2)}(\mu_W = m_t) \underset{x\to 1}{\simeq} -0.6141 - 0.8975w - 0.03492w^2 + 0.06791w^3 + 0.07966w^4 + 0.07226w^5 + 0.06132w^6 + 0.05096w^7 + 0.04216w^8 + \mathcal{O}(w^9)\ . \tag{51}$$

Type	λ_{UU}	λ_{DD}	λ_{LL}
I	$\cot\beta$	$\cot\beta$	$\cot\beta$
II	$\cot\beta$	$-\tan\beta$	$-\tan\beta$
III	$\cot\beta$	$-\tan\beta$	$\cot\beta$
IV	$\cot\beta$	$\cot\beta$	$-\tan\beta$

Table 4: Yukawa couplings for the four types of 2HDM. U, D and L stand respectively for the up-type quarks, the down-type quarks and the leptons.

For $n = 0, 1, 2$ and $i = 1, \cdots, 8$, the Wilson coefficients read:

$$C_i^{(n)} = C_i^{t(n)} - C_i^{c(n)} \ . \tag{52}$$

C.1.2 Charged Higgs contributions

At the Leading Order, the relevant charged Higgs contributions to the Wilson coefficients are given by [30]:

$$\delta C_{7,8}^{H(0)}(\mu_W) = \frac{\lambda_{tt}^2}{3} F_{7,8}^{(1)}(x_{tH^\pm}) - \lambda_{tt}\lambda_{bb} F_{7,8}^{(2)}(x_{tH^\pm}) \ , \tag{53}$$

where

$$x_{tH^\pm} = \frac{\overline{m}_t^2(\mu_W)}{M_{H^\pm}^2} \ , \tag{54}$$

and

$$\begin{aligned} F_7^{(1)}(x) &= \frac{x(7-5x-8x^2)}{24(x-1)^3} + \frac{x^2(3x-2)}{4(x-1)^4} \ln x \ , \\ F_8^{(1)}(x) &= \frac{x(2+5x-x^2)}{8(x-1)^3} - \frac{3x^2}{4(x-1)^4} \ln x \ , \\ F_7^{(2)}(x) &= \frac{x(3-5x)}{12(x-1)^2} + \frac{x(3x-2)}{6(x-1)^3} \ln x \ , \\ F_8^{(2)}(x) &= \frac{x(3-x)}{4(x-1)^2} - \frac{x}{2(x-1)^3} \ln x \ . \end{aligned} \tag{55}$$

$\lambda_{tt}, \lambda_{bb}$ are the Yukawa couplings. In Supersymmetry, they read:

$$\lambda_{tt} = -\frac{1}{\lambda_{bb}} = \frac{1}{\tan\beta} \ . \tag{56}$$

For the different types of 2HDM, the Yukawa couplings are summarized in Table 4.

At the NLO, the charged Higgs contributions can be written in the form [30]:

$$\delta C_4^{H(1)}(\mu_W) = E_4^H(x_{tH^\pm}) \,, \quad (57)$$

$$\delta C_7^{H(1)}(\mu_W) = G_7^H(x_{tH^\pm}) + \Delta_7^H(x_{tH^\pm}) \ln\left(\frac{\mu_W^2}{M_{H^\pm}^2}\right) - \frac{4}{9}\, E_4^H(x_{tH^\pm}) \,, \quad (58)$$

$$\delta C_8^{H(1)}(\mu_W) = G_8^H(x_{tH^\pm}) + \Delta_8^H(x_{tH^\pm}) \ln\left(\frac{\mu_W^2}{M_{H^\pm}^2}\right) - \frac{1}{6}\, E_4^H(x_{tH^\pm}) \,, \quad (59)$$

The NNLO contributions for C_{3-6} read [31]:

$$\delta C_3^{H(2)}(\mu_W) = G_3^H(x_{tH^\pm}) \,, \quad (60)$$

$$\delta C_4^{H(2)}(\mu_W) = E_4^{H(2)}(x_{tH^\pm}) \,, \quad (61)$$

$$\delta C_5^{H(2)}(\mu_W) = -\frac{1}{10}\, G_3^H(x_{tH^\pm}) + \frac{2}{15}\, E_4^H(x_{tH^\pm}) \,, \quad (62)$$

$$\delta C_6^{H(2)}(\mu_W) = -\frac{3}{16}\, G_3^H(x_{tH^\pm}) + \frac{1}{4}\, E_4^H(x_{tH^\pm}) \,, \quad (63)$$

with

$$\begin{aligned} G_3^H(x) = {} & \frac{1}{27}\lambda_{tt}^2 x \left\{ \frac{-20+30x+10x^3}{(x-1)^4} \mathrm{Li}_2\left(1-\frac{1}{x}\right) + \frac{-56-66x+30x^2}{3(x-1)^4} \ln x \right. \\ & \left. + \frac{213-187x+6x^2}{3(x-1)^3} + \left[\frac{20-30x}{(x-1)^4} \ln x + \frac{-80+145x-35x^2}{3(x-1)^3}\right] \ln\left(\frac{\mu_W^2}{M_{H^\pm}^2}\right) \right\} , \end{aligned} \quad (64)$$

$$\begin{aligned} G_7^H(x) = {} & \frac{4}{3}\lambda_{tt}\lambda_{bb} x \left[\frac{4(-3+7x-2x^2)}{3(x-1)^3} \mathrm{Li}_2\left(1-\frac{1}{x}\right) + \frac{8-14x-3x^2}{3(x-1)^4} \ln^2 x \right. \\ & \left. + \frac{7-13x+2x^2}{(x-1)^3} + \frac{2(-3-x+12x^2-2x^3)}{3(x-1)^4} \ln x \right] \\ & + \frac{2}{9}\lambda_{tt}^2 x \left[\frac{x(18-37x+8x^2)}{(x-1)^4} \mathrm{Li}_2\left(1-\frac{1}{x}\right) + \frac{x(-14+23x+3x^2)}{(x-1)^5} \ln^2 x \right. \\ & \left. + \frac{-50+251x-174x^2-192x^3+21x^4}{9(x-1)^5} \ln x + \frac{797-5436x+7569x^2-1202x^3}{108(x-1)^4} \right] , \end{aligned} \quad (65)$$

$$\Delta_7^H(x) = \frac{2}{9}\lambda_{tt}\lambda_{bb}x\left[\frac{21-47x+8x^2}{(x-1)^3}+\frac{2(-8+14x+3x^2)}{(x-1)^4}\ln x\right] \tag{66}$$
$$+\frac{2}{9}\lambda_{tt}^2 x\left[\frac{-31-18x+135x^2-14x^3}{6(x-1)^4}+\frac{x(14-23x-3x^2)}{(x-1)^5}\ln x\right] ,$$

$$G_8^H(x) = \frac{1}{3}\lambda_{tt}\lambda_{bb}x\left[\frac{-36+25x-17x^2}{2(x-1)^3}\,\mathrm{Li}_2\left(1-\frac{1}{x}\right)+\frac{19+17x}{(x-1)^4}\ln^2 x\right. \tag{67}$$
$$\left.+\frac{-3-187x+12x^2-14x^3}{4(x-1)^4}\ln x+\frac{3(143-44x+29x^2)}{8(x-1)^3}\right]$$
$$+\frac{1}{6}\lambda_{tt}^2 x\left[\frac{x(30-17x+13x^2)}{(x-1)^4}\,\mathrm{Li}_2\left(1-\frac{1}{x}\right)-\frac{x(31+17x)}{(x-1)^5}\ln^2 x\right.$$
$$\left.+\frac{-226+817x+1353x^2+318x^3+42x^4}{36(x-1)^5}\ln x+\frac{1130-18153x+7650x^2-4451x^3}{216(x-1)^4}\right] ,$$

$$\Delta_8^H(x) = \frac{1}{3}\lambda_{tt}\lambda_{bb}x\left[\frac{81-16x+7x^2}{2(x-1)^3}-\frac{19+17x}{(x-1)^4}\ln x\right] \tag{68}$$
$$+\frac{1}{6}\lambda_{tt}^2 x\left[\frac{-38-261x+18x^2-7x^3}{6(x-1)^4}+\frac{x(31+17x)}{(x-1)^5}\ln x\right] ,$$

$$E_4^H(x) = \frac{1}{6}\lambda_{tt}^2 x\left[\frac{16-29x+7x^2}{6(x-1)^3}+\frac{3x-2}{(x-1)^4}\ln x\right] , \tag{69}$$

$$E_4^{H(2)}(x) = \frac{1}{54}\lambda_{tt}^2 x\left\{\frac{182+99x-906x^2+515x^3}{(x-1)^4}\,\mathrm{Li}_2\left(1-\frac{1}{x}\right)\right. \tag{70}$$
$$+\frac{980-15x-2763x^2+1030x^3}{2(x-1)^5}\ln x+\frac{-18134-6717x+68142x^2-29467x^3}{36(x-1)^4}$$
$$\left.+\left[\frac{182-95x-375x^2}{(x-1)^5}\ln x+\frac{-2320+4023x-108x^2+133x^3}{6(x-1)^4}\right]\ln\left(\frac{\mu_W^2}{M_{H^\pm}^2}\right)\right\} .$$

The NNLO contributions to C_7 and C_8 are given by [32]:

$$\begin{aligned}
\delta C_7^{H(2)}(\mu_W) \;=\;& \lambda_{tt}^2 \Bigg\{ \delta C_{7tt}^{H(2)}(\mu_W = m_t) \\
&+ \ln\left(\frac{\mu_W^2}{m_t^2}\right)\Bigg[-\frac{x\left(67930x^4 - 470095x^3 + 1358478x^2 - 700243x + 54970\right)}{2187(x-1)^5} \\
&+ \frac{x\left(10422x^4 - 84390x^3 + 322801x^2 - 146588x + 1435\right)}{729(x-1)^6}\ln x \\
&+ \frac{2x^2\left(260x^3 - 1515x^2 + 3757x - 1446\right)}{27(x-1)^5}\mathrm{Li}_2\left(1 - \frac{1}{x}\right)\Bigg] \\
&+ \ln^2\left(\frac{\mu_W^2}{m_t^2}\right)\Bigg[\frac{x\left(-518x^4 + 3665x^3 - 17397x^2 + 3767x + 1843\right)}{162(x-1)^5} \\
&+ \frac{x^2\left(-63x^3 + 532x^2 + 2089x - 1118\right)}{27(x-1)^6}\ln x\Bigg]\Bigg\} \\
&+ \lambda_{tt}\lambda_{bb}\Bigg\{ \delta C_{7tb}^{H(2)}(\mu_W = m_t) \\
&+ \ln\left(\frac{\mu_W^2}{m_t^2}\right)\Bigg[\frac{x\left(3790x^3 - 22511x^2 + 53614x - 21069\right)}{81(x-1)^4} \\
&+ \frac{2x\left(-1266x^3 + 7642x^2 - 21467x + 8179\right)}{81(x-1)^5}\ln x \\
&- \frac{8x\left(139x^3 - 612x^2 + 1103x - 342\right)}{27(x-1)^4}\mathrm{Li}_2\left(1 - \frac{1}{x}\right)\Bigg] \\
&+ \ln^2\left(\frac{\mu_W^2}{m_t^2}\right)\Bigg[\frac{x\left(284x^3 - 1435x^2 + 4304x - 1425\right)}{27(x-1)^4} \\
&+ \frac{2x\left(63x^3 - 397x^2 - 970x + 440\right)}{27(x-1)^5}\ln x\Bigg]\Bigg\},
\end{aligned} \tag{71}$$

$$
\begin{aligned}
\delta C_8^{H(2)}(\mu_W) \;=\; & \lambda_{tt}^2 \Bigg\{ \delta C_{8tt}^{H(2)}(\mu_W = m_t) \qquad (72) \\
& + \ln\left(\frac{\mu_W^2}{m_t^2}\right) \Bigg[\frac{x\left(51948x^4 - 233781x^3 + 48634x^2 - 698693x + 2452\right)}{1944(x-1)^6} \ln r \\
& - \frac{x\left(522347x^4 - 2423255x^3 + 2706021x^2 - 5930609x + 148856\right)}{11664(x-1)^5} \\
& + \frac{x^2\left(481x^3 - 1950x^2 + 1523x - 2550\right)}{18(x-1)^5} \mathrm{Li}_2\left(1 - \frac{1}{x}\right) \Bigg] \\
& + \ln^2\left(\frac{\mu_W^2}{m_t^2}\right) \Bigg[\frac{x\left(-259x^4 + 1117x^3 + 2925x^2 + 28411x + 2366\right)}{216(x-1)^5} \\
& - \frac{x^2\left(139x^2 + 2938x + 2683\right)}{36(x-1)^6} \ln x \Bigg] \\
& + \lambda_{tt}\lambda_{bb} \Bigg\{ \delta C_{8tb}^{H(2)}(\mu_W = m_t) + \ln\left(\frac{\mu_W^2}{m_t^2}\right) \Bigg[\frac{x\left(1463x^3 - 5794x^2 + 5543x - 15036\right)}{27(x-1)^4} \\
& + \frac{x\left(-1887x^3 + 7115x^2 + 2519x + 19901\right)}{54(x-1)^5} \ln x \\
& + \frac{x\left(-629x^3 + 2178x^2 - 1729x + 2196\right)}{18(x-1)^4} \mathrm{Li}_2\left(1 - \frac{1}{x}\right) \Bigg] \\
& + \ln^2\left(\frac{\mu_W^2}{m_t^2}\right) \Bigg[\frac{x\left(259x^3 - 947x^2 - 251x - 5973\right)}{36(x-1)^4} \\
& + \frac{x\left(139x^2 + 2134x + 1183\right)}{18(x-1)^5} \ln x \Bigg] \;,
\end{aligned}
$$

where the three loop quantities $\delta C_{7tt}^{H(2)}(\mu_W = m_t)$, $\delta C_{7tb}^{H(2)}(\mu_W = m_t)$, $\delta C_{8tt}^{H(2)}(\mu_W = m_t)$ and $\delta C_{8tb}^{H(2)}(\mu_W = m_t)$ are given by:

$$
\begin{aligned}
\delta C_{7tt}^{H(2)}(\mu_W = m_t) \underset{r\to 0}{\simeq} \; & 0.9225\, r \ln^2 r + 4.317\, r \ln r - 8.278\, r \qquad (73) \\
& -20.73\, r^2 \ln^3 r - 112.4\, r^2 \ln^2 r - 396.1\, r^2 \ln r - 480.9\, r^2 \\
& -34.50\, r^3 \ln^3 r - 348.2\, r^3 \ln^2 r - 1292\, r^3 \ln r - 1158\, r^3 \\
& -23.26\, r^4 \ln^3 r - 541.4\, r^4 \ln^2 r - 2540\, r^4 \ln r - 1492\, r^4 \\
& +42.30\, r^5 \ln^3 r - 412.4\, r^5 \ln^2 r - 3362\, r^5 \ln r - 823.0\, r^5 + \mathcal{O}\left(r^6\right) \;,
\end{aligned}
$$

$$\delta C_{7tt}^{H(2)}(\mu_W = m_t) \underset{r\to 1^-}{\simeq} 1.283 - 0.7158\,\bar{u} - 0.3039\,\bar{u}^2 - 0.1549\,\bar{u}^3 - 0.08625\,\bar{u}^4 \tag{74}$$
$$-0.05020\,\bar{u}^5 - 0.02970\,\bar{u}^6 - 0.01740\,\bar{u}^7 - 0.009752\,\bar{u}^8 - 0.004877\,\bar{u}^9$$
$$-0.001721\,\bar{u}^{10} + 0.0003378\,\bar{u}^{11} + 0.001679\,\bar{u}^{12} + 0.002542\,\bar{u}^{13}$$
$$+0.003083\,\bar{u}^{14} + 0.003404\,\bar{u}^{15} + 0.003574\,\bar{u}^{16} + \mathcal{O}\left(\bar{u}^{17}\right) \ ,$$

$$\delta C_{7tt}^{H(2)}(\mu_W = m_t) \underset{r\to 1^+}{\simeq} 1.283 + 0.7158\,u + 0.4119\,u^2 + 0.2629\,u^3 + 0.1825\,u^4 + 0.1347\,u^5 \tag{75}$$
$$+0.1040\,u^6 + 0.08306\,u^7 + 0.06804\,u^8 + 0.05688\,u^9 + 0.04833\,u^{10}$$
$$+0.04163\,u^{11} + 0.03625\,u^{12} + 0.03188\,u^{13} + 0.02827\,u^{14} + 0.02525\,u^{15}$$
$$+0.02269\,u^{16} + \mathcal{O}\left(u^{17}\right) \ ,$$

$$\delta C_{7tt}^{H(2)}(\mu_W = m_t) \underset{r\to \infty}{\simeq} 3.970 - 8.753\frac{\ln r}{r} + 15.35\frac{1}{r} - 38.12\frac{\ln r}{r^2} + 47.09\frac{1}{r^2} - 103.8\frac{\ln r}{r^3} \tag{76}$$
$$+79.15\frac{1}{r^3} - 168.3\frac{\ln r}{r^4} + 24.41\frac{1}{r^4} - 72.13\frac{\ln r}{r^5} - 274.2\frac{1}{r^5} + \mathcal{O}\left(\frac{1}{r^6}\right) \ ,$$

$$\delta C_{7tb}^{H(2)}(\mu_W = m_t) \underset{r\to 0}{\simeq} -20.94\,r\ln^3 r - 123.5\,r\ln^2 r - 453.5\,r\ln r - 572.2\,r \tag{77}$$
$$-8.889\,r^2\ln^3 r - 195.7\,r^2\ln^2 r - 870.3\,r^2\ln r - 524.1\,r^2$$
$$+19.73\,r^3\ln^3 r - 46.61\,r^3\ln^2 r - 826.2\,r^3\ln r + 166.7\,r^3$$
$$+36.08\,r^4\ln^3 r + 323.2\,r^4\ln^2 r + 169.9\,r^4\ln r + 1480\,r^4$$
$$-66.63\,r^5\ln^3 r + 469.4\,r^5\ln^2 r + 1986\,r^5\ln r + 2828\,r^5 + \mathcal{O}\left(r^6\right) \ ,$$

$$\delta C_{7tb}^{H(2)}(\mu_W = m_t) \underset{r\to 1^-}{\simeq} 12.82 + 1.663\,\bar{u} + 0.7780\,\bar{u}^2 + 0.3755\,\bar{u}^3 + 0.1581\,\bar{u}^4 \tag{78}$$
$$+0.03021\,\bar{u}^5 - 0.04868\,\bar{u}^6 - 0.09864\,\bar{u}^7 - 0.1306\,\bar{u}^8$$
$$-0.1510\,\bar{u}^9 - 0.1637\,\bar{u}^{10} - 0.1712\,\bar{u}^{11} - 0.1751\,\bar{u}^{12}$$
$$-0.1766\,\bar{u}^{13} - 0.1763\,\bar{u}^{14} - 0.1748\,\bar{u}^{15} - 0.1724\,\bar{u}^{16} + \mathcal{O}\left(\bar{u}^{17}\right) \ ,$$

$$\delta C_{7tb}^{H(2)}(\mu_W = m_t) \underset{r\to 1^+}{\simeq} 12.82 - 1.663\,u - 0.8852\,u^2 - 0.4827\,u^3 - 0.2976\,u^4 - 0.2021\,u^5 \tag{79}$$
$$-0.1470\,u^6 - 0.1125\,u^7 - 0.08931\,u^8 - 0.07291\,u^9 - 0.06083\,u^{10}$$
$$-0.05164\,u^{11} - 0.04446\,u^{12} - 0.03873\,u^{13} - 0.03407\,u^{14} - 0.03023\,u^{15}$$
$$-0.02702\,u^{16} + \mathcal{O}\left(u^{17}\right) \ ,$$

$$\delta C_{7tb}^{H(2)}(\mu_W = m_t) \underset{r\to\infty}{\simeq} 8.088 + 9.757\frac{\ln r}{r} - 12.91\frac{1}{r} + 38.43\frac{\ln r}{r^2} - 49.32\frac{1}{r^2} + 106.2\frac{\ln r}{r^3} \tag{80}$$
$$-78.90\frac{1}{r^3} + 168.4\frac{\ln r}{r^4} - 24.97\frac{1}{r^4} + 101.1\frac{\ln r}{r^5} + 194.3\frac{1}{r^5} + \mathcal{O}\left(\frac{1}{r^6}\right) \ ,$$

$$\delta C_{8tt}^{H(2)}(\mu_W = m_t) \underset{r\to 0}{\simeq} 0.6908\, r\ln^2 r + 3.238\, r\ln r + 0.7437\, r \tag{81}$$
$$-22.98\, r^2\ln^3 r - 169.1\, r^2\ln^2 r - 602.7\, r^2\ln r - 805.5\, r^2$$
$$-66.32\, r^3\ln^3 r - 779.6\, r^3\ln^2 r - 3077\, r^3\ln r - 3357\, r^3$$
$$-143.4\, r^4\ln^3 r - 2244\, r^4\ln^2 r - 10102\, r^4\ln r - 9016\, r^4$$
$$-226.7\, r^5\ln^3 r - 5251\, r^5\ln^2 r - 26090\, r^5\ln r - 19606\, r^5 + \mathcal{O}\left(r^6\right) \ ,$$

$$\delta C_{8tt}^{H(2)}(\mu_W = m_t) \underset{r\to 1^-}{\simeq} 1.188 - 0.4078\,\bar{u} - 0.2076\,\bar{u}^2 - 0.1265\,\bar{u}^3 - 0.08570\,\bar{u}^4 \tag{82}$$
$$-0.06204\,\bar{u}^5 - 0.04689\,\bar{u}^6 - 0.03652\,\bar{u}^7 - 0.02907\,\bar{u}^8 - 0.02354\,\bar{u}^9$$
$$-0.01933\,\bar{u}^{10} - 0.01605\,\bar{u}^{11} - 0.01345\,\bar{u}^{12} - 0.01137\,\bar{u}^{13}$$
$$-0.009678\,\bar{u}^{14} - 0.008293\,\bar{u}^{15} - 0.007148\,\bar{u}^{16} + \mathcal{O}\left(\bar{u}^{17}\right) \ ,$$

$$\delta C_{8tt}^{H(2)}(\mu_W = m_t) \underset{r\to 1^+}{\simeq} 1.188 + 0.4078\, u + 0.2002\, u^2 + 0.1190\, u^3 + 0.07861\, u^4 \tag{83}$$
$$+0.05531\, u^5 + 0.04061\, u^6 + 0.03075\, u^7 + 0.02386\, u^8 + 0.01888\, u^9$$
$$+0.01520\, u^{10} + 0.01241\, u^{11} + 0.01026\, u^{12} + 0.008575\, u^{13}$$
$$+0.007238\, u^{14} + 0.006164\, u^{15} + 0.005290\, u^{16} + \mathcal{O}\left(u^{17}\right) \ ,$$

$$\delta C_{8tt}^{H(2)}(\mu_W = m_t) \underset{r\to\infty}{\simeq} 2.278 - 5.214\frac{1}{r} + 20.02\frac{\ln r}{r^2} - 39.76\frac{1}{r^2} + 78.58\frac{\ln r}{r^3} - 66.39\frac{1}{r^3} \tag{84}$$
$$+91.89\frac{\ln r}{r^4} + 96.35\frac{1}{r^4} - 300.7\frac{\ln r}{r^5} + 826.2\frac{1}{r^5} + \mathcal{O}\left(\frac{1}{r^6}\right) \ ,$$

$$\delta C_{8tb}^{H(2)}(\mu_W = m_t) \underset{r\to 0}{\simeq} -19.80\, r\ln^3 r - 174.7\, r\ln^2 r - 658.4\, r\ln r - 929.8\, r \tag{85}$$
$$-31.83\, r^2\ln^3 r - 612.6\, r^2\ln^2 r - 2770\, r^2\ln r - 2943\, r^2$$
$$-40.68\, r^3\ln^3 r - 1439\, r^3\ln^2 r - 7906\, r^3\ln r - 6481\, r^3$$
$$+54.66\, r^4\ln^3 r - 2777\, r^4\ln^2 r - 17770\, r^4\ln r - 11684\, r^4$$
$$+1003\, r^5\ln^3 r - 2627\, r^5\ln^2 r - 29962\, r^5\ln r - 15962\, r^5 + \mathcal{O}\left(r^6\right) \ ,$$

$$\delta C_{8tb}^{H(2)}(\mu_W = m_t) \underset{r\to 1^-}{\simeq} -0.6110 + 1.095\,\bar{u} + 0.6492\,\bar{u}^2 + 0.4596\,\bar{u}^3 + 0.3569\,\bar{u}^4 \\ +0.2910\,\bar{u}^5 + 0.2438\,\bar{u}^6 + 0.2075\,\bar{u}^7 + 0.1785\,\bar{u}^8 \\ +0.1546\,\bar{u}^9 + 0.1347\,\bar{u}^{10} + 0.1177\,\bar{u}^{11} + 0.1032\,\bar{u}^{12} \\ +0.09073\,\bar{u}^{13} + 0.07987\,\bar{u}^{14} + 0.07040\,\bar{u}^{15} + 0.06210\,\bar{u}^{16} + \mathcal{O}\left(\bar{u}^{17}\right)\;, \tag{86}$$

$$\delta C_{8tb}^{H(2)}(\mu_W = m_t) \underset{r\to 1^+}{\simeq} -0.6110 - 1.095\,u - 0.4463\,u^2 - 0.2568\,u^3 - 0.1698\,u^4 \\ -0.1197\,u^5 - 0.08761\,u^6 - 0.06595\,u^7 - 0.05079\,u^8 - 0.03987\,u^9 \\ -0.03182\,u^{10} - 0.02577\,u^{11} - 0.02114\,u^{12} - 0.01754\,u^{13} - 0.01471\,u^{14} \\ -0.01244\,u^{15} - 0.01062\,u^{16} + \mathcal{O}\left(u^{17}\right)\;, \tag{87}$$

$$\delta C_{8tb}^{H(2)}(\mu_W = m_t) \underset{r\to\infty}{\simeq} -3.174 + 10.89\frac{1}{r} - 35.42\frac{\ln r}{r^2} + 63.74\frac{1}{r^2} - 110.7\frac{\ln r}{r^3} \\ +62.26\frac{1}{r^3} - 71.62\frac{\ln r}{r^4} - 205.7\frac{1}{r^4} + 476.9\frac{\ln r}{r^5} - 1003\frac{1}{r^5} + \mathcal{O}\left(\frac{1}{r^6}\right)\;. \tag{88}$$

C.1.3 Supersymmetric contributions

At leading order, the chargino contributions to the Wilson coefficients are given by [31]:

$$\delta C_7^{\chi(0)}(\mu) = -\frac{1}{2}\,A_7^{\chi(0)}\;, \tag{89}$$

$$\delta C_8^{\chi(0)}(\mu) = -\frac{1}{2}\,F_8^{\chi(0)}\;, \tag{90}$$

with

$$A_7^{\chi(0)}(\mu) = \kappa\sum_{i=1}^{2}\sum_{a=1}^{6}\frac{M_W^2}{m_{\chi_i^\pm}^2}\times\left\{[X_i^{U_L\,\dagger}]_{2a}[X_i^{U_L}]_{a3}\;h_1^{(0)}(y_{ai}) + \frac{m_{\chi_i^\pm}}{m_b}[X_i^{U_L\,\dagger}]_{2a}[X_i^{U_R}]_{a3}\;h_2^{(0)}(y_{ai})\right\}\;, \tag{91}$$

$$F_8^{\chi(0)}(\mu) = \kappa\sum_{i=1}^{2}\sum_{a=1}^{6}\frac{M_W^2}{m_{\chi_i^\pm}^2}\times\left\{[X_i^{U_L\,\dagger}]_{2a}[X_i^{U_L}]_{a3}\;h_5^{(0)}(y_{ai}) + \frac{m_{\chi_i^\pm}}{m_b}[X_i^{U_L\,\dagger}]_{2a}[X_i^{U_R}]_{a3}\;h_6^{(0)}(y_{ai})\right\}\;, \tag{92}$$

where the h_i functions are given in section C.6, and

$$\kappa = \frac{1}{g_2^2 V_{tb}V_{ts}^*}\;, \qquad y_{ai} = \frac{m_{\tilde{u}_a}^2}{m_{\chi_i^\pm}^2}\;, \tag{93}$$

$$X_i^{U_L} \;=\; -g_2\left[a_g\, V_{i1}^*\,\Gamma^{U_L} - a_Y\, V_{i2}^*\,\Gamma^{U_R}\frac{M_U}{\sqrt{2}M_W\sin\beta}\right]V_{\rm CKM}\;, \tag{94}$$

$$X_i^{U_R} \;=\; g_2\, a_Y\, U_{i2}\Gamma^{U_L}\, V_{\rm CKM}\frac{M_D}{\sqrt{2}M_W\cos\beta}\;, \tag{95}$$

with $M_U = {\rm diag}(m_u, m_c, m_t)$, $M_D = {\rm diag}(m_d, m_s, m_b)$ and

$$a_g = 1 - \frac{\alpha_s(\mu_{\tilde g})}{4\pi}\left[\frac{7}{3} + 2\ln\left(\frac{\mu_{\tilde g}^2}{M_{\tilde g}^2}\right)\right]\;, \quad a_Y = 1 + \frac{\alpha_s(\mu_{\tilde g})}{4\pi}\left[1 + 2\ln\left(\frac{\mu_{\tilde g}^2}{M_{\tilde g}^2}\right)\right]\;. \tag{96}$$

In this framework, the mixing matrices Γ^{U_L} and Γ^{U_R} take the simple form

$$(\Gamma^{U_L})^T = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \cos\theta_{\tilde t} & 0 & 0 & -\sin\theta_{\tilde t} \end{pmatrix}, \quad (\Gamma^{U_R})^T = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \sin\theta_{\tilde t} & 0 & 0 & \cos\theta_{\tilde t} \end{pmatrix}. \tag{97}$$

One defines also Γ^U as

$$(\Gamma^U)_{ai} = (\Gamma^{U_L})_{ai}\;, \qquad (\Gamma^U)_{a(i+3)} = (\Gamma^{U_R})_{ai}\;, \tag{98}$$

and

$$P^U = \Gamma^U \mathbb{1}^{\rm LR}_{6\times6}\Gamma^{U\dagger}\;, \qquad \mathbb{1}^{\rm LR}_{6\times6} = {\rm diag}(1,1,1,-1,-1,-1)\;, \tag{99}$$

The leading $\tan\beta$ corrections are contained in the following expressions for ϵ_b, ϵ_b' and ϵ_0' [33], which are evaluated at a typical SUSY scale, μ_s. ϵ_b can be split into two parts:

$$\epsilon_b = \epsilon_0 + \epsilon_2\;, \tag{100}$$

with

$$\epsilon_0 \;=\; \frac{2\,\alpha_s(\mu_s)}{3\,\pi}\frac{A_b/\tan\beta - \mu}{m_{\tilde g}}H_2(x_{\tilde b_1 \tilde g}, x_{\tilde b_2 \tilde g}) \tag{101}$$
$$+\frac{\alpha(M_Z)\mu M_2}{4\pi s_W^2}\left[\frac{c_{\tilde b}^2}{2m_{\tilde b_1}^2}H_2\left(\frac{M_2^2}{m_{\tilde b_1}^2}, \frac{\mu^2}{m_{\tilde b_1}^2}\right) + \frac{s_{\tilde b}^2}{2m_{\tilde b_2}^2}H_2\left(\frac{M_2^2}{m_{\tilde b_2}^2}, \frac{\mu^2}{m_{\tilde b_2}^2}\right)\right]\;,$$

and

$$\epsilon_2 \;=\; \frac{\tilde y_t^2(\mu_s)}{16\,\pi^2}\sum_{i=1}^{2}U_{i2}\frac{\mu/\tan\beta - A_t}{m_{\chi_i^\pm}}\,H_2(x_{\tilde t_1\chi_i^\pm}, x_{\tilde t_2\chi_i^\pm})\,V_{i2} \tag{102}$$
$$+\frac{\alpha(M_Z)\mu M_2}{4\pi s_W^2}\left[\frac{c_{\tilde t}^2}{m_{\tilde t_1}^2}H_2\left(\frac{M_2^2}{m_{\tilde t_1}^2}, \frac{\mu^2}{m_{\tilde t_1}^2}\right) + \frac{s_{\tilde t}^2}{m_{\tilde t_2}^2}H_2\left(\frac{M_2^2}{m_{\tilde t_2}^2}, \frac{\mu^2}{m_{\tilde t_2}^2}\right)\right]\;,$$

where $s_W = \sin\theta_W$, $c_{\tilde q} = \cos\theta_{\tilde q}$, $s_{\tilde q} = \sin\theta_{\tilde q}$, $x_{ab} = m_a^2/m_b^2$, and A_q is the trilinear coupling of the quark q. y_q and $\tilde{y}_q$ are the ordinary and supersymmetric Yukawa couplings of the quark q respectively. The function H_2 is defined as:

$$H_2(x,y) = \frac{x \ln x}{(1-x)(x-y)} + \frac{y \ln y}{(1-y)(y-x)} \ . \tag{103}$$

We neglect the neutralino mixing matrices and we assume that the chargino masses are given by μ and M_2.

$$\begin{aligned}
\epsilon'_b(t) \;=\; & \frac{2\,\alpha_s(\mu_s)}{3\,\pi}\frac{A_b/\tan\beta - \mu}{m_{\tilde{g}}}\Big[c_{\tilde{t}}^2 c_{\tilde{b}}^2\, H_2(x_{\tilde{t}_1\tilde{g}}, x_{\tilde{b}_2\tilde{g}}) \qquad (104)\\
& + c_{\tilde{t}}^2 s_{\tilde{b}}^2\, H_2(x_{\tilde{t}_1\tilde{g}}, x_{\tilde{b}_1\tilde{g}}) + s_{\tilde{t}}^2 c_{\tilde{b}}^2\, H_2(x_{\tilde{t}_2\tilde{g}}, x_{\tilde{b}_2\tilde{g}}) + s_{\tilde{t}}^2 s_{\tilde{b}}^2\, H_2(x_{\tilde{t}_2\tilde{g}}, x_{\tilde{b}_1\tilde{g}})\Big]\\
& + \frac{y_t^2(\mu_s)}{16\,\pi^2}\sum_{i=1}^{n_{\chi^0}} N_{i4}^* \frac{A_t - \mu/\tan\beta}{m_{\chi_i^0}}\Big[c_{\tilde{t}}^2 c_{\tilde{b}}^2\, H_2(x_{\tilde{t}_2\chi_i^0}, x_{\tilde{b}_1\chi_i^0})\\
& + c_{\tilde{t}}^2 s_{\tilde{b}}^2\, H_2(x_{\tilde{t}_2\chi_i^0}, x_{\tilde{b}_2\chi_i^0}) + s_{\tilde{t}}^2 c_{\tilde{b}}^2\, H_2(x_{\tilde{t}_1\chi_i^0}, x_{\tilde{b}_1\chi_i^0}) + s_{\tilde{t}}^2 s_{\tilde{b}}^2\, H_2(x_{\tilde{t}_1\chi_i^0}, x_{\tilde{b}_2\chi_i^0})\Big] N_{i3}\\
& + \frac{\alpha(M_Z)\mu M_2}{4\pi s_W^2}\left[\frac{c_{\tilde{b}}^2}{m_{\tilde{b}_1}^2} H_2\left(\frac{M_2^2}{m_{\tilde{b}_1}^2}, \frac{\mu^2}{m_{\tilde{b}_1}^2}\right) + \frac{s_{\tilde{b}}^2}{m_{\tilde{b}_2}^2} H_2\left(\frac{M_2^2}{m_{\tilde{b}_2}^2}, \frac{\mu^2}{m_{\tilde{b}_2}^2}\right)\right.\\
& \left. + \frac{c_{\tilde{t}}^2}{2m_{\tilde{t}_1}^2} H_2\left(\frac{M_2^2}{m_{\tilde{t}_1}^2}, \frac{\mu^2}{m_{\tilde{t}_1}^2}\right) + \frac{s_{\tilde{t}}^2}{2m_{\tilde{t}_2}^2} H_2\left(\frac{M_2^2}{m_{\tilde{t}_2}^2}, \frac{\mu^2}{m_{\tilde{t}_2}^2}\right)\right] \ .
\end{aligned}$$

In the above equation, N represents the neutralino mixing matrix and n_{χ^0} the number of neutralinos, *i.e.* four in the MSSM and five in the NMSSM. The last ϵ correction reads:

$$\begin{aligned}
\epsilon'_0 \;=\; & -\frac{2\,\alpha_s(\mu_s)}{3\,\pi}\frac{\mu + A_t/\tan\beta}{m_{\tilde{g}}}\Big[c_{\tilde{t}}^2\, H_2(x_{\tilde{t}_2\tilde{g}}, x_{\tilde{s}\tilde{g}}) \ + \ s_{\tilde{t}}^2 H_2(x_{\tilde{t}_1\tilde{g}}, x_{\tilde{s}\tilde{g}})\Big] \qquad (105)\\
& + \frac{y_b^2(\mu_s)}{16\,\pi^2}\sum_{i=1}^{n_{\chi^0}} N_{i4}^* \frac{\mu/\tan\beta}{m_{\chi_i^0}}\ \Big[c_{\tilde{t}}^2 c_{\tilde{b}}^2\, H_2(x_{\tilde{t}_1\chi_i^0}, x_{\tilde{b}_2\chi_i^0}) + c_{\tilde{t}}^2 s_{\tilde{b}}^2\, H_2(x_{\tilde{t}_1\chi_i^0}, x_{\tilde{b}_1\chi_i^0})\\
& + s_{\tilde{t}}^2 c_{\tilde{b}}^2\, H_2(x_{\tilde{t}_2\chi_i^0}, x_{\tilde{b}_2\chi_i^0}) + s_{\tilde{t}}^2 s_{\tilde{b}}^2\, H_2(x_{\tilde{t}_2\chi_i^0}, x_{\tilde{b}_1\chi_i^0})\Big] N_{i3} \ .
\end{aligned}$$

The SM and charged Higgs contributions at the μ_W scale are affected by ϵ_b, ϵ'_b and ϵ'_0 as the following:

$$\delta C_{7,8}^{(SM,\tan\beta)}(\mu_W) \;=\; \frac{\big(\epsilon_b - \epsilon'_b(t)\big)\tan\beta}{1+\epsilon_b\tan\beta} F_{7,8}^{(2)}(x_{tW}) \ , \tag{106}$$

$$\delta C_{7,8}^{(H,\tan\beta)}(\mu_W) \;=\; -\frac{\big(\epsilon'_0 + \epsilon_b\big)\tan\beta}{1+\epsilon_b\tan\beta} F_{7,8}^{(2)}(x_{tH^\pm}) \ . \tag{107}$$

Higher order charged Higgs contributions are expressed as [34]:

$$\delta C_{7,8}^{(H,\tan^2\beta)}(\mu_W) = -\frac{\epsilon_2 \epsilon_1' \tan^2\beta}{(1+\epsilon_0\tan\beta)(1+\epsilon_b\tan\beta)} F_{7,8}^{(2)}(x_{tH^\pm}) \;, \tag{108}$$

where $F_{7,8}^{(2)}(x)$ are given in Eq. (55) and ϵ_1' reads [35]:

$$\epsilon_1' \;=\; \frac{1}{16\pi^2}\left[\frac{A_b y_b^2}{\mu} H_2\left(\frac{m_{\tilde{t}_L}^2}{\mu^2}, \frac{m_{\tilde{b}_R}^2}{\mu^2}\right) - g^2 \frac{M_2}{\mu} H_2\left(\frac{m_{\tilde{t}_L}^2}{\mu^2}, \frac{M_2^2}{\mu^2}\right)\right] \;. \tag{109}$$

Finally, we add the neutral Higgs contributions [36, 37]. In the MSSM, they are expressed as

$$\begin{aligned}\delta C_{7(8)}^{H^0(0)}(\mu_W) \;&=\; \frac{a_{7(8)}}{72} \frac{\epsilon_2}{\cos^2\beta(1+\epsilon_0\tan\beta)(1+\epsilon_b\tan\beta)^2} \\ &\times \sum_{S=h^0,H^0,A^0} \frac{\overline{m}_b^2}{M_S^2}(x_u^{S*} - x_d^{S*}\tan\beta)(x_d^S + x_u^S \epsilon_b) \;,\end{aligned} \tag{110}$$

where $a_7 = 1$, $a_8 = -3$ and, for $S = (h^0, H^0, A^0)$,

$$x_d^S = (-\sin\alpha, \cos\alpha, i\sin\beta) \;, \qquad x_u^S = (\cos\alpha, \sin\alpha, -i\cos\beta) \;. \tag{111}$$

In the NMSSM, they are generalized to

$$\begin{aligned}\delta C_{7(8)}^{H^0(0)}(\mu_W) \;&=\; \frac{a_{7(8)}}{72} \frac{\epsilon_2}{\cos^2\beta(1+\epsilon_0\tan\beta)(1+\epsilon_b\tan\beta)^2} \\ &\times \sum_{i=1}^{3} \frac{\overline{m}_b^2}{m_i^2}(U_{i1}^{H*} + U_{i2}^{H*}\tan\beta)(U_{i2}^H + U_{i1}^H \epsilon_b) \;,\end{aligned} \tag{112}$$

$$\begin{aligned}\delta C_{7(8)}^{A^0(0)}(\mu_W) \;&=\; \frac{a_{7(8)}}{72} \frac{\epsilon_2}{\cos^2\beta(1+\epsilon_0\tan\beta)(1+\epsilon_b\tan\beta)^2} \\ &\times \sum_{j=1}^{2} \frac{\overline{m}_b^2}{m_j^2}(U_{j1}^{A*} + U_{j2}^{A*}\tan\beta)(U_{j2}^A + U_{j1}^A \epsilon_b) \;,\end{aligned} \tag{113}$$

where $i = (h^0, H^0, H_3^0)$ and $j = (A_1^0, A_2^0)$.

The CP-even and CP-odd Higgs mixing matrices are respectively [38]

$$U^H = \begin{pmatrix} (\cos\theta_H - \sin\theta_H v\delta_+/s)/\tan\beta & \cos\theta_H & -\sin\theta_H \\ (\sin\theta_H + \cos\theta_H v\delta_+/s) & \sin\theta_H & \cos\theta_H \\ 1 & -1/\tan\beta & -v\delta_+/s\tan\beta \end{pmatrix} \;, \tag{114}$$

and

$$U^A = \left(\begin{array}{ccc} \cos\theta_A \sin\beta & \cos\theta_A \cos\beta & \sin\theta_A \\ -\sin\theta_A \sin\beta & -\sin\theta_A \cos\beta & \cos\theta_A \\ -\cos\beta & \sin\beta & 0 \end{array} \right) . \tag{115}$$

where θ_H and θ_A are the mixing angles, s is the scalar VEV, $v \approx 246$ GeV and

$$\delta_\pm = \frac{\sqrt{2}A_\lambda \pm 2\kappa s}{\sqrt{2}A_\lambda + \kappa s} . \tag{116}$$

At the NLO, the chargino contributions to the Wilson coefficients can be written in the form [31]:

$$\begin{aligned} \delta C_4^{\chi(1)}(\mu_W) &= E_4^{\chi(1)} , & (117)\\ \delta C_7^{\chi(1)}(\mu_W) &= -\frac{1}{2} A_7^{\chi(1)} , & (118)\\ \delta C_8^{\chi(1)}(\mu_W) &= -\frac{1}{2} F_8^{\chi(1)} , & (119) \end{aligned}$$

with

$$E_4^{\chi(1)}(\mu_W) = \kappa \sum_{i=1}^{2} \sum_{a=1}^{6} \frac{M_W^2}{m_{\chi_i^\pm}^2} [X_i^{U_L\dagger}]_{2a} [X_i^{U_L}]_{a3}\, h_4^{(0)}(y_{ai}) , \tag{120}$$

$$\begin{aligned} A_7^{\chi(1)}(\mu_W) = \kappa \sum_{i=1}^{2} \sum_{a=1}^{6} \frac{M_W^2}{m_{\chi_i^\pm}^2} \Bigg\{ & [X_i^{U_L\dagger}]_{2a} [X_i^{U_L}]_{a3}\, h_1^{(1)}(y_{ai}, L_{\tilde{u}_a}) \\ & + \frac{m_{\chi_i^\pm}}{m_b} [X_i^{U_L\dagger}]_{2a} [X_i^{U_R}]_{a3}\, h_2^{(1)}(y_{ai}, L_{\tilde{u}_a}) \Bigg\} , \end{aligned} \tag{121}$$

$$\begin{aligned} F_8^{\chi(1)}(\mu_W) = \kappa \sum_{i=1}^{2} \sum_{a=1}^{6} \frac{M_W^2}{m_{\chi_i^\pm}^2} \Bigg\{ & [X_i^{U_L\dagger}]_{2a} [X_i^{U_L}]_{a3}\, h_5^{(1)}(y_{ai}, L_{\tilde{u}_a}) \\ & + \frac{m_{\chi_i^\pm}}{m_b} [X_i^{U_L\dagger}]_{2a} [X_i^{U_R}]_{a3}\, h_6^{(1)}(y_{ai}, L_{\tilde{u}_a}) \Bigg\} , \end{aligned} \tag{122}$$

and

$$L_{\tilde{u}_a} = \ln\left(\frac{\mu_W^2}{m_{\tilde{u}_a}^2} \right) , \tag{123}$$

The quartic chargino-up squark contributions are given by [31]:

$$\begin{aligned} \delta C_7^{4(1)}(\mu_W) &= -\frac{1}{2} A_7^{4(1)} , & (124)\\ \delta C_8^{4(1)}(\mu_W) &= -\frac{1}{2} F_8^{4(1)} , & (125) \end{aligned}$$

with

$$A_7^{4(1)}(\mu_W) = \kappa \sum_{i=1}^{2} \sum_{a,b,c=1}^{6} \frac{M_W^2}{m^2_{\chi_i^\pm}} \, P^U_{ab} \, y_{bi} \, P^U_{bc} \, (1+L_{\tilde{u}_b}) \tag{126}$$

$$\times \Bigg\{ [X_i^{U_L\dagger}]_{2a}[X_i^{U_L}]_{c3} \Big[-q_1^{(1)}(y_{ai},y_{ci}) + \frac{2}{3} q_2^{(1)}(y_{ai},y_{ci}) \Big]$$

$$+ \frac{m_{\chi_i^\pm}}{m_b} [X_i^{U_L\dagger}]_{2a}[X_i^{U_R}]_{c3} \Big[-q_3^{(1)}(y_{ai},y_{ci}) + \frac{2}{3} q_4^{(1)}(y_{ai},y_{ci}) \Big] \Bigg\} ,$$

$$F_8^{4(1)}(\mu_W) = \kappa \sum_{i=1}^{2} \sum_{a,b,c=1}^{6} \frac{M_W^2}{m^2_{\chi_i^\pm}} \, P^U_{ab} \, y_{bi} \, P^U_{bc} \, (1+L_{\tilde{u}_b}) \tag{127}$$

$$\times \Bigg\{ [X_i^{U_L\dagger}]_{2a}[X_i^{U_L}]_{c3} \; q_2^{(1)}(y_{ai},y_{ci}) + \frac{m_{\chi_i^\pm}}{m_b} [X_i^{U_L\dagger}]_{2a}[X_i^{U_R}]_{c3} \; q_4^{(1)}(y_{ai},y_{ci}) \Bigg\} ,$$

where P^U is given in Eq. (99) and the q_i functions are given in Appendix C.6.

At the NNLO, the following contributions are considered [31]:

$$\delta C_3^{\chi(2)}(\mu_W) = G_3^{\chi(2)} , \tag{128}$$

$$\delta C_4^{\chi(2)}(\mu_W) = E_4^{\chi(2)} , \tag{129}$$

$$\delta C_5^{\chi(2)}(\mu_W) = -\frac{1}{10} \, G_3^{\chi(2)} + \frac{2}{15} \, E_4^{\chi(1)} , \tag{130}$$

$$\delta C_6^{\chi(2)}(\mu_W) = -\frac{3}{16} \, G_3^{\chi(2)} + \frac{1}{4} \, E_4^{\chi(1)} , \tag{131}$$

and

$$\delta C_4^{4(2)}(\mu_W) = E_4^{4(2)} , \tag{132}$$

where

$$E_4^{\chi(2)} = \kappa \sum_{i=1}^{2} \sum_{a=1}^{6} \frac{M_W^2}{m^2_{\chi_i^\pm}} [X_i^{U_L\dagger}]_{2a}[X_i^{U_L}]_{a3} \; h_4^{(1)}(y_{ai}, L_{\tilde{u}_a}) , \tag{133}$$

$$G_3^{\chi(2)} = \kappa \sum_{i=1}^{2} \sum_{a=1}^{6} \frac{M_W^2}{m^2_{\chi_i^\pm}} [X_i^{U_L\dagger}]_{2a}[X_i^{U_L}]_{a3} \; h_7^{(1)}(y_{ai}, L_{\tilde{u}_a}) , \tag{134}$$

$$E_4^{4(2)} = \kappa \sum_{i=1}^{2} \sum_{a,b,c=1}^{6} \frac{M_W^2}{m^2_{\chi_i^\pm}} \, P^U_{ab} \, y_{bi} \, P^U_{bc} \, (1+L_{\tilde{u}_b}) \, [X_i^{U_L\dagger}]_{2a}[X_i^{U_L}]_{c3} \; q_6^{(1)}(y_{ai},y_{ci}) , \tag{135}$$

and $E_4^{\chi(1)}$ is given in Eq. (120).
In the charm sector, the NNLO contribution from the squarks is embedded in:

$$\begin{aligned}\delta C_1^{\tilde{q}(2)} &= -\sum_{a=1}^{6}\sum_{q=u,d}\left\{2(4x_{\tilde{q}_a}-1)^{\frac{3}{2}}\ \mathrm{Cl}_2\left(2\arcsin\frac{1}{2\sqrt{x_{\tilde{q}_a}}}\right)\right. \\ &\left. -8\left(x_{\tilde{q}_a}-\frac{1}{3}\right)\ln x_{\tilde{q}_a}-16x_{\tilde{q}_a}\right\}-\frac{208}{3}\ , \end{aligned} \tag{136}$$

where $x_{\tilde{q}_a}=\dfrac{m^2_{\tilde{q}_a}}{M_W^2}$ and Cl_2 is given in Eq. (39).

The complete Wilson coefficients $C_i^{(n)}$ at a given order are obtained by adding the different contributions given in this appendix.

C.2 Wilson coefficients $C_9 - C_{10}$

The effective Hamiltonian describing the $b\to s\,\ell^+\ell^-$ transitions has the following generic structure [39]:

$$\mathcal{H}_{\rm eff}=-\frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^*\left(\sum_{i=1}^{10}C_i(\mu)O_i(\mu)+\sum_{i=1}^{10}C_{Q_i}(\mu)Q_i(\mu)\right)\ . \tag{137}$$

where $O_1 - O_8$ are given in Eq. (25) and

$$O_9 = \frac{e^2}{(4\pi)^2}(\bar{s}\gamma^\mu b_L)(\bar{l}\gamma_\mu l)\ , \tag{138}$$

$$O_{10} = \frac{e^2}{(4\pi)^2}(\bar{s}\gamma^\mu b_L)(\bar{l}\gamma_\mu\gamma_5 l)\ , \tag{139}$$

and the relevant Q_i's are given in the next section.

C.2.1 Standard Model contributions

The one and two loop SM Wilson coefficients are given below [28]:

$$C_9^{c(0)} = -\frac{1}{4s_W^2}-\frac{38}{27}+\frac{4}{9}L\ , \tag{140}$$

$$C_{10}^{c(0)} = \frac{1}{4s_W^2}\ , \tag{141}$$

$$C_9^{t(0)} = \frac{1-4s_W^2}{s_W^2}\ \mathcal{C}^{t(0)}(x_{tW})-\frac{1}{s_W^2}\ \mathcal{B}^{t(0)}(x_{tW})-\mathcal{D}^{t(0)}(x_{tW})\ , \tag{142}$$

$$C_{10}^{t(0)} = \frac{1}{s_W^2}\left[\mathcal{B}^{t(0)}(x_{tW})-\mathcal{C}^{t(0)}(x_{tW})\right]\ , \tag{143}$$

$$C_9^{c(1)} = -\frac{1}{s_W^2} - \frac{524}{729} + \frac{128}{243}\,\pi^2 + \frac{16}{3}\,L + \frac{128}{81}\,L^2\,, \tag{144}$$

$$C_{10}^{c(1)} = \frac{1}{s_W^2}\,, \tag{145}$$

$$C_9^{t(1)} = \frac{1-4s_W^2}{s_W^2}\,\mathcal{C}^{t(1)}(x_{tW}) - \frac{1}{s_W^2}\,\mathcal{B}^{t(1)}(x_{tW},-\frac{1}{2}) - \mathcal{D}^{t(1)}(x_{tW})\,, \tag{146}$$

$$C_{10}^{t(1)} = \frac{1}{s_W^2}\Big[\mathcal{B}^{t(1)}(x_{tW},-\frac{1}{2}) - \mathcal{C}^{t(1)}(x_{tW})\Big]\,, \tag{147}$$

where x_{tW} and L are given in Eqs. (29) and (30), and $s_W = \sin\theta_W$. The involved Green functions for the W-boson box ($\mathcal{B}^{t,c}$) and Z-boson penguin ($\mathcal{C}^{t,c}$) diagrams are given by:

$$\mathcal{B}^{t(0)}(x) = \frac{x}{4(1-x)^2}\ln x + \frac{1}{4(1-x)}\,, \tag{148}$$

$$\mathcal{C}^{t(0)}(x) = \frac{3x^2+2x}{8(1-x)^2}\ln x + \frac{-x^2+6x}{8(1-x)}\,, \tag{149}$$

$$\mathcal{D}^{t(0)}(x) = \frac{-3x^4+30x^3-54x^2+32x-8}{18(1-x)^4}\ln x + \frac{-47x^3+237x^2-312x+104}{108(1-x)^3}\,, \tag{150}$$

and

$$\begin{aligned}\mathcal{B}^{t(1)}(x,-\frac{1}{2}) &= \frac{-2x}{(1-x)^2}\,\mathrm{Li}_2\left(1-\frac{1}{x}\right) + \frac{-x^2+17x}{3(1-x)^3}\ln x \\ &\quad + \frac{13x+3}{3(1-x)^2} + \left[\frac{2x^2+2x}{(1-x)^3}\ln x + \frac{4x}{(1-x)^2}\right]\ln\frac{\mu_W^2}{m_t^2}\,,\end{aligned} \tag{151}$$

$$\begin{aligned}\mathcal{C}^{t(1)}(x) &= \frac{-x^3-4x}{(1-x)^2}\,\mathrm{Li}_2\left(1-\frac{1}{x}\right) + \frac{3x^3+14x^2+23x}{3(1-x)^3}\ln x \\ &\quad + \frac{4x^3+7x^2+29x}{3(1-x)^2} + \left[\frac{8x^2+2x}{(1-x)^3}\ln x + \frac{x^3+x^2+8x}{(1-x)^2}\right]\ln\frac{\mu_W^2}{m_t^2}\,,\end{aligned} \tag{152}$$

$$\begin{aligned}\mathcal{D}^{t(1)}(x) &= \frac{380x^4-1352x^3+1656x^2-784x+256}{81(1-x)^4}\,\mathrm{Li}_2\left(1-\frac{1}{x}\right) \\ &\quad + \frac{304x^4+1716x^3-4644x^2+2768x-720}{81(1-x)^5}\ln x \\ &\quad + \frac{-6175x^4+41608x^3-66723x^2+33106x-7000}{729(1-x)^4} \\ &\quad + \left[\frac{648x^4-720x^3-232x^2-160x+32}{81(1-x)^5}\ln x \right. \\ &\quad \left. + \frac{-352x^4+4912x^3-8280x^2+3304x-880}{243(1-x)^4}\right]\ln\frac{\mu_W^2}{m_t^2}\,.\end{aligned} \tag{153}$$

The three loop QCD corrections to the C_{10} SM Wilson coefficient are given below [40]

$$C_{10}^{(2)}(\mu_W) = -\frac{2}{s_W^2}\Big[C_{10}^{W,t,(2)}(x_{tW},\mu_W)+C_{10}^{Z,t,(2)}(x_{tW},\mu_W)+C_{10}^{Z,t,\mathrm{tri}}(x_{tW}) -C_{10}^{W,c,(2)}(x_{tW},\mu_W)-C_{10}^{Z,c,\mathrm{tri}}(x_{tW})\Big] , \tag{154}$$

where

$$C_{10}^{W,t,(2)}(x,\mu_W) = C_{10}^{W,t,(2)}(\mu_W=m_t)+\ln\left(\frac{\mu_W^2}{m_t^2}\right)\left[\frac{69+1292x-209x^2}{18(x-1)^3} -\frac{521x+105x^2-50x^3}{9(x-1)^4}\ln x-\frac{47x+x^2}{3(x-1)^3}\,\mathrm{Li}_2\left(1-\frac{1}{x}\right)\right] +\ln^2\left(\frac{\mu_W^2}{m_t^2}\right)\left[\frac{61x+11x^2}{3(x-1)^3}-\frac{49x+96x^2-x^3}{6(x-1)^4}\ln x\right] , \tag{155}$$

$$C_{10}^{W,c,(2)}(x,\mu_W) = C_{10}^{W,c,(2)}(\mu_W=M_W)-\frac{23}{6}\ln\left(\frac{\mu_W^2}{M_W^2}\right) ,$$

$$C_{10}^{Z,t,(2)}(x,\mu_W) = C_{10}^{Z,t,(2)}(x,\mu_W=m_t)+\ln\left(\frac{\mu_W^2}{m_t^2}\right)\left[\frac{188x+4x^2+95x^3-47x^4}{6(x-1)^3}\,\mathrm{Li}_2\left(1-\frac{1}{x}\right) + \frac{1468x+1578x^2-25x^3-141x^4}{18(x-1)^4}\ln x-\frac{4622x+1031x^2+582x^3-475x^4}{36(x-1)^3}\right] + \ln^2\left(\frac{\mu_W^2}{m_t^2}\right)\left[\frac{49x+315x^2-4x^3}{6(x-1)^4}\ln x-\frac{440x+257x^2+72x^3-49x^4}{12(x-1)^3}\right] , \tag{156}$$

For the three-loop quantities $C_{10}^{W,t,(2)}(\mu_W=m_t)$, $C_{10}^{W,c,(2)}(\mu_W=M_W)$ and $C_{10}^{Z,t,(2)}(\mu_W=m_t)$, we have access to their expansions at $x\to 1$ and $x\to\infty$. Denoting $z=1/x$, $y=\sqrt{z}$ and $w=1-z$, the coefficients become:

$$C_{10}^{W,t,(2)}(\mu_W=m_t) \underset{x\to\infty}{\simeq} 2.710\,y^2+6.010\,y^2\ln y-8.156\,y^4-1.131\,y^4\ln y -0.5394\,y^6-13.97\,y^6\ln y+35.32\,y^8+15.64\,y^8\ln y+103.9\,y^{10} +149.2\,y^{10}\ln y+207.7\,y^{12}+454.8\,y^{12}\ln y+\mathcal{O}\left(y^{14}\right) , \tag{157}$$

$$C_{10}^{W,t,(2)}(\mu_W=m_t) \underset{x\to 1}{\simeq} -0.4495-0.5845\,w+0.1330\,w^2+0.1563\,w^3+0.1233\,w^4 +0.09333\,w^5+0.07134\,w^6+0.05561\,w^7+0.04425\,w^8 +0.03589\,w^9+0.02960\,w^{10}+0.02478\,w^{11}+0.02102\,w^{12} +0.01803\,w^{13}+0.01562\,w^{14}+0.01366\,w^{15}+0.01204\,w^{16}+\mathcal{O}\left(w^{17}\right) , \tag{158}$$

$$C_{10}^{W,c,(2)}(\mu_W = M_W) \underset{x\to\infty}{\simeq} -5.222 - 0.2215\,y^2 + 0.1244\,y^2\ln y - 0.08889\,y^2\ln^2 y \tag{159}$$
$$+0.04146\,y^4 - 0.02955\,y^4\ln y + 0.009524\,y^4\ln^2 y - 0.001092\,y^6$$
$$+0.0006349\,y^6\ln y - 0.00004286\,y^8 + 0.00003207\,y^8\ln y$$
$$-3.109\cdot 10^{-6}\,y^{10} + 2.643\cdot 10^{-6}\,y^{10}\ln y - 3.009\cdot 10^{-7}\,y^{12}$$
$$+2.775\cdot 10^{-7}\,y^{12}\ln y + \mathcal{O}\left(y^{14}\right)\ ,$$
$$C_{10}^{W,c,(2)}(\mu_W = M_W) \underset{x\to 1}{\simeq} -5.403 + 0.09422\,w + 0.02786\,w^2 + 0.01355\,w^3 + 0.008129\,w^4 \tag{160}$$
$$+0.005469\,w^5 + 0.003957\,w^6 + 0.003009\,w^7 + 0.002373\,w^8$$
$$+0.001925\,w^9 + 0.001596\,w^{10} + 0.001346\,w^{11} + 0.001153\,w^{12}$$
$$+0.0009996\,w^{13} + 0.0008757\,w^{14} + 0.0007742\,w^{15}$$
$$+0.0006898\,w^{16} + \mathcal{O}\left(w^{17}\right)\ .$$

$$C_{10}^{Z,t,(2)}(\mu_W = m_t) \underset{x\to\infty}{\simeq} \frac{0.1897}{y^2} + 2.139 + 28.59\,y^2 + 33.85\,y^2\ln y + 28.01\,y^4 \tag{161}$$
$$+97.98\,y^4\ln y - 31.41\,y^6 + 106.2\,y^6\ln y - 167.0\,y^8 - 78.59\,y^8\ln y$$
$$-387.4\,y^{10} - 618.3\,y^{10}\ln y - 697.9\,y^{12} - 1688.\,y^{12}\ln y + \mathcal{O}\left(y^{14}\right)\ ,$$
$$C_{10}^{Z,t,(2)}(\mu_W = m_t) \underset{x\to 1}{\simeq} -1.934 + 0.8966\,w + 0.7399\,w^2 + 0.6058\,w^3 + 0.5113\,w^4 \tag{162}$$
$$+0.4439\,w^5 + 0.3948\,w^6 + 0.3582\,w^7 + 0.3303\,w^8 + 0.3087\,w^9$$
$$+0.2916\,w^{10} + 0.2778\,w^{11} + 0.2667\,w^{12} + 0.2575\,w^{13} + 0.2498\,w^{14}$$
$$+0.2433\,w^{15} + 0.2379\,w^{16} + \mathcal{O}\left(w^{17}\right)\ .$$

Similarly, the fermion triangle contributions are expanded as

$$C_{10}^{Z,t,\mathrm{tri}} \underset{x\to\infty}{\simeq} -\frac{0.9871}{y^2} - 2.388 - 1.627\,y^2 - 3.516\,y^2\ln y - 1.830\,y^4 - 6.959\,y^4\ln y \tag{163}$$
$$-2.038\,y^6 - 10.83\,y^6\ln y - 2.210\,y^8 - 15.09\,y^8\ln y - 2.353\,y^{10} - 19.65\,y^{10}\ln y$$
$$-2.473\,y^{12} - 24.48\,y^{12}\ln y + \mathcal{O}\left(y^{14}\right)\ ,$$
$$C_{10}^{Z,t,\mathrm{tri}} \underset{x\to 1}{\simeq} -2.418 - 1.334\,w - 1.147\,w^2 - 1.080\,w^3 - 1.048\,w^4 - 1.030\,w^5 \tag{164}$$
$$-1.019\,w^6 - 1.012\,w^7 - 1.007\,w^8 - 1.003\,w^9 - 1.001\,w^{10} - 0.9984\,w^{11}$$
$$-0.9968\,w^{12} - 0.9955\,w^{13} - 0.9944\,w^{14} - 0.9936\,w^{15} - 0.9928\,w^{16} + \mathcal{O}\left(w^{17}\right)\ ,$$

$$
\begin{aligned}
C_{10}^{Z,c,\mathrm{tri}} \underset{x\to\infty}{\simeq} & -1.250 + 1.500 \ln y - 0.5331\, y^2 + 0.2778\, y^2 \ln y - 0.2222\, y^2 \ln^2 y \\
& +0.1144\, y^4 - 0.08194\, y^4 \ln y + 0.02778\, y^4 \ln^2 y - 0.003538\, y^6 + 0.002143\, y^6 \ln y \\
& -0.0001573\, y^8 + 0.0001235\, y^8 \ln y - 0.00001283\, y^{10} + 0.00001145\, y^{10} \ln y \\
& -1.383 \cdot 10^{-6}\, y^{12} + 1.338 \cdot 10^{-6}\, y^{12} \ln y + \mathcal{O}\left(y^{14}\right) \ ,
\end{aligned} \tag{165}
$$

$$
\begin{aligned}
C_{10}^{Z,c,\mathrm{tri}} \underset{x\to 1}{\simeq} & -1.672 - 0.5336\, w - 0.3100\, w^2 - 0.2181\, w^3 - 0.1683\, w^4 - 0.1370\, w^5 \\
& -0.1156\, w^6 - 0.09997\, w^7 - 0.08808\, w^8 - 0.07873\, w^9 - 0.07118\, w^{10} \\
& -0.06495\, w^{11} - 0.05973\, w^{12} - 0.05529\, w^{13} - 0.05147\, w^{14} - 0.04814\, w^{15} \\
& -0.04522\, w^{16} + \mathcal{O}\left(w^{17}\right) \ .
\end{aligned} \tag{166}
$$

C.2.2 Charged Higgs contributions

The charged Higgs contributions to the Wilson coefficients are written in the form [31]:

$$
\delta C_9^{H(0,1)} = \frac{1-4s_W^2}{s_W^2}\, \mathcal{C}^{H(0,1)}(x_{tH^\pm}) - \mathcal{D}^{H(0,1)}(x_{tH^\pm}) \ , \tag{167}
$$

$$
\delta C_{10}^{H(0,1)} = -\frac{1}{s_W^2}\, \mathcal{C}^{H(0,1)}(x_{tH^\pm}) \ , \tag{168}
$$

with $x_{tH^\pm} = \dfrac{m_t^2}{M_{H^\pm}^2}$.

The involved Green functions are given in the following:

$$\mathcal{C}^{H(0)}(x) = \frac{M_{H^\pm}^2}{8M_W^2}\lambda_{tt}^2\, x^2 \left\{ \frac{-1}{(x-1)^2}\ln x + \frac{1}{x-1} \right\} , \tag{169}$$

$$\mathcal{D}^{H(0)}(x) = \frac{1}{18}\lambda_{tt}^2\, x \left\{ \frac{-3x^3+6x-4}{(x-1)^4}\ln x + \frac{47x^2-79x+38}{6(x-1)^3} \right\} , \tag{170}$$

$$\begin{aligned}\mathcal{C}^{H(1)}(x) &= \frac{M_{H^\pm}^2}{8M_W^2}\lambda_{tt}^2\, x^2 \left\{ \frac{-8x+16}{(x-1)^2}\,\mathrm{Li}_2\left(1-\frac{1}{x}\right) + \frac{-24x+88}{3(x-1)^3}\,\ln x + \frac{32x-96}{3(x-1)^2} \right. \\ &\left. + \left[\frac{16}{(x-1)^3}\,\ln x + \frac{8x-24}{(x-1)^2}\right]\ln\frac{\mu_W^2}{M_{H^\pm}^2} \right\} ,\end{aligned} \tag{171}$$

$$\begin{aligned}\mathcal{D}^{H(1)}(x) &= \frac{1}{81}\lambda_{tt}^2\, x \left\{ \frac{380x^3-528x^2+72x+128}{(x-1)^4}\,\mathrm{Li}_2\left(1-\frac{1}{x}\right) \right. \\ &+ \frac{596x^3-672x^2+64x+204}{(x-1)^5}\ln x + \frac{-6175x^3+9138x^2-3927x-764}{9(x-1)^4} \\ &\left. + \left[\frac{432x^3-456x^2+40x+128}{(x-1)^5}\ln x + \frac{-352x^3-972x^2+1944x-1052}{3(x-1)^4}\right]\ln\frac{\mu_W^2}{M_{H^\pm}^2} \right\} .\end{aligned} \tag{172}$$

C.2.3 Supersymmetric contributions

The chargino contributions to the Wilson coefficients read [31].

$$\delta C_9^{\chi(0,1)} = \frac{1-4s_W^2}{s_W^2}\,\mathcal{C}^{\chi(0,1)} - \frac{1}{s_W^2}\,\mathcal{B}_9^{\chi(0,1)} - \mathcal{D}^{\chi(0,1)} , \tag{173}$$

$$\delta C_{10}^{\chi(0,1)} = \frac{1}{s_W^2}\left[\mathcal{B}_{10}^{\chi(0,1)} - \mathcal{C}^{\chi(0,1)}\right] . \tag{174}$$

The contributing chargino-up squark loop Green functions are:

$$\begin{aligned}\mathcal{B}_{9,10}^{\chi(0)} &= \mp\kappa\frac{M_W^2}{2g_2^2}\sum_{i,j=1}^{2}\sum_{a=1}^{6}\sum_{b=1}^{3}\frac{[X_j^{U_L\dagger}]_{2a}[X_i^{U_L}]_{a3}}{m_{\chi_i^\pm}^2} \\ &\times\left\{\frac{1}{2}[X_i^{N_L\dagger}]_{lb}[X_j^{N_L}]_{bl}f_5^{(0)}(x_{ji},y_{ai},v_{bi}) \mp [X_i^{N_R\dagger}]_{lb}[X_j^{N_R}]_{bl}\sqrt{x_{ji}}f_6^{(0)}(x_{ji},y_{ai},v_{bi})\right\} ,\end{aligned} \tag{175}$$

$$\mathcal{C}^{\chi(0)} = -\frac{\kappa}{8}\sum_{i,j=1}^{2}\sum_{a,b=1}^{6}[X_j^{U_L\dagger}]_{2b}[X_i^{U_L}]_{a3} \tag{176}$$

$$\times\left\{2\sqrt{x_{ji}}\, f_3^{(0)}(x_{ji},y_{ai})U_{j1}U_{i1}^*\delta_{ab} - f_4^{(0)}(x_{ji},y_{ai})V_{j1}^*V_{i1}\delta_{ab} + f_4^{(0)}(y_{ai},y_{bi})(\Gamma^{U_L}\Gamma^{U_L\dagger})_{ba}\delta_{ij}\right\},$$

$$\mathcal{D}^{\chi(0)} = \kappa\sum_{i=1}^{2}\sum_{a=1}^{6}\frac{M_W^2}{m_{\chi_i^\pm}^2}[X_i^{U_L\dagger}]_{2a}[X_i^{U_L}]_{a3}\, h_3^{(0)}(y_{ai}), \tag{177}$$

$$\mathcal{B}_{9,10}^{\chi(1)} = \mp\kappa\frac{M_W^2}{2g_2^2}\sum_{i,j=1}^{2}\sum_{a=1}^{6}\sum_{b=1}^{3}\frac{[X_j^{U_L\dagger}]_{2a}[X_i^{U_L}]_{a3}}{m_{\chi_i^\pm}^2} \tag{178}$$

$$\times\left\{\frac{1}{2}[X_i^{N_L\dagger}]_{lb}[X_j^{N_L}]_{bl}\left[f_8^{(1)}(x_{ji},y_{ai},v_{bi}) + 4\left(1+y_{ai}\frac{\partial}{\partial y_{ai}}\right)f_5^{(0)}(x_{ji},y_{ai},v_{bi})\, L_{\tilde{u}_a}\right]\right.$$

$$\left.\mp[X_i^{N_R\dagger}]_{lb}[X_j^{N_R}]_{bl}\sqrt{x_{ji}}\left[f_9^{(1)}(x_{ji},y_{ai},v_{bi}) + 4\left(1+y_{ai}\frac{\partial}{\partial y_{ai}}\right)f_6^{(0)}(x_{ji},y_{ai},v_{bi})\, L_{\tilde{u}_a}\right]\right\},$$

$$\mathcal{C}^{\chi(1)} = -\frac{\kappa}{8}\sum_{i,j=1}^{2}\sum_{a,b=1}^{6}[X_j^{U_L\dagger}]_{2b}[X_i^{U_L}]_{a3} \tag{179}$$

$$\times\left\{2\sqrt{x_{ji}}\left[f_3^{(1)}(x_{ji},y_{ai}) + 4\left(1+y_{ai}\frac{\partial}{\partial y_{ai}}\right)f_3^{(0)}(x_{ji},y_{ai})L_{\tilde{u}_a}\right]U_{j1}U_{i1}^*\delta_{ab}\right.$$

$$-\left[f_4^{(1)}(x_{ji},y_{ai}) + 4\left(1+y_{ai}\frac{\partial}{\partial y_{ai}}\right)f_4^{(0)}(x_{ji},y_{ai})L_{\tilde{u}_a}\right]V_{j1}^*V_{i1}\delta_{ab}$$

$$\left.+\left[f_5^{(1)}(y_{ai},y_{bi}) + 4\left(1+y_{ai}\frac{\partial}{\partial y_{ai}}+y_{bi}\frac{\partial}{\partial y_{bi}}\right)f_4^{(0)}(y_{ai},y_{bi})L_{\tilde{u}_a}\right](\Gamma^{U_L}\Gamma^{U_L\dagger})_{ba}\delta_{ij}\right\},$$

$$\mathcal{D}^{\chi(1)} = \kappa\sum_{i=1}^{2}\sum_{a=1}^{6}\frac{M_W^2}{m_{\chi_i^\pm}^2}[X_i^{U_L\dagger}]_{2a}[X_i^{U_L}]_{a3}\, h_3^{(1)}(y_{ai},L_{\tilde{u}_a}), \tag{180}$$

where $C^{\chi(0)}$ and $C^{\chi(1)}$ are taken from [44].

The chargino-up squark contributions containing the quartic squark vertex are

$$\mathcal{B}_{9,10}^{4(1)} = \pm\frac{\kappa}{2g_2^2}\frac{4}{3}\sum_{i,j=1}^{2}\sum_{f=1}^{3}\sum_{a,b,c=1}^{6}\frac{M_W^2}{m_{\chi_i^\pm}^2}\,P_{ab}^U\,y_{bi}\,P_{bc}^U\,(1+L_{\tilde{u}_b})\,[X_j^{U_L\dagger}]_{2a}[X_i^{U_L}]_{c3} \tag{181}$$
$$\times\Bigg\{\frac{1}{2}f_9^{(0)}(x_{ji},y_{ai},y_{ci},v_{fi})[X_i^{N_L\dagger}]_{lf}[X_j^{N_L}]_{fl}$$
$$\mp\sqrt{x_{ji}}f_{10}^{(0)}(x_{ji},y_{ai},y_{ci},v_{fi})[X_i^{N_R\dagger}]_{lf}[X_j^{N_R}]_{fl}\Bigg\}\,,$$

$$\mathcal{C}^{4(1)} = \frac{\kappa}{6}\sum_{i,j=1}^{2}\sum_{a,\ldots,e,g,k=1}^{6}P_{gk}^U\,y_{ki}\,P_{ke}^U\,(1+L_{\tilde{u}_k})\,[X_j^{U_L\dagger}]_{2d}[X_i^{U_L}]_{a3} \tag{182}$$
$$\times\Bigg\{2\sqrt{x_{ji}}f_6^{(0)}(x_{ji},y_{ai},y_{di})U_{j1}U_{i1}^*\delta_{ae}\delta_{gd}\delta_{b1}\delta_{c1}$$
$$-f_5^{(0)}(x_{ji},y_{ai},y_{di})V_{j1}^*V_{i1}\delta_{ae}\delta_{gd}\delta_{b1}\delta_{c1}$$
$$+f_5^{(0)}(y_{ai},y_{bi},y_{ci})(\Gamma^{U_L}\Gamma^{U_L\dagger})_{cb}\delta_{ij}\delta_{ae}\delta_{bg}\delta_{cd}$$
$$+f_5^{(0)}(y_{ai},y_{ci},y_{di})(\Gamma^{U_L}\Gamma^{U_L\dagger})_{cb}\delta_{ij}\delta_{ab}\delta_{ce}\delta_{dg}\Bigg\}\,,$$

$$\mathcal{D}^{4(1)} = \kappa\sum_{i=1}^{2}\sum_{a,b,c=1}^{6}\frac{M_W^2}{m_{\chi_i^\pm}^2}\,P_{ab}^U\,y_{bi}\,P_{bc}^U\,(1+L_{\tilde{u}_b})\,[X_i^{U_L\dagger}]_{2a}[X_i^{U_L}]_{c3}\;q_5^{(1)}(y_{ai},y_{ci})\,. \tag{183}$$

In the above equations:

$$\kappa = \frac{1}{g_2^2V_{tb}V_{ts}^*}\,,\qquad L_{\tilde{u}_a} = \ln\frac{\mu_W^2}{m_{\tilde{u}_a}^2}\,, \tag{184}$$

$$x_{ij} = \frac{m_{\chi_i^\pm}^2}{m_{\chi_j^\pm}^2}\,,\qquad y_{ai} = \frac{m_{\tilde{u}_a}^2}{m_{\chi_i^\pm}^2}\,,\qquad v_{fi} = \frac{m_{\tilde{\nu}_f}^2}{m_{\chi_i^\pm}^2}\,, \tag{185}$$

and P^U is given in Eq. (99), and the auxiliary functions f_i, h_i and q_i are given in section C.6. $X_i^{U_L}$ and $X_i^{U_R}$ are defined in Eq. (94), and

$$X_i^{N_L} = -g_2\,V_{i1}^*\,\Gamma^N\,,\qquad X_i^{N_R} = g_2\,U_{i2}\,\Gamma^N\frac{M_E}{\sqrt{2}M_W\cos\beta}\,, \tag{186}$$

where Γ^N is the sneutrino mixing matrix and $M_E = \mathrm{diag}(m_e,m_\mu,m_\tau)$.

The NMSSM contributions to $C_{9,10}$ are the same as in the MSSM.

C.3 Wilson coefficients $C_{Q_1} - C_{Q_2}$

The scalar and pseudoscalar operators Q_1 and Q_2 read [45]:

$$Q_1 = \frac{e^2}{(4\pi)^2}(\bar{s}^\alpha_L b^\alpha_R)(\bar{l}\,l) \;, \tag{187}$$

$$Q_2 = \frac{e^2}{(4\pi)^2}(\bar{s}^\alpha_L b^\alpha_R)(\bar{l}\gamma_5 l) \;, \tag{188}$$

The used convention here is slightly different from the one with O_S and O_P operators in [44]. However the Wilson coefficients in the two bases are related by the simple relation:

$$C_{Q_1,Q_2} = m_b C_{S,P} \,. \tag{189}$$

There is no SM contribution to C_{Q_1} and C_{Q_2}. In the following we give the 2HDM, MSSM and NMSSM contributions separately.

C.3.1 2HDM contributions

We extend the results of [46] to general Yukawa couplings. The most important contributions (for large Yukawa couplings) are given in the following.

The first contribution to C_{Q_1} and C_{Q_2} is from box diagrams involving H^+ and W^+:

$$C^a_{Q_1} = -C^a_{Q_2} = -\frac{m_\mu}{4M_W^2 s_W^2}\left(m_b\lambda_{bb} + m_s\lambda_{ss} - 2m_t\lambda_{tt}\right)\lambda_{\mu\mu}\, B_+(x_{H^\pm W}, x_{tW}) \;, \tag{190}$$

where $x_{H^\pm W} = M^2_{H^\pm}/M^2_W$, $x_{tW} = \overline{m}_t^2/M_W^2$, and

$$B_+(x,y) = \frac{y}{x-y}\left(\frac{\ln y}{y-1} - \frac{\ln x}{x-1}\right) \;. \tag{191}$$

The second contribution comes from penguin diagrams mediated by neutral Higgs bosons with H^+ and W^+ in the loop:

$$\begin{aligned} C^b_{Q_1} &= -\frac{m_\mu}{4s_W^2}\left(\frac{\sin^2\alpha}{M^2_{h^0}} + \frac{\cos^2\alpha}{M^2_{H^0}}\right)\left(m_b\lambda_{bb} + m_s\lambda_{ss} - 2m_t\lambda_{tt}\right)\lambda_{\mu\mu}\, P_+(x_{H^\pm W}, x_{tW}) \;, \\ C^b_{Q_2} &= \frac{m_\mu}{4M^2_{A_0} s_W^2}\left(m_b\lambda_{bb} + m_s\lambda_{ss} - 2m_t\lambda_{tt}\right)\lambda_{\mu\mu}\, P_+(x_{H^\pm W}, x_{tW}) \;. \end{aligned} \tag{192}$$

where α is the Higgs mixing angle and the loop function is given by:

$$P_+(x,y) = \frac{y}{x-y}\left(\frac{x\,\ln x}{x-1} - \frac{y\,\ln y}{y-1}\right) \;. \tag{193}$$

The third contribution originates from penguin diagrams mediated by neutral Higgs bosons, involving H^+ and G^+ in the loops:

$$\begin{aligned} C^c_{Q_1} &= \frac{m_\mu}{4s_W^2}\left[\frac{\sin^2\alpha}{M_{h^0}^2}\frac{(M_{H^+}^2-M_{h^0}^2)}{M_W^2}+\frac{\cos^2\alpha}{M_{H^0}^2}\frac{(M_{H^+}^2-M_{H^0}^2)}{M_W^2}\right] \\ &\times (m_b\lambda_{bb}+m_s\lambda_{ss}-2m_t\lambda_{tt})\,\lambda_{\mu\mu}P_+(x_{H^\pm W},x_{tW})\;, \qquad (194) \\ C^c_{Q_2} &= -\frac{m_\mu}{4M_{A^0}^2 s_W^2}\left(\frac{M_{H^+}^2-M_{A^0}^2}{M_W^2}\right)(m_b\lambda_{bb}+m_s\lambda_{ss}-2m_t\lambda_{tt})\,\lambda_{\mu\mu}P_+(x_{H^\pm W},x_{tW})\;. \end{aligned}$$

The last contribution is from self-energy diagrams:

$$\begin{aligned} C^d_{Q_1} &= -\frac{m_\mu}{4s_W^2}\left(\frac{\sin^2\alpha}{M_{h^0}^2}+\frac{\cos^2\alpha}{M_{H^0}^2}\right)(m_b\lambda_{bb}+m_s\lambda_{ss})\,\lambda_{\mu\mu} \\ &\times\left[x_{H^\pm W}+\left(\lambda_{bb}+\frac{m_s}{m_b}\lambda_{ss}\right)\lambda_{tt}\right]\;P_+(x_{H^\pm W},x_{tW})\;, \qquad (195) \end{aligned}$$

$$C^d_{Q_2} = \frac{m_\mu}{4M_{A^0}^2 s_W^2}\;(m_b\lambda_{bb}+m_s\lambda_{ss})\,\lambda_{\mu\mu}\;\left[x_{H^\pm W}+\left(\lambda_{bb}-\frac{m_s}{m_b}\lambda_{ss}\right)\lambda_{tt}\right]\;P_+(x_{H^\pm W},x_{tW})\;.$$

The Yukawa couplings λ_{ii} are given in Table 4 for the four types of 2HDM Yukawa sectors. Adding the four contributions, the total 2HDM contribution to the Wilson coefficients can be obtained.

C.3.2 MSSM contributions

The Wilson coefficients can be written as [44]:

$$C^{(0,1)}_{Q_1,Q_2} = \frac{1}{(1+\epsilon_0\tan\beta)(1+\epsilon_b\tan\beta)}\sum_{J=H,\tilde{\chi},4}\left[\mathcal{N}^{J(0,1)}_{Q_1,Q_2}+\mathcal{B}^{J(0,1)}_{Q_1,Q_2}\right]\;. \qquad (196)$$

The charged Higgs contributions can be split into two parts:

Box-diagram contributions:

$$\begin{aligned} \mathcal{B}^{H(0)}_{Q_1,Q_2} &= \pm\frac{m_l m_b\tan^2\beta}{4M_W^2 s_W^2}f_7^{(0)}(x_{tW},x_{H^\pm W})\;, \qquad (197) \\ \mathcal{B}^{H(1)}_{Q_1,Q_2} &= \pm\frac{m_l m_b\tan^2\beta}{4M_W^2 s_W^2}\left[f_{11}^{(1)}(x_{tW},x_{H^\pm W})+8x_{tW}\frac{\partial}{\partial x_{tW}}f_7^{(0)}(x_{tW},x_{H^\pm W})\ln\frac{\mu_W^2}{M_{H^\pm}^2}\right]\;, \end{aligned}$$

Neutral Higgs boson penguin diagrams:

$$\begin{aligned} \mathcal{N}^{H(0)}_{Q_1,Q_2} &= \mp\frac{m_l m_b\tan^2\beta}{4M_W^2 s_W^2}xf_3^{(0)}(x_{tW},x_{H^\pm W})\;, \qquad (198) \\ \mathcal{N}^{H(1)}_{Q_1,Q_2} &= \mp\frac{m_l m_b\tan^2\beta}{4M_W^2 s_W^2}\left\{f_{14}^{(1)}(x_{tW},x_{H^\pm W})+8x_{tW}\frac{\partial}{\partial x_{tW}}\left[x_{tW}f_3^{(0)}(x_{tW},x_{H^\pm W})\right]\ln\frac{\mu_W^2}{M_{H^\pm}^2}\right\}\;. \end{aligned}$$

The SUSY contributions are also split into Box-diagram contributions and neutral Higgs boson penguin diagrams, and are provided in the following.

Box-diagram contributions:

$$\begin{aligned}
\mathcal{B}^{\chi(0)}_{Q_1,Q_2} &= \pm\frac{\kappa M_W^2}{2g_2^2 s_W^2}\sum_{i,j=1}^{2}\sum_{f=1}^{3}\sum_{a=1}^{6}\frac{[X_j^{U_L\dagger}]_{2a}[X_i^{U_R}]_{a3}}{m^2_{\chi_i^\pm}} \\
&\times \left[f_5^{(0)}(x_{ji},y_{ai},v_{fi})[X_i^{N_R\dagger}]_{lf}[X_j^{N_L}]_{fl} \pm \sqrt{x_{ji}}f_6^{(0)}(x_{ji},y_{ai},v_{fi})[X_i^{N_L\dagger}]_{lf}[X_j^{N_R}]_{fl}\right],
\end{aligned} \tag{199}$$

$$\begin{aligned}
\mathcal{B}^{\chi(1)}_{Q_1,Q_2} &= \pm\frac{\kappa M_W^2}{2g_2^2 s_W^2}\sum_{i,j=1}^{2}\sum_{f=1}^{3}\sum_{a=1}^{6}\frac{[X_j^{U_L\dagger}]_{2a}[X_i^{U_R}]_{a3}}{m^2_{\chi_i^\pm}} \\
&\times \Bigg\{\left[f_{12}^{(1)}(x_{ji},y_{ai},v_{fi}) + 4y_{ai}\frac{\partial}{\partial y_{ai}}f_5^{(0)}(x_{ji},y_{ai},v_{fi})L_{\tilde{u}_a}\right][X_i^{N_R\dagger}]_{lf}[X_j^{N_L}]_{fl} \\
&\pm\sqrt{x_{ji}}\left[f_{13}^{(1)}(x_{ji},y_{ai},v_{fi}) + 4y_{ai}\frac{\partial}{\partial y_{ai}}f_6^{(0)}(x_{ji},y_{ai},v_{fi})L_{\tilde{u}_a}\right][X_i^{N_L\dagger}]_{lf}[X_j^{N_R}]_{fl}\Bigg\}.
\end{aligned} \tag{200}$$

$X_i^{U_L}$ and $X_i^{U_R}$ are given in Eq. (94), $X_i^{N_L}$ and $X_i^{N_R}$ in Eq. (186), κ, $L_{\tilde{u}_a}$, x_{ij}, y_{ai}, v_{fi} in Eqs. (767) and (185), P^U in Eq. (99), and the f_i functions are given in section C.6.

The contributions from the quartic couplings read:

$$\begin{aligned}
\mathcal{B}^{4(1)}_{Q_1,Q_2} &= \mp\frac{2\kappa M_W^2}{3g_2^2 s_W^2}\sum_{i,j=1}^{2}\sum_{f=1}^{3}\sum_{a,b,c=1}^{6}\frac{[X_j^{U_L\dagger}]_{2b}[X_i^{U_R}]_{a3}}{m^2_{\chi_i^\pm}}\left[P^U_{ac}\,y_{ci}\,P^U_{cb}(1+L_{\tilde{u}_c})\right] \\
&\times\left\{f_9^{(0)}(x_{ji},y_{ai},y_{bi},v_{fi})[X_i^{N_R\dagger}]_{lf}[X_j^{N_L}]_{fl} \pm \sqrt{x_{ji}}f_{10}^{(0)}(x_{ji},y_{ai},y_{bi},v_{fi})[X_i^{N_L\dagger}]_{lf}[X_j^{N_R}]_{fl}\right\}.
\end{aligned} \tag{201}$$

Neutral Higgs boson penguin diagrams:

$$\begin{aligned}
\mathcal{N}^{\chi(0)}_{Q_1,Q_2} &= \pm\frac{m_l m_b\tan^2\beta}{M_W s_W^2 M^2_{A^0}}\sum_{i,j=1}^{2}\sum_{a,b=1}^{6}\sum_{m,n=1}^{3}\Gamma^a_{imn}(\Gamma^{U_L})_{bm}U_{j2}a_Y \\
&\times\left[a^{(0)}_{0,Q_1,Q_2} + a_1^{(0)}\tan\beta\right],
\end{aligned} \tag{202}$$

$$\begin{aligned}
\mathcal{N}^{\chi(1)}_{Q_1,Q_2} &= \pm\frac{m_l m_b\tan^2\beta}{M_W s_W^2 M^2_{A^0}}\sum_{i,j=1}^{2}\sum_{a,b=1}^{6}\sum_{m,n=1}^{3}\Gamma^a_{imn}(\Gamma^{U_L})_{bm}U_{j2}a_Y \\
&\times\left[a^{(1)}_{0,Q_1,Q_2} + a_1^{(1)}\tan\beta + a_2\, m_s^2\tan^2\beta\right],
\end{aligned} \tag{203}$$

with

$$\Gamma^a_{imn} \equiv \frac{1}{2\sqrt{2}}\Big[\sqrt{2}M_W V_{i1}(\Gamma^{U_L\dagger})_{na}a_g - (M_U)_{nn}V_{i2}(\Gamma^{U_R\dagger})_{na}a_Y\Big]\lambda_{mn} \ , \tag{204}$$

$$\lambda_{mn} \equiv \frac{V_{mb}V^*_{nq}}{V_{tb}V^*_{tq}} \ . \tag{205}$$

The coefficients a_{0,Q_1,Q_2}, a_1, a_2 are given by

$$\begin{aligned} a^{(0)}_{0,Q_1,Q_2} &= \mp\Big[\sqrt{x_{ij}}f^{(0)}_3(x_{ij},y_{aj})U_{i2}V_{j1} \pm f^{(0)}_4(x_{ij},y_{aj})U^*_{j2}V^*_{i1}\Big]\delta_{ab} \\ &\quad + \frac{(\Delta^\pm_i)_{ab}}{M_W}f^{(0)}_3(y_{ai},y_{bi})\,\delta_{ij} \ , \end{aligned} \tag{206}$$

$$\begin{aligned} a^{(1)}_{0,Q_1,Q_2} &= \mp\Bigg\{\sqrt{x_{ij}}\Big[f^{(1)}_{18}(x_{ij},y_{aj}) + 4y_{ai}\frac{\partial}{\partial y_{ai}}f^{(0)}_3(x_{ij},y_{aj})L_{\tilde{u}_a}\Big]U_{i2}V_{j1} \\ &\quad \pm\Big[f^{(1)}_{19}(x_{ij},y_{aj}) + 4y_{ai}\frac{\partial}{\partial y_{ai}}f^{(0)}_4(x_{ij},y_{aj})L_{\tilde{u}_a}\Big]U^*_{j2}V^*_{i1}\Bigg\}\delta_{ab} \\ &\quad + \frac{(\Delta^\pm_i)_{ab}}{M_W}\left[f^{(1)}_{17}(y_{ai},y_{bi}) + 4\Big(1 + y_{ai}\frac{\partial}{\partial y_{ai}} + y_{bi}\frac{\partial}{\partial y_{bi}}\Big)f^{(0)}_3(y_{ai},y_{bi})L_{\tilde{u}_a}\right]\delta_{ij} \\ &\quad + \frac{4\Gamma^{a*}_{imn}}{M_W\,(\Gamma^{U_L})_{bm}\,\lambda^*_{mn}U_{j2}}f^{(1)}_{15}(y_{ai})\delta_{ij}\delta_{ab}\delta_{mn} \ , \end{aligned} \tag{207}$$

$$a^{(0)}_1 = \frac{m_{\chi^\pm_i}}{\sqrt{2}M_W}f^{(0)}_8(y_{ai})\,\delta_{ij}\delta_{ab} \ , \tag{208}$$

$$a^{(1)}_1 = \frac{m_{\chi^\pm_i}}{\sqrt{2}M_W}\Big[f^{(1)}_{16}(y_{ai}) + 4y_{ai}\frac{\partial}{\partial y_{ai}}f^{(0)}_8(y_{ai})L_{\tilde{u}_a}\Big]\delta_{ij}\delta_{ab} \ , \tag{209}$$

$$a_2 = \frac{(\Gamma^{U_L\dagger})_{mb}\lambda_{mn}U^*_{j2}}{2M_W\Gamma^a_{imn}}f^{(1)}_{15}(y_{ai})\delta_{ij}\delta_{ab}\delta_{mn} \ , \tag{210}$$

with

$$(\Delta^\pm_i)_{ab} \equiv \sum_{f=1}^{3}\frac{(M_U)_f}{\sqrt{2}\,m_{\chi^\pm_i}}\Big[\mu^*(\Gamma^{U_R})_{af}(\Gamma^{U_L\dagger})_{fb} \pm \mu(\Gamma^{U_L})_{af}(\Gamma^{U_R\dagger})_{fb}\Big] \ . \tag{211}$$

The contributions from the quartic couplings read:

$$\mathcal{N}^4_{Q_1,Q_2} = \mp\frac{4m_l m_b \tan^2\beta}{3M_W^2 s_W^2 M_{A^0}^2}\sum_{i,j=1}^{2}\sum_{m,n=1}^{3}\sum_{a,\ldots,e,g,k=1}^{6}\Gamma^a_{imn}(\Gamma^{U_L})_{dm}U_{j2}a_Y \tag{212}$$

$$\times\Bigg\{P^U_{ek}\,y_{kj}\,P^U_{kg}(1+L_{\tilde{u}_k})\Bigg\{\tan\beta\frac{m_{\chi_i^\pm}}{\sqrt{2}}f_3^{(0)}(y_{ai},y_{di})\delta_{ij}\delta_{ae}\delta_{gd}$$

$$+(\Delta_i^\pm)_{bc}\Big[\delta_{ae}\delta_{gb}\delta_{cd}f_6^{(0)}(y_{ai},y_{bi},y_{ci})+\delta_{ab}\delta_{ce}\delta_{gd}f_6^{(0)}(y_{ai},y_{ci},y_{di})\Big]\delta_{ij}$$

$$\mp M_W\Big[\sqrt{x_{ij}}f_6^{(0)}(x_{ij},y_{aj},y_{dj})U_{i2}V_{j1}\pm f_5^{(0)}(x_{ij},y_{aj},y_{dj})U^*_{j2}V^*_{i1}\Big]\delta_{ae}\delta_{dg}\Bigg\}$$

$$-P^U_{ae}[1+L_{\tilde{u}_g}-f_{11}^{(0)}(y_{ej},y_{gj})]P^U_{gd}\,(\Delta_i^\pm)_{eg}\delta_{ij}f_3^{(0)}(y_{ai},y_{di})\Bigg\}\,.$$

The additional auxiliary functions are given in section C.6.

C.3.3 NMSSM contributions

In this section, we give the NMSSM specific contributions to C_{Q_1} and C_{Q_2} following [47]. The NMSSM results cannot explicitly reduce to the MSSM results by simply dropping out the singlet.

For a light A_1 ($M_{A_1} \ll M_W$), a new operator is introduced since in this case A_1 becomes an active field:

$$O_A = i\frac{g_2}{16\pi^2}\,m_b M_W\,\bar{s}^\alpha_L b^\alpha_R A_1\,, \tag{213}$$

associated to the Wilson coefficient C_A, which is not changed by changing the scale from μ_W to M_{A_1}. At the M_{A_1} scale, C_{Q_2} receives a contribution from C_A:

$$\delta C_{Q_2}(M_{A_1}) = -\frac{\delta_-}{2}\frac{v}{s}\frac{m_b m_l}{m_{A_1}^2 s_W^2}C_A(M_{A_1})\,, \tag{214}$$

where δ_- is given in Eq. (116). For a very light A_1 ($M_{A_1} < m_b$), the effects of O_A are represented by a change in C_{Q_2} at the m_b scale:

$$\delta C_{Q_2}(m_b) = \frac{\delta_-}{2}\frac{v}{s}\frac{1}{s_W^2}\frac{m_b m_l}{\left(p^2-M_{A_1}^2+i m_{A_1}\Gamma_{A_1}\right)}C_A(m_b)\,, \tag{215}$$

where p is the momentum transfer and Γ_{A_1} is the total width of A_1. Since A_1 can be on-shell in this case, the effects of A_1 can be sizeable even without $\tan\beta$ enhancement.

The charged Higgs contributions read:

$$\delta C_A^H = -\frac{i\lambda A_\lambda}{g_2 M_W}\tan\beta f_3^{(0)}(x_{H^\pm t}, x_{Wt}) \,, \tag{216}$$

$$\delta C_{Q_1}^H = -\sum_{a=1}^{3}\frac{m_b m_l}{4M_{H_a}^2 s_W^2}\tan^2\beta\Bigg[\frac{M_{H^\pm}^2}{M_W^2}U_{a1}^H U_{a1}^H f_3^{(0)}(x_{H^\pm t}, x_{Wt}) + \frac{m_t^2 M_{H_a}^2}{M_W^2 M_{H^\pm}^2} f_3^{(0)}(x_{tH^\pm}, x_{tW})\Bigg] \,, \tag{217}$$

$$\delta C_{Q_2}^H = \sum_{\alpha=1}^{2}\frac{m_b m_l}{4M_{A_\alpha}^2 s_W^2}\tan^2\beta\Bigg[\left(\frac{M_{H^\pm}^2}{M_W^2}U_{\alpha 1}^A U_{\alpha 1}^A + \delta_{\alpha 2}U_{\alpha 1}^A\right) f_3^{(0)}(x_{H^\pm t}, x_{Wt}) + \frac{m_t^2 M_{A_\alpha}^2}{M_W^2 M_{H^\pm}^2} f_3^{(0)}(x_{tH^\pm}, x_{tW})\Bigg] \,. \tag{218}$$

The chargino contributions can be written as:

$$\delta C_A^\chi = i\frac{\tan\beta}{\sqrt{2}}\sum_{i=1}^{3}\sum_{j,l=1}^{2}\Gamma_1(i,i,j,l)\Bigg\{\delta_- \delta_{lj}\frac{v}{s}\sqrt{x_{\chi_j^\pm W}} f_8^{(0)}\left(x_{\tilde{t}_{i-1}\chi_j^\pm}\right) - \Bigg[R_{1jl}\sqrt{x_{\chi_j^\pm \chi_l^\pm}} f_3^{(0)}\left(x_{\tilde{t}_{i-1}\chi_l^\pm}, x_{\chi_j^\pm \chi_l^\pm}\right) - R_{1lj}^* f_4^{(0)}\left(x_{\tilde{t}_{i-1}\chi_l^\pm}, x_{\chi_j^\pm \chi_l^\pm}\right)\Bigg]\Bigg\}, \tag{219}$$

$$\begin{aligned} C_{Q_1}^\chi = {} & \sum_{a=1}^{3}\frac{m_b m_l}{4M_{H_a}^2 s_W^2}\tan^2\beta\sum_{i,k=1}^{3}\sum_{j,l=1}^{2}\Gamma_1(i,k,j,l)\Bigg\{\frac{\sqrt{2}U_{a1}^H U_{a1}^H m_{\chi_j^\pm}}{M_W\cos\beta}\delta_{ik}\delta_{lj} f_8^{(0)}(x_{\tilde{t}_{i-1}\chi_j^\pm}) \\ & - \frac{2\sqrt{2}U_{a1}^H}{g_2}\delta_{ik}\Bigg[Q_{alj}^* f_4^{(0)}(x_{\tilde{t}_{i-1}\chi_l^\pm}, x_{\chi_j^\pm\chi_l^\pm}) + \frac{m_{\chi_j^\pm}}{m_{\chi_l^\pm}}Q_{ajl} f_3^{(0)}(x_{\tilde{t}_{i-1}\chi_l^\pm}, x_{\chi_j^\pm\chi_l^\pm})\Bigg] \\ & + \frac{2\sqrt{2}U_{a1}^H T_2^{aik} m_{\chi_j^\pm}}{m_{\tilde{t}_{k-1}}^2}\delta_{lj} f_3^{(0)}(x_{\tilde{t}_{i-1}\tilde{t}_{k-1}}, x_{\chi_j^\pm \tilde{t}_{k-1}}) \\ & + \frac{M_{H_a}^2}{m_{\chi_j^\pm}^2}\delta_{ik}\Bigg[U^{2j}V^{1l} f_5^{(0)}(x_{\tilde{t}_{i-1}\chi_j^\pm}, x_{\chi_l^\pm\chi_j^\pm}, x_{\tilde{\nu}\chi_l^\pm}) \\ & - \frac{m_{\chi_l^\pm}}{m_{\chi_j^\pm}}U^{2l^*}V^{1j^*} f_6^{(0)}(x_{\tilde{t}_{i-1}\chi_j^\pm}, x_{\chi_l^\pm\chi_j^\pm}, x_{\tilde{\nu}\chi_l^\pm})\Bigg]\Bigg\} \,, \end{aligned} \tag{220}$$

$$
\begin{aligned}
C^{\chi}_{Q_2} \;=\; & -\sum_{\alpha=1}^{2}\frac{m_b m_l}{4M^2_{A_\alpha}s^2_W}\tan^2\beta\sum_{i,k=1}^{3}\sum_{j,l=1}^{2}\Gamma_1(i,k,j,l)\Bigg\{\frac{\sqrt{2}U^A_{\alpha 1}U^A_{\alpha 1}m_{\chi^\pm_j}}{M_W\cos\beta}\delta_{ik}\delta_{lj}f^{(0)}_8(x_{\tilde{t}_{i-1}\chi^\pm_j}) \\
& -\frac{2\sqrt{2}U^A_{\alpha 1}}{g_2}\delta_{ik}\left[-R^*_{alj}f^{(0)}_4(x_{\tilde{t}_{i-1}\chi^\pm_l},x_{\chi^\pm_j\chi^\pm_l})+\frac{m_{\chi^\pm_j}}{m_{\chi^\pm_l}}R_{ajl}f^{(0)}_3(x_{\tilde{t}_{i-1}\chi^\pm_l},x_{\chi^\pm_j\chi^\pm_l})\right] \\
& -\frac{\sqrt{2}U^A_{\alpha 1}T^{\alpha ik}_1 m_t m_{\chi^\pm_j}}{M_W m^2_{\tilde{t}_{k-1}}}\delta_{lj}f^{(0)}_3(x_{\tilde{t}_{i-1}\tilde{t}_{k-1}},x_{\chi^\pm_j\tilde{t}_{k-1}}) \qquad (221) \\
& +\frac{M^2_{A_\alpha}}{m^2_{\chi^\pm_j}}\delta_{ik}\Bigg[U^{2j}V^{1l}f^{(0)}_5(x_{\tilde{t}_{i-1}\chi^\pm_j},x_{\chi^\pm_l\chi^\pm_j},x_{\tilde{\nu}\chi^\pm_l}) \\
& \qquad -\frac{m_{\chi^\pm_l}}{m_{\chi^\pm_j}}U^{2l^*}V^{1j^*}f^{(0)}_6(x_{\tilde{t}_{i-1}\chi^\pm_j},x_{\chi^\pm_l\chi^\pm_j},x_{\tilde{\nu}\chi^\pm_l})\Bigg]\Bigg\}\,,
\end{aligned}
$$

with

$$
R_{\alpha lj} \;=\; -\frac{g_2}{\sqrt{2}}\left(U^A_{\alpha 1}U^{2l}V^{1j}+U^A_{\alpha 2}U^{1l}V^{2j}\right)-\frac{\lambda}{\sqrt{2}}U^A_{\alpha 3}U^{2l}V^{2j}\,, \qquad (222)
$$

$$
Q_{alj} \;=\; \frac{g_2}{\sqrt{2}}\left(U^H_{a1}U^{2l}V^{1j}+U^H_{a2}U^{1l}V^{2j}\right)-\frac{\lambda}{\sqrt{2}}U^H_{a3}U^{2l}V^{2j}\,, \qquad (223)
$$

$$
\Gamma_1(i,k,j,l) \;=\; \left(T^{i2}_U T^{k2^*}_U-\delta_{i1}\delta_{k1}\right)V^{1l^*}U^{2j^*}-\frac{m_t}{\sqrt{2}M_W\sin\beta}T^{i3}_U T^{k2^*}_U V^{2l^*}U^{2j^*}\,, \qquad (224)
$$

$$
T^{\alpha ik}_1 \;=\; A_3\,T^{i3^*}_U T^{k2}_U - A^*_3 T^{i2^*}_U T^{k3}_U\,, \qquad (225)
$$

$$
\begin{aligned}
T^{aik}_2 \;=\; & A_5\left(T^{i1^*}_U T^{k1}_U+T^{i2^*}_U T^{k2}_U\right)+A_6 T^{i3^*}_U T^{k3}_U \qquad (226) \\
& -\frac{m_t}{2M_W}\left[2m_t U^H_{a2}\left(T^{i2^*}_U T^{k2}_U+T^{i3^*}_U T^{k3}_U\right)+\left(A_4 T^{i3^*}_U T^{k2}_U+A^*_4 T^{i2^*}_U T^{k3}_U\right)\right]\,,
\end{aligned}
$$

and

$$
A_3 \;=\; \frac{\lambda}{\sqrt{2}}\left(v_d U^A_{\alpha 3}+sU^A_{\alpha 1}\right)-A_U U^A_{\alpha 2}\,, \qquad (227)
$$

$$
A_4 \;=\; \frac{\lambda}{\sqrt{2}}\left(v_d U^H_{a3}+sU^H_{a1}\right)+A_U U^H_{a2}\,, \qquad (228)
$$

$$
A_5 \;=\; \frac{M_Z}{2\cos\theta_W}\left(1-\frac{4}{3}\sin^2\theta_W\right)U^H_{a2}\,, \qquad (229)
$$

$$
A_6 \;=\; \frac{2}{3}M_W\tan^2\theta_W U^H_{a2}\,. \qquad (230)
$$

In the above equations, T_U is the stop mixing matrix, U^H and U^A are respectively the CP-even and CP-odd Higgs mixing matrices, U and V are the chargino mixing matrices, and $m_{\tilde{t}_0} \equiv m_{\tilde{u}_R}$. The f_i functions are given in section C.6.

C.4 Prime Wilson coefficients

In the following, we use the results obtained in [44]. However, the conventions are changed as follows:

$$C'_{Q_1,Q_2} = m_s C'_{S,P} \tag{231}$$

to be consistent with the previous sections.

C.4.1 Prime Wilson coefficients $C'_{7,8}$

The SM contributions are:

$$C_7^{\prime SM}(\mu_W) = \frac{m_s}{m_b}\left(-\frac{1}{2}A_0^t(x_{tW}) - \frac{23}{36}\right) , \tag{232}$$

$$C_8^{\prime SM}(\mu_W) = \frac{m_s}{m_b}\left(-\frac{1}{2}F_0^t(x_{tW}) - \frac{1}{3}\right) . \tag{233}$$

where

$$x_{tW^\pm} = \frac{\overline{m}_t^2(\mu_W)}{M_W^2} , \tag{234}$$

and A_0^t and F_0^t are given in Eqs. (31) and (33).

The charged Higgs contributions are:

$$\delta C_{7,8}^{\prime H}(\mu_W) = \frac{1}{3}\frac{m_s m_b}{\overline{m}_t^2(\mu_W)} \tan^2\beta \; F_{7,8}^{(1)}(x_{tH^\pm}) , \tag{235}$$

where

$$x_{tH^\pm} = \frac{\overline{m}_t^2(\mu_W)}{M_{H^\pm}^2} , \tag{236}$$

and $F_{7,8}^{(1)}$ is given in Eq. (55).

The supersymmetric contributions are:

$$\delta C_7^{\prime\chi}(\mu_W) = -\frac{1}{2} A_7^{\prime\chi} , \tag{237}$$

$$\delta C_8^{\prime\chi}(\mu_W) = -\frac{1}{2} F_8^{\prime\chi} , \tag{238}$$

with

$$A_7'^{\chi}(\mu) = \kappa \sum_{i=1}^{2} \sum_{a=1}^{6} \frac{M_W^2}{m_{\chi_i^\pm}^2} \times \left\{ [X_i^{U_R \dagger}]_{2a} [X_i^{U_R}]_{a3} \, h_1^{(0)}(y_{ai}) \right. \tag{239}$$

$$\left. + \frac{m_{\chi_i^\pm}}{m_b} [X_i^{U_R \dagger}]_{2a} [X_i^{U_L}]_{a3} \, h_2^{(0)}(y_{ai}) \right\} ,$$

$$F_8'^{\chi}(\mu) = \kappa \sum_{i=1}^{2} \sum_{a=1}^{6} \frac{M_W^2}{m_{\chi_i^\pm}^2} \times \left\{ [X_i^{U_R \dagger}]_{2a} [X_i^{U_R}]_{a3} \, h_5^{(0)}(y_{ai}) \right. \tag{240}$$

$$\left. + \frac{m_{\chi_i^\pm}}{m_b} [X_i^{U_R \dagger}]_{2a} [X_i^{U_L}]_{a3} \, h_6^{(0)}(y_{ai}) \right\} ,$$

where $X_i^{U_{L,R}}$ are defined in Eq. (94), the h_i functions are given in section C.6, and

$$\kappa = \frac{1}{g_2^2 V_{tb} V_{ts}^*} , \qquad y_{ai} = \frac{m_{\tilde{u}_a}^2}{m_{\chi_i^\pm}^2} . \tag{241}$$

C.4.2 Prime Wilson coefficients $C'_{9,10}$

$C'_{9,10}$ do not receive relevant SM contributions. The charged Higgs contributions are:

$$\delta C_9'^H = -\frac{1}{s_W^2} \mathcal{C}'^H , \tag{242}$$

$$\delta C_{10}'^H = \frac{1 - s_W^2}{s_W^2} \mathcal{C}'^H - \mathcal{D}'^H , \tag{243}$$

$$\tag{244}$$

where

$$\mathcal{C}'^H = \frac{m_s m_b \tan^2\beta}{8 M_W^2} \left(1 + \frac{m_\ell^2 \tan^2\beta}{2 M_{H^\pm}^2} \right) f_2^{(0)}(x_{tH^\pm}) , \tag{245}$$

$$\mathcal{D}'^H = \frac{1}{18} \tan^2\beta \frac{m_s m_b \tan^2\beta}{\overline{m}_t^2(\mu_W)} x_{tH^\pm} \tag{246}$$

$$\times \left\{ \frac{-3 x_{tH^\pm}^3 + 6 x_{tH^\pm} - 4}{(x_{tH^\pm} - 1)^4} \ln x_{tH^\pm} + \frac{47 x_{tH^\pm}^2 - 79 x_{tH^\pm} + 38}{6 (x_{tH^\pm} - 1)^3} \right\} ,$$

with

$$x_{tH^\pm} = \frac{\overline{m}_t^2(\mu_W)}{M_{H^\pm}^2} . \tag{247}$$

The supersymmetric contributions are:

$$\delta C_9'^{\chi(0)} = \frac{1-4s_W^2}{s_W^2}\,\mathcal{C}^{\chi(0)} - \frac{1}{s_W^2}\,\mathcal{B}_9^{\chi(0,1)} - \mathcal{D}^{\chi(0)}\,, \tag{248}$$

$$\delta C_{10}'^{\chi(0)} = \frac{1}{s_W^2}\left[\mathcal{B}_{10}^{\chi(0)} - \mathcal{C}^{\chi(0)}\right]. \tag{249}$$

The contributing chargino-up squark loop Green functions are:

$$\mathcal{B}_{9,10}'^{\chi(0)} = \mp\kappa\frac{M_W^2}{2g_2^2}\sum_{i,j=1}^{2}\sum_{a=1}^{6}\sum_{b=1}^{3}\frac{[X_j^{U_R\dagger}]_{2a}[X_i^{U_R}]_{a3}}{m^2_{\chi_i^\pm}} \tag{250}$$
$$\times\left\{\frac{1}{2}[X_i^{N_R\dagger}]_{lb}[X_j^{N_R}]_{bl}f_5^{(0)}(x_{ji},y_{ai},v_{bi}) \mp [X_i^{N_L\dagger}]_{lb}[X_j^{N_L}]_{bl}\sqrt{x_{ji}}f_6^{(0)}(x_{ji},y_{ai},v_{bi})\right\},$$

$$\mathcal{C}'^{\chi(0)} = -\frac{\kappa}{8}\sum_{i,j=1}^{2}\sum_{a,b=1}^{6}[X_j^{U_R\dagger}]_{2b}[X_i^{U_R}]_{a3} \tag{251}$$
$$\times\left\{2\sqrt{x_{ji}}\,f_3^{(0)}(x_{ji},y_{ai})U_{j1}U_{i1}^*\delta_{ab} - f_4^{(0)}(x_{ji},y_{ai})V_{j1}^*V_{i1}\delta_{ab} + f_4^{(0)}(y_{ai},y_{bi})(\Gamma^{U_R}\Gamma^{U_R\dagger})_{ba}\delta_{ij}\right\},$$

$$\mathcal{D}'^{\chi(0)} = \kappa\sum_{i=1}^{2}\sum_{a=1}^{6}\frac{M_W^2}{m^2_{\chi_i^\pm}}[X_i^{U_R\dagger}]_{2a}[X_i^{U_R}]_{a3}\;h_3^{(0)}(y_{ai})\,, \tag{252}$$

where $X_i^{U_{L,R}}$ are defined in Eq. (94), the h_i functions are given in section C.6, and κ and y_{ai} in Eq. (241).

C.4.3 Prime Wilson coefficients C'_{Q_1,Q_2}

The Wilson coefficients $C'_{Q_{1,2}}$ do not receive relevant SM contributions. They can be written as:

$$C'_{Q_1} = \frac{1}{(1+\epsilon_0\tan\beta)(1+\epsilon_b\tan\beta)}\sum_{J=H,\tilde{\chi}}\left[\mathcal{N}^J_{Q_1{}'} + \mathcal{B}^J_{Q_1{}'}\right], \tag{253}$$

$$C'_{Q_2} = \frac{1}{(1+\epsilon_0\tan\beta)(1+\epsilon_b\tan\beta)}\sum_{J=H,\tilde{\chi}}\left[\mathcal{N}^J_{Q_2{}'} + \mathcal{B}^J_{Q_2{}'}\right]. \tag{254}$$

The charged Higgs contributions are:

$$\mathcal{B}^H_{Q_1{}',Q_2{}'} = \frac{m_l m_s\tan^2\beta}{4M_W^2 s_W^2}f_7^{(0)}(x,z)\,, \tag{255}$$

and

$$\mathcal{N}^H_{Q_1{}',Q_2{}'} = -\frac{m_l m_s\tan^2\beta}{4M_W^2 s_W^2}x f_3^{(0)}(x,z)\,. \tag{256}$$

The supersymmetric contributions are:

$$\mathcal{B}^{\chi}_{Q_1{}',Q_2{}'} = \frac{\kappa M_W^2}{2g_2^2 s_W^2}\sum_{i,j=1}^{2}\sum_{f=1}^{3}\sum_{a=1}^{6}\frac{[X_j^{U_R\dagger}]_{2a}[X_i^{U_L}]_{a3}}{m^2_{\chi_i^\pm}} \times\Bigg[f_5^{(0)}(x_{ji},y_{ai},v_{fi})[X_i^{N_L\dagger}]_{lf}[X_j^{N_R}]_{fl} \pm\sqrt{x_{ji}}f_6^{(0)}(x_{ji},y_{ai},v_{fi})[X_i^{N_R\dagger}]_{lf}[X_j^{N_L}]_{fl}\Bigg]\,. \tag{257}$$

and

$$\mathcal{N}^{\chi}_{Q_1{}',Q_2{}'} = \frac{m_l m_s\tan^2\beta}{M_W s_W^2 M_{A^0}^2}\sum_{i,j=1}^{2}\sum_{a,b=1}^{6}\sum_{m,n=1}^{3}\Gamma^{a\prime}_{imn}(\Gamma^{U_L})_{bm}U_{j2}a_Y\left[a_{0,Q_1{}',Q_2{}'}+a_1'\tan\beta\right]\,, \tag{258}$$

where

$$\Gamma^{a\prime}_{imn} \equiv \frac{1}{2\sqrt{2}}[\sqrt{2}M_W V_{i1}(\Gamma^{U_L\dagger})_{na}a_g - (M_U)_{nn}V_{i2}(\Gamma^{U_R\dagger})_{na}a_Y]\lambda^*_{mn}\,, \tag{259}$$

and $a_{0,Q_1{}',Q_2{}'} = a^{(0)}_{0,Q_1,Q_2}$ is given in Eq. (207) and $a_1' = a_1^{(0)}$ is given in Eq. (208). The $X_i^{U_{L,R}}$ are given in Eq. (94), $X_i^{N_{L,R}}$ in Eq. (186), κ, x_{ij}, y_{ai}, v_{fi} in Eqs. (767) and (185), and the f_i functions are given in section C.6.

C.5 Wilson coefficients C_L, C_R

The $s\to d$ transitions can be parametrised by the following effective hamiltonian:

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}}\lambda_t^{sd}\frac{\alpha_e}{4\pi}\sum_k C_k^{sd,\ell}O_k^{sd,\ell} \tag{260}$$

where $\lambda_u^{sd}\equiv V^*_{Us}V_{Ud}$ with $U=u,c,t$, and the relevant effective operators are

$$O_9^{s,\ell} = (\bar{s}\gamma_\mu P_L d)\,(\bar{\ell}\gamma^\mu\ell), \qquad Q_1^{dd,\ell} = (\bar{s}P_R d)\,(\bar{\ell}\ell), \tag{261}$$

$$O_{10}^{dd,\ell} = (\bar{s}\gamma_\mu P_L d)\,(\bar{\ell}\gamma^\mu\gamma_5\ell), \qquad Q_2^{dd,\ell} = (\bar{s}P_R d)\,(\bar{\ell}\gamma_5\ell), \tag{262}$$

$$O_L^{dd,\ell} = (\bar{s}\gamma_\mu P_L d)\,(\bar{\nu}\gamma^\mu(1-\gamma_5)\nu). \tag{263}$$

In general, there are also the primed version of the above operators ($O'_{9,10}, Q'_{1,2}$) where the chirality of the quark currents are right-handed and there is also O_R which is the chirality-flipped version of O_L. The Wilson coefficients $C_k^{sd,\ell}$ include any potential flavour violating New Physics contributions and are parametrised as

$$C_k^{sd,\ell} = C_{k,\text{SM}}^{sd,\ell} + C_{k,\text{NP}}^{sd,\ell}. \tag{264}$$

C.5.1 Standard Model contributions

The one and two loop SM Wilson coefficients involving the top-quark in the loop is given by [138] (extracted from the original papers [134–137])

$$C_L^{\nu_\ell} = -\frac{1}{s_W^2} X(x_t) \tag{265}$$

with

$$X(x_{tW}) = X^{(0)}(x_{tW}) + \frac{\alpha_s(\mu_{tW})}{4\pi} X^{(1)}(x_{tW}) + \frac{\alpha}{4\pi} X^{(\mathrm{EW})}(x_{tW}), \tag{266}$$

where $X^{(0)}$ is the leading order result, and $X^{(1)}$, $X^{(\mathrm{EW})}$ are the NLO QCD and EW corrections, respectively. The coupling constants α_s and α, as well as the parameter $x_{tW} = m_t^2/m_W^2$ have to be evaluated at scale $\mu \sim \mathcal{O}(M_t)$. The LO expression is the gauge-independent linear combination $X_0(x_{tW}) \equiv C(x_{tW}) - 4B(x_{tW})$ [139, 140]

$$X^{(0)}(x) = \frac{x}{8}\left[\frac{x+2}{x-1} + \frac{3x-6}{(x-1)^2}\log x\right]. \tag{267}$$

The NLO QCD correction [134–136], in the $\overline{\mathrm{MS}}$ scheme reads,

$$\begin{aligned} X^{(1)}(x) = &-\frac{29x - x^2 - 4x^3}{3(1-x)^2} - \frac{x + 9x^2 - x^3 - x^4}{(1-x)^3}\log x \\ &+ \frac{8x + 4x^2 + x^3 - x^4}{2(1-x)^3}\log^2 x - \frac{4x - x^3}{(1-x)^2}\mathrm{Li}_2(1-x) \\ &+ 8x\frac{\partial X^{(0)}}{\partial x}\log\frac{\mu^2}{M_W^2}, \end{aligned} \tag{268}$$

where μ is the renormalisation scale and the EW correction $X^{(\mathrm{EW})}$ is given in [137].

C.5.2 MSSM contributions

The MSSM contributions to the Wilson coefficients are written in the form [44]:

$$\delta C_L^{J(0,1)} = -\frac{1}{s_W^2}\left(\mathcal{B}_L^{J(0,1)} + \mathcal{C}_L^{J(0,1)}\right), \tag{269}$$

$$\delta C_R^{J(0,1)} = -\frac{1}{s_W^2}\left(\mathcal{B}_R^{J(0,1)} + \mathcal{C}_R^{J(0,1)}\right), \tag{270}$$

with J is corresponds to charged Higgs ($J = H$), chargino ($J = \chi$) and the quartic coupling ($J = 4$). In the following q corresponds to d-quark, $f = e, \mu, \tau$, and $\kappa_{sd} = 1/(g_2^2\ V_{td}V_{ts}^*)$. For $b \to s$ or $b \to d$ transitions, the appropriate quark mass replacements should be done and in the following formulas we should replace the subscripts $a2 \to a3$ and $\kappa_{sd} \to \kappa_{bq} = 1/(g_2^2\ V_{tb}V_{tq}^*)$ where q correspond to the s- or the d-quark.

The Charged Higgs contributions are:

$$\mathcal{B}_L^{H(0)} = 0 \; , \tag{271}$$

$$\mathcal{C}_L^{H(0)} = \mathcal{C}^{H(0)} = -\frac{M_H^2 \cot^2\beta}{8M_W^2} y f_2^{(0)}(y) \; , \tag{272}$$

$$\mathcal{B}_L^{H(1)} = 0 \; , \tag{273}$$

$$\mathcal{C}_L^{H(1)} = \mathcal{C}^{H(1)} = -\frac{M_H^2 \cot^2\beta}{8M_W^2} \left[y f_2^{(1)}(y) + 8y\frac{\partial}{\partial y}\left(y f_2^{(0)}(y)\right) L_t \right] \; , \tag{274}$$

$$\mathcal{B}_R^{H(0)} = -\frac{m_s m_q m_f^2 \tan^4\beta}{16 M_H^2 M_W^2} f_2^{(0)}(y) \; , \tag{275}$$

$$\mathcal{C}_R^{H(0)} = \mathcal{C}'^{H(0)} = \frac{m_s m_q \tan^2\beta}{8M_W^2} f_2^{(0)}(y) \; , \tag{276}$$

$$\mathcal{B}_R^{H(1)} = -\frac{m_s m_q m_f^2 \tan^4\beta}{16 M_H^2 M_W^2} \left[f_7^{(1)}(y) + 8\left(1 + y\frac{\partial}{\partial y}\right) f_2^{(0)}(y) L_t \right] \; , \tag{277}$$

$$\mathcal{C}_R^{H(1)} = \mathcal{C}'^{H(1)} = \frac{m_s m_q \tan^2\beta}{8M_W^2} \left[f_2^{(1)}(y) + 8\left(1 + y\frac{\partial}{\partial y}\right) f_2^{(0)}(y) L_t \right] \; , \tag{278}$$

The chargino contributions to the Wilson coefficients read [44]:

$$\mathcal{B}_L^{\chi(0)} = \frac{\kappa_{sd} M_W^2}{4 g_2^2} \sum_{i,j=1}^{2} \sum_{a,b=1}^{6} \frac{1}{M_{\tilde{\chi}_i}^2} \left[X_j^{E_L}\right]_{bf} \left[X_i^{E_L \dagger}\right]_{fb} \left[X_j^{U_L \dagger}\right]_{qa} \left[X_i^{U_L}\right]_{a2} f_5^{(0)}(x_{ji}, y_{ai}, z_{bi}) \,, \tag{279}$$

$$\begin{aligned} \mathcal{C}_L^{\chi(0)} &= \mathcal{C}^{\chi(0)} = -\frac{\kappa_{sd}}{8} \sum_{i,j=1}^{2} \sum_{a,b=1}^{6} \left[X_j^{U_L \dagger}\right]_{qb} \left[X_i^{U_L}\right]_{a2} \\ &\times \left\{ 2\sqrt{x_{ji}} f_3^{(0)}(x_{ji}, y_{ai}) U_{j1} U_{i1}^* \delta_{ab} - f_4^{(0)}(x_{ji}, y_{ai}) V_{j1}^* V_{i1} \delta_{ab} + f_4^{(0)}(y_{ai}, y_{bi}) (\Gamma^{U_L} \Gamma^{U_L \dagger})_{ba} \delta_{ij} \right\} , \end{aligned} \tag{280}$$

$$\begin{aligned} \mathcal{B}_L^{\chi(1)} &= \frac{\kappa_{sd} M_W^2}{4 g_2^2} \sum_{i,j=1}^{2} \sum_{a,b=1}^{6} \frac{1}{M_{\tilde{\chi}_i}^2} \left[X_j^{E_L}\right]_{bf} \left[X_i^{E_L \dagger}\right]_{fb} \left[X_j^{U_L \dagger}\right]_{qa} \left[X_i^{U_L}\right]_{a2} \\ &\times \left[f_8^{(1)}(x_{ji}, y_{ai}, z_{bi}) + 4 \left(1 + y_{ai} \frac{\partial}{\partial y_{ai}}\right) f_5^{(0)}(x_{ji}, y_{ai}, z_{bi}) L_{\tilde{u}_a} \right] \,, \end{aligned} \tag{281}$$

$$\begin{aligned} \mathcal{C}_L^{\chi(1)} &= \mathcal{C}^{\chi(1)} = -\frac{\kappa_{sd}}{8} \sum_{i,j=1}^{2} \sum_{a,b=1}^{6} \left[X_j^{U_L \dagger}\right]_{qb} \left[X_i^{U_L}\right]_{a2} \\ &\times \left\{ 2\sqrt{x_{ji}} \left[f_3^{(1)}(x_{ji}, y_{ai}) + 4 \left(1 + y_{ai} \frac{\partial}{\partial y_{ai}}\right) f_3^{(0)}(x_{ji}, y_{ai}) L_{\tilde{u}_a} \right] U_{j1} U_{i1}^* \delta_{ab} \right. \\ &- \left[f_4^{(1)}(x_{ji}, y_{ai}) + 4 \left(1 + y_{ai} \frac{\partial}{\partial y_{ai}}\right) f_4^{(0)}(x_{ji}, y_{ai}) L_{\tilde{u}_a} \right] V_{j1}^* V_{i1} \delta_{ab} \\ &+ \left. \left[f_5^{(1)}(y_{ai}, y_{bi}) + 4 \left(1 + y_{ai} \frac{\partial}{\partial y_{ai}} + y_{bi} \frac{\partial}{\partial y_{bi}}\right) f_4^{(0)}(y_{ai}, y_{bi}) L_{\tilde{u}_a} \right] (\Gamma^{U_L} \Gamma^{U_L \dagger})_{ba} \delta_{ij} \right\} , \end{aligned} \tag{282}$$

$$\mathcal{B}_R^{\chi(0)} = \frac{-\kappa_{sd} M_W^2}{2g_2^2} \sum_{i,j=1}^{2} \sum_{a,b=1}^{6} \frac{\sqrt{x_{ji}}}{M^2_{\tilde{\chi}_i}} \left[X_j^{E_L}\right]_{bf} \left[X_i^{E_L\dagger}\right]_{fb} \left[X_j^{U_R\dagger}\right]_{qa} \left[X_i^{U_R}\right]_{a2} f_6^{(0)}(x_{ji}, y_{ai}, z_{bi}) \,, \tag{283}$$

$$\begin{aligned}
\mathcal{C}_R^{\chi(0)} &= \mathcal{C}'^{\chi(0)} \\
&= \left[C_L^{\chi(0)}\right]_L \left(X^{U_L} \to X^{U_R}; U \to V^*; V \to U^*; \Gamma^{U_L}\Gamma^{U_L\dagger} \to -\Gamma^{U_R}\Gamma^{U_R\dagger}\right) \\
&= -\frac{\kappa_{sd}}{8} \sum_{i,j=1}^{2} \sum_{a,b=1}^{6} \left[X_j^{U_R\dagger}\right]_{qb} \left[X_i^{U_R}\right]_{a2} \Bigg\{ 2\sqrt{x_{ji}} f_3^{(0)}(x_{ji}, y_{ai}) V^*_{j1} V_{i1} \delta_{ab} \\
&\quad - f_4^{(0)}(x_{ji}, y_{ai}) U_{j1} U^*_{i1} \delta_{ab} - f_4^{(0)}(y_{ai}, y_{bi}) (\Gamma^{U_R}\Gamma^{U_R\dagger})_{ba} \delta_{ij} \Bigg\} \,,
\end{aligned} \tag{284}$$

$$\begin{aligned}
\mathcal{B}_R^{\chi(1)} &= \frac{-\kappa_{sd} M_W^2}{2g_2^2} \sum_{i,j=1}^{2} \sum_{a,b=1}^{6} \frac{\sqrt{x_{ji}}}{M^2_{\tilde{\chi}_i}} \left[X_j^{E_L}\right]_{bf} \left[X_i^{E_L\dagger}\right]_{fb} \left[X_j^{U_R\dagger}\right]_{qa} \left[X_i^{U_R}\right]_{a2} \\
&\quad \times \left[f_9^{(1)}(x_{ji}, y_{ai}, z_{bi}) + 4\left(1 + y_{ai}\frac{\partial}{\partial y_{ai}}\right) f_6^{(0)}(x_{ji}, y_{ai}, z_{bi}) L_{\tilde{u}_a} \right] \,,
\end{aligned} \tag{285}$$

$$\begin{aligned}
\mathcal{C}_R^{\chi(1)} &= \mathcal{C}'^{\chi(1)} \\
&= \left[C_L^{\chi(1)}\right]_L \left(X^{U_L} \to X^{U_R}; U \to V^*; V \to U^*; \Gamma^{U_L}\Gamma^{U_L\dagger} \to -\Gamma^{U_R}\Gamma^{U_R\dagger}\right) \\
&= -\frac{\kappa_{sd}}{8} \sum_{i,j=1}^{2} \sum_{a,b=1}^{6} \left[X_j^{U_R\dagger}\right]_{qb} \left[X_i^{U_R}\right]_{a2} \\
&\quad \times \Bigg\{ 2\sqrt{x_{ji}} \left[f_3^{(1)}(x_{ji}, y_{ai}) + 4\left(1 + y_{ai}\frac{\partial}{\partial y_{ai}}\right) f_3^{(0)}(x_{ji}, y_{ai}) L_{\tilde{u}_a} \right] V^*_{j1} V_{i1} \delta_{ab} \\
&\quad - \left[f_4^{(1)}(x_{ji}, y_{ai}) + 4\left(1 + y_{ai}\frac{\partial}{\partial y_{ai}}\right) f_4^{(0)}(x_{ji}, y_{ai}) L_{\tilde{u}_a} \right] U_{j1} U^*_{i1} \delta_{ab} \\
&\quad - \left[f_5^{(1)}(y_{ai}, y_{bi}) + 4\left(1 + y_{ai}\frac{\partial}{\partial y_{ai}} + y_{bi}\frac{\partial}{\partial y_{bi}}\right) f_4^{(0)}(y_{ai}, y_{bi}) L_{\tilde{u}_a} \right] (\Gamma^{U_R}\Gamma^{U_R\dagger})_{ba} \delta_{ij} \Bigg\}
\end{aligned} \tag{286}$$

The contributions from the quartic couplings read [44]:

$$\mathcal{B}_L^{4(0)} = 0 \,, \tag{287}$$

$$\mathcal{C}_L^{4(0)} = \mathcal{C}^{4(0)} = 0 \,, \tag{288}$$

$$\mathcal{B}_L^{4(1)} = -\frac{\kappa_{sd} M_W^2}{3 g_2^2} \sum_{i,j=1}^{2} \sum_{a,\dots,d=1}^{6} \frac{1}{M_{\tilde{\chi}_i}^2} [X_j^{E_L}]_{cf} [X_i^{E_L \dagger}]_{fc} [X_j^{U_L \dagger}]_{qb} [X_i^{U_L}]_{a2}$$
$$\times [P_U]_{ad}\, y_{di}\, [P_U]_{db} (1 + L_{\tilde{u}_d}) f_9^{(0)}(x_{ji}, y_{ai}, y_{bi}, z_{ci}) \,, \tag{289}$$

$$\mathcal{C}_L^{4(1)} = \mathcal{C}^{4(1)} = \frac{\kappa_{sd}}{6} \sum_{i,j=1}^{2} \sum_{a,\dots,e,g,k=1}^{6} [X_j^{U_L \dagger}]_{qd} [X_i^{U_L}]_{a2} [P_U]_{ek}\, y_{ki}\, [P_U]_{kg} (1 + L_{\tilde{u}_k})$$
$$\times \Bigg\{ 2\sqrt{x_{ji}} f_6^{(0)}(x_{ji}, y_{ai}, y_{di}) U_{j1} U_{i1}^* \delta_{ae} \delta_{gd} - f_5^{(0)}(x_{ji}, y_{ai}, y_{di}) V_{j1}^* V_{i1} \delta_{ae} \delta_{gd} \tag{290}$$
$$+ f_5^{(0)}(y_{ai}, y_{bi}, y_{ci}) (\Gamma^{U_L} \Gamma^{U_L \dagger})_{bc} \delta_{ij} \delta_{ae} \delta_{bg} \delta_{cd} + f_5^{(0)}(y_{ai}, y_{ci}, y_{di}) (\Gamma^{U_L} \Gamma^{U_L \dagger})_{bc} \delta_{ij} \delta_{ab} \delta_{ce} \delta_{dg} \Bigg\} \,,$$

$$\mathcal{B}_R^{4(0)} = 0 \,, \tag{291}$$

$$\mathcal{C}_R^{4(0)} = \mathcal{C}'^{4(0)} = 0 \,, \tag{292}$$

$$\mathcal{B}_R^{4(1)} = \frac{2\kappa_{sd} M_W^2}{3 g_2^2} \sum_{i,j=1}^{2} \sum_{a,\dots,d=1}^{6} \frac{\sqrt{x_{ji}}}{M_{\tilde{\chi}_i}^2} (X_j^{E_L})_{cf} (X_i^{E_L \dagger})_{fc} (X_j^{U_R \dagger})_{qb} (X_i^{U_R})_{a2}$$
$$\times [(P_U)_{ad} y_{di} (P_U)_{db} (1 + L_{\tilde{u}_d})] f_{10}^{(0)}(x_{ji}, y_{ai}, y_{bi}, z_{ci}) \,, \tag{293}$$

$$\mathcal{C}_R^{4(1)} = \mathcal{C}'^{4(1)} = \left[C_L^{4(1)} \right]_4 \left(X^{U_L} \to X^{U_R}; U \to V^*; V \to U^*; \Gamma^{U_L} \Gamma^{U_L \dagger} \to -\Gamma^{U_R} \Gamma^{U_R \dagger} \right)$$
$$\frac{\kappa_{sd}}{6} \sum_{i,j=1}^{2} \sum_{a,\dots,e,g,k=1}^{6} [X_j^{U_R \dagger}]_{qd} [X_i^{U_R}]_{a2} [P_U]_{ek}\, y_{ki}\, [P_U]_{kg} (1 + L_{\tilde{u}_k})$$
$$\times \Bigg\{ 2\sqrt{x_{ji}} f_6^{(0)}(x_{ji}, y_{ai}, y_{di}) V_{j1}^* V_{i1} \delta_{ae} \delta_{gd} - f_5^{(0)}(x_{ji}, y_{ai}, y_{di}) U_{j1} U_{i1}^* \delta_{ae} \delta_{gd} \tag{294}$$
$$- f_5^{(0)}(y_{ai}, y_{bi}, y_{ci}) (\Gamma^{U_R} \Gamma^{U_R \dagger})_{bc} \delta_{ij} \delta_{ae} \delta_{bg} \delta_{cd} - f_5^{(0)}(y_{ai}, y_{ci}, y_{di}) (\Gamma^{U_R} \Gamma^{U_R \dagger})_{bc} \delta_{ij} \delta_{ab} \delta_{ce} \delta_{dg} \Bigg\} \,.$$

C.6 Auxiliary functions

The auxiliary functions h_i and q_i are listed below [31]:

$$h_1^{(0)}(x) = \frac{3x^2-2x}{3(x-1)^4}\ln x + \frac{-8x^2-5x+7}{18(x-1)^3} \ , \tag{295}$$

$$h_2^{(0)}(x) = \frac{-6x^2+4x}{3(x-1)^3}\ln x + \frac{7x-5}{3(x-1)^2} \ , \tag{296}$$

$$h_3^{(0)}(x) = \frac{-6x^3+9x^2-2}{9(x-1)^4}\ln x + \frac{52x^2-101x+43}{54(x-1)^3} \ , \tag{297}$$

$$h_4^{(0)}(x) = \frac{-1}{3(x-1)^4}\ln x + \frac{2x^2-7x+11}{18(x-1)^3} \ , \tag{298}$$

$$h_5^{(0)}(x) = \frac{-x}{(x-1)^4}\ln x + \frac{-x^2+5x+2}{6(x-1)^3} \ , \tag{299}$$

$$h_6^{(0)}(x) = \frac{2x}{(x-1)^3}\ln x + \frac{-x-1}{(x-1)^2}\,, \tag{300}$$

$$\begin{aligned} h_1^{(1)}(x,y) &= \frac{-48x^3-104x^2+64x}{9(x-1)^4}\,\mathrm{Li}_2\left(1-\frac{1}{x}\right) \\ &+\frac{-378x^3-1566x^2+850x+86}{81(x-1)^5}\ln x + \frac{2060x^3+3798x^2-2664x-170}{243(x-1)^4} \\ &+\left[\frac{12x^3-124x^2+64x}{9(x-1)^5}\ln x + \frac{-56x^3+258x^2+24x-82}{27(x-1)^4}\right] y\,, \end{aligned} \tag{301}$$

$$\begin{aligned} h_2^{(1)}(x,y) &= \frac{224x^2-96x}{9(x-1)^3}\,\mathrm{Li}_2\left(1-\frac{1}{x}\right) \\ &+\frac{-24x^3+352x^2-128x-32}{9(x-1)^4}\ln x + \frac{-340x^2+132x+40}{9(x-1)^3} \\ &+\left[\frac{-24x^3+176x^2-80x}{9(x-1)^4}\ln x + \frac{-28x^2-108x+64}{9(x-1)^3}\right] y\,, \end{aligned} \tag{302}$$

$$\begin{aligned} h_3^{(1)}(x,y) &= \frac{32x^3+120x^2-384x+128}{81(x-1)^4}\,\mathrm{Li}_2\left(1-\frac{1}{x}\right) \\ &+\frac{-108x^4+1058x^3-898x^2-1098x+710}{81(x-1)^5}\ln x \\ &+\frac{-304x^3-13686x^2+29076x-12062}{729(x-1)^4} \\ &+\left[\frac{540x^3-972x^2+232x+56}{81(x-1)^5}\ln x + \frac{-664x^3+54x^2+1944x-902}{243(x-1)^4}\right] y\,, \end{aligned} \tag{303}$$

$$\begin{aligned} h_4^{(1)}(x,y) &= \frac{-562x^3+1101x^2-420x+101}{54(x-1)^4}\,\mathrm{Li}_2\left(1-\frac{1}{x}\right) \\ &+\frac{-562x^3+1604x^2-799x+429}{54(x-1)^5}\ln x + \frac{17470x^3-47217x^2+31098x-13447}{972(x-1)^4} \\ &+\left[\frac{89x+55}{27(x-1)^5}\ln x + \frac{38x^3-135x^2+54x-821}{162(x-1)^4}\right] y\,, \end{aligned} \tag{304}$$

$$h_5^{(1)}(x,y) = \frac{9x^3+46x^2+49x}{6(x-1)^4}\,\mathrm{Li}_2\left(1-\frac{1}{x}\right) + \frac{81x^3+594x^2+1270x+71}{54(x-1)^5}\ln x + \frac{-923x^3-3042x^2-6921x-1210}{324(x-1)^4} + \left[\frac{10x^2+38x}{3(x-1)^5}\ln x + \frac{-7x^3+30x^2-141x-26}{9(x-1)^4}\right] y\,, \tag{305}$$

$$h_6^{(1)}(x,y) = \frac{-32x^2-24x}{3(x-1)^3}\,\mathrm{Li}_2\left(1-\frac{1}{x}\right) + \frac{-52x^2-109x-7}{3(x-1)^4}\ln x + \frac{95x^2+180x+61}{6(x-1)^3} + \left[\frac{-20x^2-52x}{3(x-1)^4}\ln x + \frac{-2x^2+60x+14}{3(x-1)^3}\right] y\,, \tag{306}$$

$$h_7^{(1)}(x,y) = \frac{-20x^3+60x^2-60x-20}{27(x-1)^4}\,\mathrm{Li}_2\left(1-\frac{1}{x}\right) + \frac{-60x^2+240x+4}{81(x-1)^4}\ln x + \frac{132x^2-382x+186}{81(x-1)^3} + \left[\frac{20}{27(x-1)^4}\ln x + \frac{-20x^2+70x-110}{81(x-1)^3}\right] y\,, \tag{307}$$

$$q_1^{(1)}(x,y) = \frac{4}{3(x-y)}\left[\frac{x^2\ln x}{(x-1)^4} - \frac{y^2\ln y}{(y-1)^4}\right] + \frac{4x^2y^2+10xy^2-2y^2+10x^2y-44xy+10y-2x^2+10x+4}{9(x-1)^3(y-1)^3}\,, \tag{308}$$

$$q_2^{(1)}(x,y) = \frac{4}{3(x-y)}\left[\frac{x\ln x}{(x-1)^4} - \frac{y\ln y}{(y-1)^4}\right] + \frac{-2x^2y^2+10xy^2+4y^2+10x^2y-20xy-14y+4x^2-14x+22}{9(x-1)^3(y-1)^3}\,, \tag{309}$$

$$q_3^{(1)}(x,y) = \frac{8}{3(x-y)}\left[\frac{-x^2\ln x}{(x-1)^3} + \frac{y^2\ln y}{(y-1)^3}\right] + \frac{-12xy+4y+4x+4}{3(x-1)^2(y-1)^2}\,,$$

$$q_4^{(1)}(x,y) = \frac{8}{3(x-y)}\left[\frac{-x\ln x}{(x-1)^3} + \frac{y\ln y}{(y-1)^3}\right] + \frac{-4xy-4y-4x+12}{3(x-1)^2(y-1)^2}\,, \tag{310}$$

$$q_5^{(1)}(x,y) = \frac{4}{27(x-y)}\left[\frac{(6x^3-9x^2+2)\ln x}{(x-1)^4} - \frac{(6y^3-9y^2+2)\ln y}{(y-1)^4}\right] \tag{311}$$
$$+\frac{104x^2y^2-202xy^2+86y^2-202x^2y+380xy-154y+86x^2-154x+56}{81(x-1)^3(y-1)^3}\ ,$$

$$q_6^{(1)}(x,y) = \frac{4}{9(x-y)}\left[\frac{\ln x}{(x-1)^4} - \frac{\ln y}{(y-1)^4}\right] \tag{312}$$
$$+\frac{4x^2y^2-14xy^2+22y^2-14x^2y+52xy-62y+22x^2-62x+52}{27(x-1)^3(y-1)^3}\ .$$

The functions f_i read [44]:

$$f_1^{(0)}(x) = -\frac{x(6-x)}{2(x-1)} + \frac{x(2+3x)}{2(x-1)^2}\ln x\ , \tag{313}$$

$$f_2^{(0)}(x) = -\frac{x}{x-1} + \frac{x}{(x-1)^2}\ln x\ , \tag{314}$$

$$f_3^{(0)}(x,y) = \frac{x\ln x}{(x-1)(x-y)} + \frac{y\ln y}{(y-1)(y-x)}\ , \tag{315}$$

$$f_4^{(0)}(x,y) = \frac{x^2\ln x}{(x-1)(x-y)} + \frac{y^2\ln y}{(y-1)(y-x)}\ , \tag{316}$$

$$f_5^{(0)}(x,y,z) = \frac{x^2\ln x}{(x-1)(x-y)(x-z)} + (x\leftrightarrow y) + (x\leftrightarrow z)\ , \tag{317}$$

$$f_6^{(0)}(x,y,z) = \frac{x\ln x}{(x-1)(x-y)(x-z)} + (x\leftrightarrow y) + (x\leftrightarrow z)\ , \tag{318}$$

$$f_7^{(0)}(x,y) = \frac{x\ln x}{(x-1)(x-y)} + \frac{x\ln y}{(y-1)(y-x)}\ , \tag{319}$$

$$f_8^{(0)}(x) = \frac{x\ln x}{x-1}\ , \tag{320}$$

$$f_9^{(0)}(w,x,y,z) = \frac{w^2\ln w}{(w-1)(w-x)(w-y)(w-z)} \tag{321}$$
$$+(w\leftrightarrow x) + (w\leftrightarrow y) + (w\leftrightarrow z)\ ,$$

$$f^{(0)}_{10}(w,x,y,z) = \frac{w\ln w}{(w-1)(w-x)(w-y)(w-z)} + (w\leftrightarrow x) + (w\leftrightarrow y) + (w\leftrightarrow z)\ , \tag{322}$$

$$f^{(0)}_{11}(x,y) = \frac{x\ln x}{(x-y)} + \frac{x\ln y}{(y-x)}\ , \tag{323}$$

$$f^{(1)}_{1}(x) = \frac{4x(29+7x+4x^2)}{3(x-1)^2} - \frac{4x(23+14x+3x^2)}{3(x-1)^3}\ln x - \frac{4x(4+x^2)}{(x-1)^2}\,\mathrm{Li}_2\left(1-\frac{1}{x}\right) \tag{324}$$

$$f^{(1)}_{2}(x) = \frac{32x(3-x)}{3(x-1)^2} - \frac{8x(11-3x)}{3(x-1)^3}\ln x - \frac{8x(2-x)}{(x-1)^2}\,\mathrm{Li}_2\left(1-\frac{1}{x}\right)\ , \tag{325}$$

$$f^{(1)}_{3}(x,y) = -\frac{28y}{3(x-y)(y-1)} + \frac{2x(11x+3y)}{3(x-1)(x-y)^2}\ln x + \frac{2y\big[x(25-11y)-y(11+3y)\big]}{3(x-y)^2(y-1)^2}\ln y + \frac{4(1+y)}{(x-1)(y-1)}\,\mathrm{Li}_2\left(1-\frac{1}{y}\right) + \frac{4(x+y)}{(x-1)(x-y)}\,\mathrm{Li}_2\left(1-\frac{x}{y}\right)\ , \tag{326}$$

$$f^{(1)}_{4}(x,y) = \frac{59x(1-y)-y(59-3y)}{6(y-1)(x-y)} + \frac{4x(7x^2-3xy+3y^2)}{3(x-1)(x-y)^2}\ln x + 2\ln^2 y + \frac{4y^2\big[x(18-11y)-y(11-4y)\big]}{3(x-y)^2(y-1)^2}\ln y + \frac{4(1+y^2)}{(x-1)(y-1)}\,\mathrm{Li}_2\left(1-\frac{1}{y}\right) + \frac{4(x^2+y^2)}{(x-1)(x-y)}\,\mathrm{Li}_2\left(1-\frac{x}{y}\right)\ , \tag{327}$$

$$f_5^{(1)}(x,y) = -\frac{83+27x(y-1)-27y}{6(x-1)(y-1)} - \Bigg\{ \frac{4x\big[1+x(12+y)-y-6x^2\big]}{3(x-1)^2(x-y)} \ln x \tag{328}$$

$$-\frac{2\big[1+6x^2(y-1)-3x^3(y-1)+x(3y-4)\big]}{3(x-1)^2(x-y)(y-1)} \ln^2 x$$

$$+\frac{4y\big[3x^2(y-1)+xy(3-2y)+y^2(y-2)\big]}{3(x-1)(x-y)^2(y-1)} \mathrm{Li}_2\left(1-\frac{x}{y}\right)$$

$$+\frac{4\big[1-3x-x^2(3-6y)-x^3\big]}{3(x-1)(x-y)(y-1)} \mathrm{Li}_2\left(1-\frac{1}{x}\right) + (x \leftrightarrow y)\Bigg\}$$

$$+4\ln x \left(1 + x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}\right) f_4^{(0)}(x,y) \; ,$$

$$f_6^{(1)}(x) = \frac{2x(29+3x)}{3(x-1)^2} - \frac{2x(25+7x)}{3(x-1)^3} \ln x - \frac{8x}{(x-1)^2} \mathrm{Li}_2\left(1-\frac{1}{x}\right) \; , \tag{329}$$

$$f_7^{(1)}(x) = \frac{4x\big[27-11x+(x-1)^2\pi^2\big]}{3(x-1)^2} - \frac{4x(37-33x+12x^2)}{3(x-1)^3} \ln x \tag{330}$$

$$-\frac{8x(2-2x+x^2)}{(x-1)^2} \mathrm{Li}_2\left(1-\frac{1}{x}\right) \; ,$$

$$f_8^{(1)}(x,y,z) = -\frac{28y^2}{3(x-y)(y-1)(y-z)} + \left[\frac{4x(7x^2-3xy+3y^2)}{3(x-1)(x-y)^2(x-z)} \ln x + (x \leftrightarrow z)\right]$$

$$-\frac{4y^2\Big\{x\big[4y^2+18z-11y(1+z)\big]+y\big[3y^2-11z+4y(1+z)\big]\Big\}}{3(x-y)^2(y-1)^2(y-z)^2} \ln y$$

$$-\frac{4(1+y^2)}{(x-1)(y-1)(z-1)} \mathrm{Li}_2\left(1-\frac{1}{y}\right) \tag{331}$$

$$+\left[\frac{4(x^2+y^2)}{(x-1)(x-y)(x-z)} \mathrm{Li}_2\left(1-\frac{x}{y}\right) + (x \leftrightarrow z)\right] \; ,$$

$$f_9^{(1)}(x,y,z) = -\frac{28y}{3(x-y)(y-1)(y-z)} \tag{332}$$

$$+\left[\frac{2x(11x+3y)}{3(x-1)(x-y)^2(x-z)}\ln x + (x\leftrightarrow z)\right]$$

$$+\frac{2y\Big\{x\big[3y^2-25z+11y(1+x)\big]+y\big[11z-17y^2+3y(1+z)\big]\Big\}}{3(x-y)^2(y-1)^2(y-z)^2}\ln y$$

$$-\frac{4(1+y)}{(x-1)(y-1)(z-1)}\,\mathrm{Li}_2\left(1-\frac{1}{y}\right)$$

$$+\left[\frac{4(x+y)}{(x-1)(x-y)(x-z)}\,\mathrm{Li}_2\left(1-\frac{x}{y}\right)+(x\leftrightarrow z)\right],$$

$$f_{10}^{(1)}(x) = \frac{4x(19-3x)}{3(x-1)^2}-\frac{4x(17-x)}{3(x-1)^3}\ln x-\frac{8x}{(x-1)^2}\,\mathrm{Li}_2\left(1-\frac{1}{x}\right), \tag{333}$$

$$f_{11}^{(1)}(x,y) = \frac{4x\big[8y+(x-1)(x-y)\pi^2\big]}{3y(x-1)(x-y)}-\frac{8x\big[x^2-7y+3x(1+y)\big]}{3(x-y)^2(x-1)^2}\ln x \tag{334}$$

$$-\frac{8x(3x-7y)}{3(x-y)^2(y-1)}\ln y-\frac{8x}{y-1}\,\mathrm{Li}_2\left(1-\frac{1}{x}\right)$$

$$+\frac{8x}{y(y-1)}\,\mathrm{Li}_2\left(1-\frac{y}{x}\right),$$

$$f_{12}^{(1)}(x,y,z) = -\frac{28y^2}{3(x-y)(y-1)(y-z)} \tag{335}$$

$$+\left[\frac{4x^2(6x+y)}{3(x-1)(x-y)^2(x-z)}\ln x + (x\leftrightarrow z)\right]$$

$$-\frac{4y^2\Big\{x\big[6y^2+20z-13y(1+z)\big]+y\big[y^2-13z+6y(1+z)\big]\Big\}}{3(x-y)^2(y-1)^2(y-z)^2}\ln y,$$

$$f_{13}^{(1)}(x,y,z) = -\frac{28y}{3(x-y)(y-1)(y-z)} + \left[\frac{4x(6x+y)}{3(x-1)(x-y)^2(x-z)}\ln x + (x \leftrightarrow z)\right] + \frac{4y\Big\{x\big[y^2-13z+6y(1+z)\big]+y\big[y-8y^2+6z+yz\big]\Big\}}{3(x-y)^2(y-1)^2(y-z)^2}\ln y\,, \tag{336}$$

$$f_{14}^{(1)}(x,y) = \frac{32x^2}{3(x-1)(x-y)} - \frac{8x^2\big[7x(1+y)-11y-3x^2\big]}{3(x-1)^2(x-y)^2}\ln x - \frac{8xy(3x-7y)}{3(x-y)^2(y-1)}\ln y - \frac{8x}{y-1}\,\mathrm{Li}_2\left(1-\frac{1}{x}\right) + \frac{8x}{y-1}\,\mathrm{Li}_2\left(1-\frac{y}{x}\right)\,, \tag{337}$$

$$f_{15}^{(1)}(x) = \frac{1-3x}{x-1} + \frac{2x}{(x-1)^2}\ln x + \frac{2x}{(x-1)}\,\mathrm{Li}_2\left(1-\frac{1}{x}\right)\,, \tag{338}$$

$$f_{16}^{(1)}(x) = \frac{28}{3(x-1)} - \frac{4x(13-6x)}{3(x-1)^2}\ln x\,, \tag{339}$$

$$f_{17}^{(1)}(x,y) = -\frac{28}{3(x-1)(y-1)} + \frac{4y(10-3y)}{3(x-y)(y-1)^2}\ln y - \frac{4y}{(x-y)(y-1)^2}\ln^2 y + \left[\frac{4(13x-6x^2-3y-7xy+3x^2y)}{3(x-1)^2(x-y)(y-1)} + \frac{4y\ln y}{(x-y)(y-1)^2}\right]\ln x\,, \tag{340}$$

$$f_{18}^{(1)}(x,y) = -\frac{28y}{3(x-y)(y-1)} + \frac{4x(6x+y)}{3(x-1)(x-y)^2}\ln x - \frac{4y\big[y(6+y)-x(13-6y)\big]}{3(x-y)^2(y-1)^2}\ln y\,, \tag{341}$$

$$f_{19}^{(1)}(x,y) = -\frac{28\big[x(y-1)+y\big]}{3(x-y)(y-1)} + \frac{4x^2(6x+y)}{3(x-1)(x-y)^2}\ln x + \frac{4y^2\big[x(20-13y)-y(13-6y)\big]}{3(x-y)^2(y-1)^2}\ln y\,. \tag{342}$$

Appendix D Renormalization group equations

D.1 RGE for the $C_{1\cdots 10}$ Wilson coefficients in the standard operator basis

In the standard operator basis given in Eq. (25), the effective Wilson coefficients are defined as [48]:

$$C_i^{\rm eff}(\mu) = \begin{cases} C_i(\mu), & \text{for } i = 1,...,6 \ , \\ C_7(\mu) + \displaystyle\sum_{j=1}^{6} y_j C_j(\mu), & \text{for } i = 7 \ , \\ C_8(\mu) + \displaystyle\sum_{j=1}^{6} z_j C_j(\mu), & \text{for } i = 8 \ , \\ C_9(\mu), & \text{for } i = 9 \ , \end{cases} \tag{343}$$

where $\vec{y} = (0, 0, -\frac{1}{3}, -\frac{4}{9}, -\frac{20}{3}, -\frac{80}{9})$ and $\vec{z} = (0, 0, 1, -\frac{1}{6}, 20, -\frac{10}{3})$.

The transformations of the Wilson coefficients from the matching scale μ_W to the scale μ_b in the standard operator basis (Eq. (25)) are given by [49]:

$$\vec{C}^{(0)\rm eff}(\mu_b) = U^{(0)}\vec{C}^{(0)\rm eff}(\mu_W) \ , \tag{344}$$

$$\vec{C}^{(1)\rm eff}(\mu_b) = \eta \left[U^{(0)}\vec{C}^{(1)\rm eff}(\mu_W) + U^{(1)}\vec{C}^{(0)\rm eff}(\mu_W)\right] \ , \tag{345}$$

$$\vec{C}^{(2)\rm eff}(\mu_b) = \eta^2 \left[U^{(0)}\vec{C}^{(2)\rm eff}(\mu_W) + U^{(1)}\vec{C}^{(1)\rm eff}(\mu_W) + U^{(2)}\vec{C}^{(0)\rm eff}(\mu_W)\right] \ , \tag{346}$$

where $\eta = \alpha_s(\mu_W)/\alpha_s(\mu_b)$ and $\vec{C} = \{C_1, \cdots, C_9\}$. The $U^{(n)}$ matrix elements read

$$U_{kl}^{(n)} = \sum_{j=0}^{n}\sum_{i=1}^{9} m_{kli}^{(nj)} \eta^{a_i - j} \ . \tag{347}$$

The powers a_i are given in Table 5. The $m_{kli}^{(nj)}$ relevant in our calculations for the $U_{kl}^{(n)}$ are given in Tables 6–10.

i	1	2	3	4	5	6	7	8	9
a_i	14/23	16/23	6/23	−12/23	0.4086	−0.4230	−0.8994	0.1456	−1

Table 5: Values of the RGE a_i numbers.

i	1	2	3	4	5	6	7	8
$m_{11i}^{(00)}$	0	0	0.3333	0.6667	0	0	0	0
$m_{12i}^{(00)}$	0	0	1	-1	0	0	0	0
$m_{21i}^{(00)}$	0	0	0.2222	-0.2222	0	0	0	0
$m_{22i}^{(00)}$	0	0	0.6667	0.3333	0	0	0	0
$m_{31i}^{(00)}$	0	0	0.0106	0.0247	-0.0129	-0.0497	0.0092	0.0182
$m_{32i}^{(00)}$	0	0	0.0317	-0.0370	-0.0659	0.0595	-0.0218	0.0335
$m_{34i}^{(00)}$	0	0	0	0	-0.1933	0.1579	0.1428	-0.1074
$m_{41i}^{(00)}$	0	0	0.0159	-0.0741	0.0046	0.0144	0.0562	-0.0171
$m_{42i}^{(00)}$	0	0	0.0476	0.1111	0.0237	-0.0173	-0.1336	-0.0316
$m_{44i}^{(00)}$	0	0	0	0	0.0695	-0.0459	0.8752	0.1012
$m_{51i}^{(00)}$	0	0	-0.0026	-0.0062	0.0018	0.0083	-0.0004	-0.0009
$m_{52i}^{(00)}$	0	0	-0.0079	0.0093	0.0094	-0.0100	0.0010	-0.0017
$m_{54i}^{(00)}$	0	0	0	0	0.0274	-0.0264	-0.0064	0.0055
$m_{61i}^{(00)}$	0	0	-0.0040	0.0185	0.0021	-0.0136	-0.0043	0.0012
$m_{62i}^{(00)}$	0	0	-0.0119	-0.0278	0.0108	0.0163	0.0103	0.0023
$m_{64i}^{(00)}$	0	0	0	0	0.0317	0.0432	-0.0675	-0.0074
$m_{71i}^{(00)}$	0.5784	-0.3921	-0.1429	0.0476	-0.1275	0.0317	0.0078	-0.0031
$m_{72i}^{(00)}$	2.2996	-1.0880	-0.4286	-0.0714	-0.6494	-0.0380	-0.0185	-0.0057
$m_{73i}^{(00)}$	8.0780	-5.2777	0	0	-2.8536	0.1281	0.1495	-0.2244
$m_{74i}^{(00)}$	5.7064	-3.8412	0	0	-1.9043	-0.1008	0.1216	0.0183
$m_{75i}^{(00)}$	202.9010	-149.4668	0	0	-55.2813	2.6494	0.7191	-1.5213
$m_{76i}^{(00)}$	86.4618	-59.6604	0	0	-25.4430	-1.2894	0.0228	-0.0917
$m_{77i}^{(00)}$	0	1	0	0	0	0	0	0
$m_{78i}^{(00)}$	2.6667	-2.6667	0	0	0	0	0	0
$m_{81i}^{(00)}$	0.2169	0	0	0	-0.1793	-0.0730	0.0240	0.0113
$m_{82i}^{(00)}$	0.8623	0	0	0	-0.9135	0.0873	-0.0571	0.0209
$m_{84i}^{(00)}$	2.1399	0	0	0	-2.6788	0.2318	0.3741	-0.0670
$m_{88i}^{(00)}$	1	0	0	0	0	0	0	0

Table 6: Values of the $m_{kli}^{(00)}$ relevant for $U_{kl}^{(0)}$ [49].

i	1	2	3	4	5	6	7	8
$m_{12i}^{(10)}$	0	0	-2.9606	-4.0951	0	0	0	0
$m_{12i}^{(11)}$	0	0	5.9606	1.0951	0	0	0	0
$m_{22i}^{(10)}$	0	0	-1.9737	1.3650	0	0	0	0
$m_{22i}^{(11)}$	0	0	1.9737	-1.3650	0	0	0	0
$m_{32i}^{(10)}$	0	0	-0.0940	-0.1517	-0.2327	0.2288	0.1455	-0.4760
$m_{32i}^{(11)}$	0	0	-0.5409	1.6332	1.6406	-1.6702	-0.2576	-0.2250
$m_{42i}^{(10)}$	0	0	-0.1410	0.4550	0.0836	-0.0664	0.8919	0.4485
$m_{42i}^{(11)}$	0	0	2.2203	2.0265	-4.1830	-0.7135	-1.8215	0.7996
$m_{52i}^{(10)}$	0	0	0.0235	0.0379	0.0330	-0.0383	-0.0066	0.0242
$m_{52i}^{(11)}$	0	0	0.0400	-0.1861	-0.1669	0.1887	0.0201	0.0304
$m_{62i}^{(10)}$	0	0	0.0352	-0.1138	0.0382	0.0625	-0.0688	-0.0327
$m_{62i}^{(11)}$	0	0	-0.2614	-0.1918	0.4197	0.0295	0.1474	-0.0640
$m_{71i}^{(10)}$	0.0021	-1.4498	0.8515	0.0521	0.6707	0.1220	-0.0578	0.0355
$m_{71i}^{(11)}$	-4.3519	3.0646	1.5169	-0.5013	0.3934	-0.6245	0.2268	0.0496
$m_{72i}^{(10)}$	9.9372	-7.4878	1.2688	-0.2925	-2.2923	-0.1461	0.1239	0.0812
$m_{72i}^{(11)}$	-17.3023	8.5027	4.5508	0.7519	2.0040	0.7476	-0.5385	0.0914
$m_{74i}^{(10)}$	-8.6840	8.5586	0	0	0.7579	0.4446	0.3093	0.4318
$m_{74i}^{(11)}$	-42.9356	30.0198	0	0	5.8768	1.9845	3.5291	-0.2929
$m_{77i}^{(10)}$	0	7.8152	0	0	0	0	0	0
$m_{77i}^{(11)}$	0	-7.8152	0	0	0	0	0	0
$m_{78i}^{(10)}$	17.9842	-18.7604	0	0	0	0	0	0
$m_{78i}^{(11)}$	-20.0642	20.8404	0	0	0	0	0	0
$m_{82i}^{(10)}$	3.7264	0	0	0	-3.2247	0.3359	0.3812	-0.2968
$m_{82i}^{(11)}$	-5.8157	0	1.4062	-3.9895	3.2850	3.6851	-0.1424	0.6492
$m_{88i}^{(10)}$	6.7441	0	0	0	0	0	0	0
$m_{88i}^{(11)}$	-6.7441	0	0	0	0	0	0	0

Table 7: Values of the $m_{kli}^{(1j)}$ relevant for $U_{kl}^{(1)}$ [49].

i	1	2	3	4	5	6	7	8
$m^{(20)}_{72i}$	-212.4136	167.6577	5.7465	-3.7262	28.8574	-2.1262	2.2903	0.1462
$m^{(21)}_{72i}$	-74.7681	58.5182	-13.4731	3.0791	7.0744	2.8757	3.5962	-1.2982
$m^{(22)}_{72i}$	31.4443	-18.1165	23.2117	13.2771	-19.8699	4.0279	-8.6259	2.6149
$m^{(20)}_{77i}$	0	44.4252	0	0	0	0	0	0
$m^{(21)}_{77i}$	0	-61.0768	0	0	0	0	0	0
$m^{(22)}_{77i}$	0	16.6516	0	0	0	0	0	0
$m^{(20)}_{78i}$	15.4051	-18.7662	0	0	0	0	0	0
$m^{(21)}_{78i}$	-135.3141	146.6159	0	0	0	0	0	0
$m^{(22)}_{78i}$	36.4636	-44.4043	0	0	0	0	0	0

Table 8: Values of the $m^{(2j)}_{kli}$ relevant for $U^{(2)}_{kl}$ [49].

i	1	2	3	4	5	6	7	8	9
$m^{(00)}_{91i}$	0	0	-0.0328	-0.0404	0.0021	-0.0289	-0.0174	-0.0010	0.1183
$m^{(00)}_{92i}$	0	0	-0.0985	0.0606	0.0108	0.0346	0.0412	-0.0018	-0.0469
$m^{(00)}_{93i}$	0	0	0	0	0.0476	-0.1167	-0.3320	-0.0718	0.4729
$m^{(00)}_{94i}$	0	0	0	0	0.0318	0.0918	-0.2700	0.0059	0.1405
$m^{(00)}_{95i}$	0	0	0	0	0.9223	-2.4126	-1.5972	-0.4870	3.57455
$m^{(00)}_{96i}$	0	0	0	0	0.4245	1.1742	-0.0507	-0.0293	-1.5186
$m^{(00)}_{99i}$	0	0	0	0	0	0	0	0	1
$m^{(10)}_{91i}$	0	0	0.1958	-0.0442	-0.0112	-0.1111	0.1283	0.0114	-0.3596
$m^{(10)}_{92i}$	0	0	0.2917	0.2482	0.0382	0.1331	-0.2751	0.0260	-0.8794
$m^{(10)}_{93i}$	0	0	0	0	-0.1041	-0.5696	9.5004	0.0396	-0.4856
$m^{(10)}_{94i}$	0	0	0	0	-0.0126	-0.4049	-0.6870	0.1382	0.4172
$m^{(10)}_{95i}$	0	0	0	0	4.7639	-35.0057	30.7862	5.5105	62.3651
$m^{(10)}_{96i}$	0	0	0	0	-1.9027	-1.8789	-43.9516	1.9612	54.4557
$m^{(11)}_{91i}$	0	0	0.2918	0.0484	-0.0331	-0.0269	0.0200	-0.1094	0
$m^{(11)}_{92i}$	0	0	0.8754	-0.0725	-0.1685	0.0323	-0.0475	-0.2018	0
$m^{(11)}_{93i}$	0	0	0	0	-0.7405	-0.1088	0.3825	-7.9139	0
$m^{(11)}_{94i}$	0	0	0	0	-0.4942	0.0856	0.3111	0.6465	0
$m^{(11)}_{95i}$	0	0	0	0	-14.3464	-2.2495	1.8402	-53.6643	0
$m^{(11)}_{96i}$	0	0	0	0	-6.6029	1.0948	0.0584	-3.2339	0

Table 9: Values of the $m^{(00)}_{9li}$ and $m^{(1j)}_{9li}$ relevant for $U^{(0)}_{9l}$ and $U^{(1)}_{9l}$ [50].

i	1	2	3	4	5	6	7	8	9
$m^{(20)}_{91i}$	0	0	0.6878	-0.9481	-0.1928	-0.8077	-0.2554	0.0562	-0.6436
$m^{(20)}_{92i}$	0	0	1.3210	3.1616	-0.4814	1.9362	-5.0873	0.0468	-13.5825
$m^{(20)}_{93i}$	0	0	0	0	-2.5758	-5.8751	0.0922	0.6433	7.7756
$m^{(20)}_{94i}$	0	0	0	0	-2.6194	1.1302	-27.7073	-0.8550	16.0333
$m^{(20)}_{95i}$	0	0	0	0	-6.4519	-555.931	35.1531	80.2925	102.043
$m^{(20)}_{96i}$	0	0	0	0	-53.3822	34.3969	-124.609	-32.7515	-98.8845
$m^{(21)}_{91i}$	0	0	-1.7394	0.0530	0.1741	-0.1036	-0.1478	1.2522	0
$m^{(21)}_{92i}$	0	0	-2.5918	-0.2971	-0.5949	0.1241	0.3170	2.8655	0
$m^{(21)}_{93i}$	0	0	0	0	1.6188	-0.5311	-10.9454	4.36311	0
$m^{(21)}_{94i}$	0	0	0	0	0.1967	-0.3775	0.7915	15.2328	0
$m^{(21)}_{95i}$	0	0	0	0	-74.1049	-32.6399	-35.4688	607.188	0
$m^{(21)}_{96i}$	0	0	0	0	29.5971	-1.7519	50.6366	216.094	0
$m^{(22)}_{91i}$	0	0	4.1531	-0.4627	-0.3404	-1.0326	0.0809	0.2167	0
$m^{(22)}_{92i}$	0	0	12.4592	0.6940	-1.7340	1.2360	-0.1921	0.3998	0
$m^{(22)}_{93i}$	0	0	0	0	-7.6198	-4.1683	1.5484	15.674	0
$m^{(22)}_{94i}$	0	0	0	0	-5.0848	3.2810	1.2592	-1.2804	0
$m^{(22)}_{95i}$	0	0	0	0	-147.615	-86.199	7.4486	106.285	0
$m^{(22)}_{96i}$	0	0	0	0	-67.9394	41.9523	0.2364	6.405	0

Table 10: Values of the $m^{(2j)}_{9li}$ relevant for $U^{(2)}_{9l}$ [50].

The NLO electroweak matching corrections to C_{10} in the SM are given as [41]

$$\delta C_{10}^{\rm ew}(\mu_b) = \frac{\alpha_e}{\alpha_s}\Big(\frac{4\pi}{\alpha_s}C_{10}^{(01)}(\mu_b) + C_{10}^{(11)}(\mu_b)\Big) + \frac{\alpha_e}{4\pi}C_{10}^{(12)}(\mu_b)\,, \tag{348}$$

where

$$C_{10}^{(01)}(\mu_b) = \sum_{i=1}^{8} b_i \eta^{a_i}\, C_2^{(0)}\,, \tag{349}$$

$$\begin{aligned} C_{10}^{(11)}(\mu_b) &= \sum_{i=1}^{8} \eta^{a_i+1}\Bigg[\Big(d_i^{(2a)}\eta^{-1} + d_i^{(2b)}\Big)\, C_2^{(0)} + d_i^{(1)} C_1^{(0)} + d_i^{(4)} C_4^{(0)}\Bigg] \\ &\quad -0.11060\frac{\ln\eta}{\eta}C_2^{(0)} + \left(\eta^{-1}-1\right)\Big(0.26087\, C_9^{(0)} + 1.15942\, C_{10}^{(0)}\Big)\,, \end{aligned} \tag{350}$$

$$\begin{aligned} C_{10}^{(12)}(\mu_b) &= \sum_{i=1}^{8} \eta^{a_i+2}\Bigg[\Big(e_i^{(1a)}\eta^{-1} + e_i^{(1b)}\Big)\, C_1^{(1)} + \Big(e_i^{(4a)}\eta^{-1} + e_i^{(4b)}\Big)\, C_4^{(1)} + \sum_{j=1}^{6} e_i^{(j)} C_j^{(2)}\Bigg] \\ &\quad + \Big(0.27924\, C_1^{(1)} + 0.33157\, C_4^{(1)} + 2.35917\, C_9^{(0)} + 3.29679\, C_{10}^{(0)}\Big)\ln\eta \\ &\quad + (1-\eta)\Big(0.26087\, C_9^{(1)} + 1.15942\, C_{10}^{(1)}\Big) + C_{10}^{(12)}(\mu_W)\,, \end{aligned} \tag{351}$$

with $C_{10}^{(12)}(\mu_W)$ given by the following interpolation:

$$\begin{aligned} C_{10}^{(12)}(\mu_W) &= \frac{1}{s_W^{2\,\rm OS}}\Big[46.9288 - 3.1023\,\log(\mu_W^2) + 0.099297\,\log^2(\mu_W^2) \\ &\quad +0.175877\,(m_t - 163.5) + 0.0173725\,(M_h - 125.9)\Big] - \frac{G_\mu^{(1)}}{G_\mu^{(0)}} C_{10}^{(0)}(\mu_W)\,, \end{aligned} \tag{352}$$

where [42, 43]

$$\frac{G_\mu^{(1)}}{G_\mu^{(0)}} = \frac{4\pi}{\alpha_e}\Delta r = \frac{4\pi}{\alpha_e}\big[\Delta\alpha + \Delta r^{\delta\rho} + \Delta r_{\rm rem}\big] \approx \frac{4\pi}{\alpha_e}\big[0.06 - 0.03 + 0.01\big]\,. \tag{353}$$

In the above equations, M_h is the Higgs mass and the powers a_i' and magic numbers b_i, $d_i^{(j)}$ and $e_i^{(j)}$ are given in Table 11.

i	1	2	3	4	5	6	7	8
a'_i	-2	-1	-0.899395	-0.521739	-0.422989	0.145649	0.260870	0.408619
b_i	0.00354	0.01223	−0.00977	−0.01070	−0.00572	0.00022	0.01137	−0.00117
$d_i^{(2a)}$	0	0	0.61602	0.44627	0.57472	0.08573	−0.48807	−0.24089
$d_i^{(2b)}$	−1.18162	0.22940	0.06522	−0.04380	−0.02201	−0.00316	−0.03366	−0.00414
$d_i^{(1)}$	0.01117	−0.03088	0.00411	0.00713	0.00478	0.00012	0.00379	−0.00023
$d_i^{(4)}$	−0.00799	−0.03666	0.06300	0	−0.01519	−0.00071	0	−0.00344
$e_i^{(1a)}$	0	0	−0.25941	−0.29751	−0.48014	0.04647	−0.16269	−0.04728
$e_i^{(1b)}$	1.13374	0.09381	−0.03041	0.00781	0.01838	−0.00138	−0.02259	0.00121
$e_i^{(4a)}$	0	0	−4.03683	0	1.52565	−0.27461	0	−0.70642
$e_i^{(4b)}$	3.38669	−0.10885	0.16283	0	0.06697	−0.01681	0	0.00137
$e_i^{(1)}$	0.01117	−0.03088	0.00411	0.00713	0.00478	0.00012	0.00379	−0.00023
$e_i^{(2)}$	0.00354	0.01223	−0.00977	−0.01070	−0.00572	0.00022	0.01137	−0.00117
$e_i^{(3)}$	0.02179	−0.12336	0.07870	0	0.01930	0.00873	0	−0.00516
$e_i^{(4)}$	−0.00799	−0.03666	0.06400	0	−0.01519	−0.00071	0	−0.00344
$e_i^{(5)}$	0.19550	−0.93249	0.37858	0	0.39909	0.05921	0	−0.09989
$e_i^{(6)}$	−0.17154	0.39616	0.01201	0	−0.19423	0.00357	0	−0.04597

Table 11: Numerical values of a'_i, b_i, $d_i^{(j)}$ and $e_i^{(j)}$ relevant for C_{10} electroweak corrections [41].

D.2 RGE the $C_{1 \cdots 8}$ Wilson coefficients in the traditional operator basis

The operators in the traditional basis can be expressed as [48]

$$\begin{aligned}
P_1 &= (\bar{s}_L^\alpha \gamma_\mu c_L^\beta)(\bar{c}_L^\beta \gamma^\mu b_L^\alpha) , \\
P_2 &= (\bar{s}_L^\alpha \gamma_\mu c_L^\alpha)(\bar{c}_L^\beta \gamma^\mu b_L^\beta) , \\
P_3 &= (\bar{s}_L^\alpha \gamma_\mu b_L^\alpha) \sum_q (\bar{q}_L^\beta \gamma^\mu q_L^\beta) , \\
P_4 &= (\bar{s}_L^\alpha \gamma_\mu b_L^\beta) \sum_q (\bar{q}_L^\beta \gamma^\mu q_L^\alpha) , \\
P_5 &= (\bar{s}_L^\alpha \gamma_\mu b_L^\alpha) \sum_q (\bar{q}_R^\beta \gamma^\mu q_R^\beta) , \\
P_6 &= (\bar{s}_L^\alpha \gamma_\mu b_L^\beta) \sum_q (\bar{q}_R^\beta \gamma^\mu q_R^\alpha) .
\end{aligned} \tag{354}$$

The operators P_7 and P_8 are identical to O_7 and O_8, and the C_7 and C_8 coefficients are therefore indistinguishable in both bases.

We derive the transition formulas from the standard basis to the traditional basis at the matching scale. At LO, they are as follows:

$$C_1^{(0)\mathrm{trad}}(\mu_W) = \frac{1}{2}\,C_1^{(0)}(\mu_W)\,, \tag{355}$$

$$C_2^{(0)\mathrm{trad}}(\mu_W) = -\frac{1}{6}\,C_1^{(0)}(\mu_W) + C_2^{(0)}(\mu_W)\,, \tag{356}$$

$$C_3^{(0)\mathrm{trad}}(\mu_W) = C_3^{(0)}(\mu_W) - \frac{1}{6}\,C_4^{(0)}(\mu_W) + 16\,C_5^{(0)}(\mu_W) - \frac{8}{3}\,C_6^{(0)}(\mu_W)\,, \tag{357}$$

$$C_4^{(0)\mathrm{trad}}(\mu_W) = \frac{1}{2}\,C_4^{(0)}(\mu_W) + 8\,C_6^{(0)}(\mu_W)\,, \tag{358}$$

$$C_5^{(0)\mathrm{trad}}(\mu_W) = C_3^{(0)}(\mu_W) - \frac{1}{6}\,C_4^{(0)}(\mu_W) + 4\,C_5^{(0)}(\mu_W) - \frac{2}{3}\,C_6^{(0)}(\mu_W)\,, \tag{359}$$

$$C_6^{(0)\mathrm{trad}}(\mu_W) = \frac{1}{2}\,C_4^{(0)}(\mu_W) + 2\,C_6^{(0)}(\mu_W)\,. \tag{360}$$

At NLO they read:

$$C_1^{(1)\mathrm{trad}}(\mu_W) = -\frac{5}{6}\,C_1^{(0)}(\mu_W) - 2\,C_2^{(0)}(\mu_W) + \frac{1}{2}C_1^{(1)}(\mu_W)\,, \tag{361}$$

$$C_2^{(1)\mathrm{trad}}(\mu_W) = -\frac{11}{18}\,C_1^{(0)}(\mu_W) + \frac{2}{3}\,C_2^{(0)}(\mu_W) - \frac{1}{6}\,C_1^{(1)}(\mu_W) + C_2^{(1)}(\mu_W)\,, \tag{362}$$

$$\begin{aligned} C_3^{(1)\mathrm{trad}}(\mu_W) &= \frac{2}{3}\,C_3^{(0)}(\mu_W) - \frac{11}{18}\,C_4^{(0)}(\mu_W) + \frac{736}{9}\,C_5^{(0)}(\mu_W) - \frac{350}{27}\,C_6^{(0)}(\mu_W) \\ &\quad + C_3^{(1)}(\mu_W) - \frac{1}{6}\,C_4^{(1)}(\mu_W) + 16\,C_5^{(1)}(\mu_W) - \frac{8}{3}\,C_6^{(1)}(\mu_W)\,, \end{aligned} \tag{363}$$

$$\begin{aligned} C_4^{(1)\mathrm{trad}}(\mu_W) &= -2\,C_3^{(0)}(\mu_W) - \frac{5}{6}\,C_4^{(0)}(\mu_W) - \frac{160}{3}\,C_5^{(0)}(\mu_W) - \frac{322}{9}\,C_6^{(0)}(\mu_W) \\ &\quad + \frac{1}{2}\,C_4^{(1)}(\mu_W) + 8\,C_6^{(1)}(\mu_W)\,, \end{aligned} \tag{364}$$

$$\begin{aligned} C_5^{(1)\mathrm{trad}}(\mu_W) &= -\frac{2}{3}\,C_3^{(0)}(\mu_W) + \frac{11}{18}\,C_4^{(0)}(\mu_W) - \frac{680}{9}\,C_5^{(0)}(\mu_W) - \frac{20}{27}\,C_6^{(0)}(\mu_W) \\ &\quad + C_3^{(1)}(\mu_W) - \frac{1}{6}\,C_4^{(1)}(\mu_W) + 4\,C_5^{(1)}(\mu_W) - \frac{2}{3}\,C_6^{(1)}(\mu_W)\,, \end{aligned} \tag{365}$$

$$\begin{aligned} C_6^{(1)\mathrm{trad}}(\mu_W) &= 2\,C_3^{(0)}(\mu_W) + \frac{5}{6}\,C_4^{(0)}(\mu_W) + \frac{104}{3}\,C_5^{(0)}(\mu_W) + \frac{404}{9}\,C_6^{(0)}(\mu_W) \\ &\quad + \frac{1}{2}\,C_4^{(1)}(\mu_W) + 2\,C_6^{(1)}(\mu_W)\,. \end{aligned} \tag{366}$$

In the following we omit the superscript "trad".

The effective coefficients in this basis are different and given below [48,51]:

$$C_i^{\text{eff}}(\mu) = \begin{cases} C_i(\mu), & \text{for } i = 1, ..., 6 \ , \\ C_7(\mu) + \sum_{j=1}^{6} y_j C_j(\mu), & \text{for } i = 7 \ , \\ C_8(\mu) + \sum_{j=1}^{6} z_j C_j(\mu), & \text{for } i = 8 \ , \end{cases} \tag{367}$$

where $\vec{y} = (0, 0, 0, 0, -\frac{1}{3}, -1)$ and $\vec{z} = (0, 0, 0, 0, 1, 0)$.

The RGE formulas for the Wilson coefficients to the scale μ_b are:

$$\vec{C}^{(0)\text{eff}}(\mu_b) = V^{(0)} \vec{C}^{(0)\text{eff}}(\mu_W) \ , \tag{368}$$

$$\vec{C}^{(1)\text{eff}}(\mu_b) = \eta \left[V^{(0)} \vec{C}^{(1)\text{eff}}(\mu_W) + V^{(1)} \vec{C}^{(0)\text{eff}}(\mu_W) \right] \ , \tag{369}$$

where $\eta = \alpha_s(\mu_W)/\alpha_s(\mu_b)$ and $\vec{C} = \{C_1, \cdots, C_8\}$. The $V^{(n)}$ matrix elements read

$$V_{kl}^{(n)} = \sum_{j=0}^{n} \sum_{i=1}^{8} l_{kli}^{(nj)} \eta^{a_i - j} \ . \tag{370}$$

The powers a_i are given in Table 5. The $l_{kli}^{(nj)}$ relevant in our calculations for the $V_{kl}^{(n)}$ are given in Tables 12–14.

D.3 Renormalization group equations for C_{Q_1,Q_2}

The RGE for C_{Q_1,Q_2} are:

$$C_{Q_1,Q_2}(\mu_b) = \eta^{-12/23} C_{Q_1,Q_2}(\mu_W) \ , \tag{371}$$

where $\eta = \alpha_s(\mu_W)/\alpha_s(\mu_b)$.

D.4 Renormalization group equations for the prime Wilson coefficients

The RGE for $C'_{7,8}$ are:

$$C'_7(\mu_b) = \eta^{16/23} C'_7(\mu_W) \ , \tag{372}$$

$$C'_8(\mu_b) = \eta^{14/23} C'_8(\mu_W) \ , \tag{373}$$

(374)

where $\eta = \alpha_s(\mu_W)/\alpha_s(\mu_b)$. $C'_{9,10}$ do not run, so that:

$$C'_{9,10}(\mu_b) = C'_{9,10}(\mu_W) \ . \tag{375}$$

Finally, the RGE for C_{Q_1,Q_2} are:

$$C'_{Q_1,Q_2}(\mu_b) = \eta^{-12/23} C_{Q_1,Q_2}(\mu_W) \ . \tag{376}$$

i	1	2	3	4	5	6	7	8
$l^{(00)}_{11i}$	0	0	0.5	0.5	0	0	0	0
$l^{(00)}_{12i}$	0	0	0.5	−0.5	0	0	0	0
$l^{(00)}_{21i}$	0	0	0.5	−0.5	0	0	0	0
$l^{(00)}_{22i}$	0	0	0.5	0.5	0	0	0	0
$l^{(00)}_{31i}$	0	0	−0.0714286	−0.166667	0.0369877	0.187713	0.00572944	0.00766546
$l^{(00)}_{32i}$	0	0	−0.0714286	0.166667	0.0509536	−0.140341	−0.011259	0.00540834
$l^{(00)}_{33i}$	0	0	0	0	0.286846	0.657881	0.00612913	0.049144
$l^{(00)}_{34i}$	0	0	0	0	0.328744	−0.32628	−0.0448363	0.0423726
$l^{(00)}_{35i}$	0	0	0	0	−0.0629326	−0.184577	0.0845972	0.162912
$l^{(00)}_{36i}$	0	0	0	0	0.0447397	−0.261048	0.222642	−0.00633396
$l^{(00)}_{41i}$	0	0	−0.0714286	0.166667	0.07142	−0.162432	−0.00795682	0.00373054
$l^{(00)}_{42i}$	0	0	−0.0714286	−0.166667	0.0983869	0.12144	0.0156361	0.00263207
$l^{(00)}_{43i}$	0	0	0	0	0.553874	−0.569279	−0.00851189	0.0239168
$l^{(00)}_{44i}$	0	0	0	0	0.634775	0.282337	0.0622669	0.0206214
$l^{(00)}_{45i}$	0	0	0	0	−0.121517	0.159718	−0.117485	0.0792842
$l^{(00)}_{46i}$	0	0	0	0	0.0863884	0.22589	−0.309196	−0.00308254
$l^{(00)}_{51i}$	0	0	0	0	−0.0287888	−0.0156196	0.00125475	0.0431536
$l^{(00)}_{52i}$	0	0	0	0	−0.0396589	0.0116778	−0.00246572	0.0304469
$l^{(00)}_{53i}$	0	0	0	0	−0.223262	−0.0547423	0.00134228	0.276662
$l^{(00)}_{54i}$	0	0	0	0	−0.255872	0.0271497	−0.00981913	0.238542
$l^{(00)}_{55i}$	0	0	0	0	0.0489825	0.0153586	0.0185267	0.917132
$l^{(00)}_{56i}$	0	0	0	0	−0.0348224	0.0217218	0.0487584	−0.0356578
$l^{(00)}_{61i}$	0	0	0	0	0.0243034	−0.0319436	0.023497	−0.0158568
$l^{(00)}_{62i}$	0	0	0	0	0.03348	0.0238823	−0.0461745	−0.0111877
$l^{(00)}_{63i}$	0	0	0	0	0.188477	−0.111954	0.0251362	−0.10166
$l^{(00)}_{64i}$	0	0	0	0	0.216007	0.0555241	−0.183878	−0.0876523
$l^{(00)}_{65i}$	0	0	0	0	−0.0413509	0.03141	0.346942	−0.337001
$l^{(00)}_{66i}$	0	0	0	0	0.029397	0.0444233	0.913077	0.0131025
$l^{(00)}_{71i}$	1.92331	−1.14694	−0.428571	0.0714286	−0.471374	0.0508102	0.00943952	−0.00810992
$l^{(00)}_{72i}$	2.29959	−1.08798	−0.428571	−0.0714286	−0.649357	−0.0379876	−0.0185498	−0.00572193
$l^{(00)}_{73i}$	14.2158	−10.6963	0	0	−3.65558	0.178076	0.010098	−0.0519935
$l^{(00)}_{74i}$	15.3446	−10.948	0	0	−4.18953	−0.0883176	−0.0738699	−0.0448295
$l^{(00)}_{75i}$	−6.13775	5.41867	0	0	0.802017	−0.0499613	0.139378	−0.172358
$l^{(00)}_{76i}$	−1.23905	1.50636	0	0	−0.570166	−0.0706605	0.366812	0.00670122
$l^{(00)}_{77i}$	0	1	0	0	0	0	0	0
$l^{(00)}_{78i}$	2.66667	-2.66667	0	0	0	0	0	0
$l^{(00)}_{81i}$	0.721243	0	0	0	−0.663114	−0.116803	0.0290479	0.0296268
$l^{(00)}_{82i}$	0.862347	0	0	0	−0.913494	0.0873264	−0.0570826	0.0209031
$l^{(00)}_{83i}$	5.33091	0	0	0	−5.14256	−0.409363	0.0310743	0.18994
$l^{(00)}_{84i}$	5.75422	0	0	0	−5.8937	0.203026	−0.227317	0.163769
$l^{(00)}_{85i}$	−2.30166	0	0	0	1.12825	0.114852	0.428902	0.62965
$l^{(00)}_{86i}$	−0.464643	0	0	0	−0.802091	0.162435	1.12878	−0.0244806
$l^{(00)}_{88i}$	1	0	0	0	0	0	0	0

Table 12: Values of the $l^{(00)}_{kli}$ relevant for $V^{(0)}_{kl}$.

i	1	2	3	4	5	6	7	8
$l^{(10)}_{11i}$	0	0	−0.813642	0.714241	0	0	0	0
$l^{(10)}_{12i}$	0	0	−0.813642	−0.714241	0	0	0	0
$l^{(10)}_{21i}$	0	0	−0.813642	−0.714241	0	0	0	0
$l^{(10)}_{22i}$	0	0	−0.813642	0.714241	0	0	0	0
$l^{(10)}_{31i}$	0	0	0.116235	−0.23808	0.091958	1.0191	−0.0429392	−0.0694057
$l^{(10)}_{32i}$	0	0	0.116235	0.23808	0.219879	−0.0708733	0.0941505	−0.0650574
$l^{(10)}_{33i}$	0	0	0	0	0.899547	4.95376	−0.0263952	−0.477143
$l^{(10)}_{34i}$	0	0	0	0	0.785234	2.92566	0.138935	−0.406354
$l^{(10)}_{35i}$	0	0	0	0	−1.0542	−3.33502	−3.05229	0.533491
$l^{(10)}_{36i}$	0	0	0	0	−0.0548816	1.82004	−1.33142	−0.449073
$l^{(10)}_{41i}$	0	0	0.116235	0.23808	0.177563	−0.881851	0.0596323	−0.0337776
$l^{(10)}_{42i}$	0	0	0.116235	−0.23808	0.424566	0.0613283	−0.130752	−0.0316614
$l^{(10)}_{43i}$	0	0	0	0	1.73695	−4.2866	0.0366567	−0.232211
$l^{(10)}_{44i}$	0	0	0	0	1.51622	−2.53164	−0.192947	−0.19776
$l^{(10)}_{45i}$	0	0	0	0	−2.03556	2.88586	4.2389	0.259633
$l^{(10)}_{46i}$	0	0	0	0	−0.105971	−1.57492	1.84902	−0.21855
$l^{(10)}_{51i}$	0	0	0	0	−0.071574	−0.0847995	−0.00940367	−0.390727
$l^{(10)}_{52i}$	0	0	0	0	−0.171139	0.00589737	0.0206189	−0.366248
$l^{(10)}_{53i}$	0	0	0	0	−0.700148	−0.412203	−0.00578055	−2.68613
$l^{(10)}_{54i}$	0	0	0	0	−0.611174	−0.243444	0.0304267	−2.28762
$l^{(10)}_{55i}$	0	0	0	0	0.820517	0.277507	−0.668451	3.00335
$l^{(10)}_{56i}$	0	0	0	0	0.0427162	−0.151445	−0.29158	−2.52811
$l^{(10)}_{61i}$	0	0	0	0	0.0604226	−0.173424	−0.176098	0.143573
$l^{(10)}_{62i}$	0	0	0	0	0.144475	0.0120607	0.386121	0.134578
$l^{(10)}_{63i}$	0	0	0	0	0.591063	−0.842998	−0.10825	0.987022
$l^{(10)}_{64i}$	0	0	0	0	0.515952	−0.497869	0.569787	0.840587
$l^{(10)}_{65i}$	0	0	0	0	−0.692679	0.56753	−12.5178	−1.10358
$l^{(10)}_{66i}$	0	0	0	0	−0.0360609	−0.309721	−5.46029	0.928955
$l^{(10)}_{71i}$	10.104	−9.15977	0.697408	0.102034	−1.17192	0.275851	−0.0707444	0.0734299
$l^{(10)}_{72i}$	12.2508	−9.05632	0.697408	−0.102034	−2.80215	−0.019184	0.155117	0.0688296
$l^{(10)}_{73i}$	133.186	−109.059	0	0	−11.4639	1.34089	−0.0434874	0.504809
$l^{(10)}_{74i}$	56.7737	−31.5638	0	0	−10.0071	0.79192	0.228902	0.429915
$l^{(10)}_{75i}$	−118.662	102.581	0	0	13.4348	−0.902725	−5.02879	−0.564424
$l^{(10)}_{76i}$	−16.7842	6.06809	0	0	0.699415	0.492649	−2.19357	0.475111
$l^{(10)}_{77i}$	0	7.81516	0	0	0	0	0	0
$l^{(10)}_{78i}$	17.9842	−18.7604	0	0	0	0	0	0
$l^{(10)}_{81i}$	3.789	0	0	0	−1.64862	−0.634131	−0.217699	−0.268251
$l^{(10)}_{82i}$	4.59403	0	0	0	−3.94197	0.0441005	0.477337	−0.251445
$l^{(10)}_{83i}$	49.9448	0	0	0	−16.127	−3.08245	−0.133822	−1.84414
$l^{(10)}_{84i}$	21.2901	0	0	0	−14.0776	−1.82048	0.704391	−1.57055
$l^{(10)}_{85i}$	−44.4981	0	0	0	18.8996	2.0752	−15.4749	2.06193
$l^{(10)}_{86i}$	−6.29409	0	0	0	0.983914	−1.13251	−6.75021	−1.73565
$l^{(10)}_{88i}$	6.74407	0	0	0	0	0	0	0

Table 13: Values of the $l^{(1j)}_{kli}$ relevant for $V^{(1)}_{kl}$.

i	1	2	3	4	5	6	7	8
$l^{(11)}_{11i}$	0	0	0.813642	−0.714241	0	0	0	0
$l^{(11)}_{12i}$	0	0	0.813642	0.714241	0	0	0	0
$l^{(11)}_{21i}$	0	0	0.813642	0.714241	0	0	0	0
$l^{(11)}_{22i}$	0	0	0.813642	−0.714241	0	0	0	0
$l^{(11)}_{31i}$	0	0	−0.076552	0.145488	−0.64231	−0.55328	0.0057761	0.244008
$l^{(11)}_{32i}$	0	0	−0.076552	−0.145488	−0.884835	0.413653	−0.0113507	0.172159
$l^{(11)}_{33i}$	0	0	0	0	−4.98122	−1.93909	0.00617904	1.56436
$l^{(11)}_{34i}$	0	0	0	0	−5.70879	0.961704	−0.0452015	1.34881
$l^{(11)}_{35i}$	0	0	0	0	1.09285	0.544036	0.0852862	5.18584
$l^{(11)}_{36i}$	0	0	0	0	−0.776927	0.769433	0.224455	−0.201624
$l^{(11)}_{41i}$	0	0	−0.235282	0.0396975	0.357177	0.368846	−0.000986616	−0.205332
$l^{(11)}_{42i}$	0	0	−0.235282	−0.0396975	0.49204	−0.275763	0.00193882	−0.144871
$l^{(11)}_{43i}$	0	0	0	0	2.76997	1.2927	−0.00105544	−1.3164
$l^{(11)}_{44i}$	0	0	0	0	3.17456	−0.641123	0.00772086	−1.13502
$l^{(11)}_{45i}$	0	0	0	0	−0.607717	−0.362684	−0.0145677	−4.36387
$l^{(11)}_{46i}$	0	0	0	0	0.432035	−0.512945	−0.0383392	0.169666
$l^{(11)}_{51i}$	0	0	0.0396825	−0.0925926	0.532957	0.168728	0.0615379	−0.153808
$l^{(11)}_{52i}$	0	0	0.0396825	0.0925926	0.734191	−0.126148	−0.120929	−0.108519
$l^{(11)}_{53i}$	0	0	0	0	4.13317	0.591346	0.0658307	−0.986079
$l^{(11)}_{54i}$	0	0	0	0	4.73687	−0.293281	−0.481571	−0.850211
$l^{(11)}_{55i}$	0	0	0	0	−0.906796	−0.165909	0.908628	−3.26884
$l^{(11)}_{56i}$	0	0	0	0	0.644655	−0.234646	2.39132	0.127092
$l^{(11)}_{61i}$	0	0	−0.119048	0.277778	−0.40242	−0.256146	0.139621	0.505742
$l^{(11)}_{62i}$	0	0	−0.119048	−0.277778	−0.554367	0.191504	−0.274371	0.356824
$l^{(11)}_{63i}$	0	0	0	0	−3.12083	−0.897721	0.149361	3.24236
$l^{(11)}_{64i}$	0	0	0	0	−3.57667	0.44523	−1.09262	2.7956
$l^{(11)}_{65i}$	0	0	0	0	0.684696	0.251867	2.06155	10.7484
$l^{(11)}_{66i}$	0	0	0	0	−0.486761	0.356216	5.42556	−0.417894
$l^{(11)}_{71i}$	−14.4711	8.96349	4.55076	−0.7519	1.45474	−0.999901	0.274046	0.129599
$l^{(11)}_{72i}$	−17.3023	8.50271	4.55076	0.7519	2.00402	0.747564	−0.538534	0.0914379
$l^{(11)}_{73i}$	−106.96	83.5937	0	0	11.2817	−3.50438	0.293164	0.830869
$l^{(11)}_{74i}$	−115.454	85.5607	0	0	12.9296	1.73802	−2.14458	0.716387
$l^{(11)}_{75i}$	46.1808	−42.3478	0	0	−2.47516	0.983197	4.04639	2.75433
$l^{(11)}_{76i}$	9.32267	−11.7725	0	0	1.75963	1.39054	10.6492	−0.107087
$l^{(11)}_{77i}$	0	−7.81516	0	0	0	0	0	0
$l^{(11)}_{78i}$	−20.0642	20.8404	0	0	0	0	0	0
$l^{(11)}_{81i}$	−4.86411	0	1.40619	3.98945	2.38464	−4.92905	0.0724692	0.920088
$l^{(11)}_{82i}$	−5.81573	0	1.40619	−3.98945	3.28504	3.68514	−0.142411	0.649165
$l^{(11)}_{83i}$	−35.952	0	0	0	18.4933	−17.2749	0.0775246	5.89877
$l^{(11)}_{84i}$	−38.8068	0	0	0	21.1945	8.56761	−0.567115	5.086
$l^{(11)}_{85i}$	15.5225	0	0	0	−4.05734	4.8467	1.07003	19.5544
$l^{(11)}_{86i}$	3.13358	0	0	0	2.88442	6.85471	2.8161	−0.760267
$l^{(11)}_{88i}$	−6.74407	0	0	0	0	0	0	0

Table 14: Values of the $l^{(1j)}_{kli}$ relevant for $V^{(1)}_{kl}$.

Appendix E Calculation of flavour observables

We provide here the detailed calculation of all the implemented observables for reference. The meson masses, lifetimes and form factors, as well as the CKM matrix elements which appear in the following are given in Appendix G. At the end, in Appendix H, we provide also some suggested limits for the observables which can be used in order to constrain supersymmetric models.

E.1 Branching ratio of $\bar{B} \to X_s\gamma$

The decay $\bar{B} \to X_s\gamma$ proceeds through electromagnetic penguin loops, involving W boson in the Standard Model, in addition to charged Higgs boson, chargino, neutralino and gluino loops in supersymmetric models. The contribution of neutralino and gluino loops is negligible in minimal flavour violating scenarios.
In `SuperIso`, the full NNLO calculation of the $\bar{B} \to X_s\gamma$ branching ratio is implemented based on [52, 53]. The branching ratio reads

$$\mathrm{BR}(\bar{B} \to X_s\gamma) = \mathrm{BR}(\bar{B} \to X_c e\bar{\nu})_{\exp} \left|\frac{V_{ts}^* V_{tb}}{V_{cb}}\right|^2 \frac{6\alpha}{\pi C}\left[P(E_0) + N(E_0) + \epsilon_{em}\right] , \tag{377}$$

with

$$C = \left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{\Gamma[\bar{B} \to X_c e\bar{\nu}]}{\Gamma[\bar{B} \to X_u e\bar{\nu}]} . \tag{378}$$

$P(E_0)$ and $N(E_0)$ denote respectively the perturbative and non perturbative contributions, where E_0 is a cut on the photon energy, taken to be 1.6 GeV. C can be obtained from a fit to the semileptonic moments [54]

$$\begin{aligned} C &= g(\rho)\Big[0.849 - 0.92\,\delta_{\alpha_s} + 0.0596\,\delta_b - 0.2237\big(\bar{m}_c(3\,\mathrm{GeV}) - 1\big) \\ &\quad -0.0167\,\mu_G^2 - 0.203\,\rho_D^3 + 0.004\,\rho_{LS}^3\Big] , \end{aligned} \tag{379}$$

where

$$g(\rho) = 1 - 8\rho + 8\rho^3 - \rho^4 - 12\rho^2 \ln\rho , \tag{380}$$

with $\rho = (m_c/m_b)^2$, $\delta_{\alpha_s} = \alpha_s(4.6\,\mathrm{GeV}) - 0.22$, and $\delta_b = \bar{m}_b(\bar{m}_b) - 4.18\,\mathrm{GeV}$. The numerical values of μ_G, ρ_D and ρ_{LS} are given in Appendix G.

Following [53], we can expand $P(E_0)$ as

$$\begin{aligned} P(E_0) &= P^{(0)}(\mu_b) + \left(\frac{\alpha_s(\mu_b)}{4\pi}\right)\left[P_1^{(1)}(\mu_b) + P_2^{(1)}(E_0,\mu_b)\right] \\ &+ \left(\frac{\alpha_s(\mu_b)}{4\pi}\right)^2\left[P_1^{(2)}(\mu_b) + P_2^{(2)}(E_0,\mu_b) + P_3^{(2)}(E_0,\mu_b)\right] + \mathcal{O}\left(\alpha_s^3(\mu_b)\right) , \end{aligned} \tag{381}$$

where

$$
\begin{aligned}
P^{(0)}(\mu_b) &= \left[C_7^{(0)\mathrm{eff}}(\mu_b)\right]^2 + \left[C_7'(\mu_b)\right]^2 \ , \\
P_1^{(1)}(\mu_b) &= 2\, C_7^{(0)\mathrm{eff}}(\mu_b)\, C_7^{(1)\mathrm{eff}}(\mu_b) \ , \\
P_1^{(2)}(\mu_b) &= \left[C_7^{(1)\mathrm{eff}}(\mu_b)\right]^2 + 2\, C_7^{(0)\mathrm{eff}}(\mu_b)\, C_7^{(2)\mathrm{eff}}(\mu_b) \ ,
\end{aligned}
\tag{382}
$$

and

$$
P_2^{(1)}(E_0,\mu_b) = \sum_{i,j=1}^{8} C_i^{(0)\mathrm{eff}}(\mu_b)\, C_j^{(0)\mathrm{eff}}(\mu_b)\, K_{ij}^{(1)}(E_0,\mu_b) \ + \left(C_i \leftrightarrow C_i'\right) \ , \tag{383}
$$

$$
P_3^{(2)}(E_0,\mu_b) = 2 \sum_{i,j=1}^{8} C_i^{(0)\mathrm{eff}}(\mu_b)\, C_j^{(1)\mathrm{eff}}(\mu_b)\, K_{ij}^{(1)}(E_0,\mu_b) \ , \tag{384}
$$

where the $K_{ij}^{(1)}$ can be written in the form:

$$
K_{i7}^{(1)} = \mathrm{Re}\, r_i^{(1)} - \frac{1}{2}\gamma_{i7}^{(0)\mathrm{eff}} \ln\left(\frac{\mu_b}{m_b^{1S}}\right)^2 + 2\phi_{i7}^{(1)}(\delta) \ , \qquad \text{for } i \le 6 \ , \tag{385}
$$

$$
K_{77}^{(1)} = -\frac{182}{9} + \frac{8}{9}\pi^2 - \gamma_{77}^{(0)\mathrm{eff}} \ln\left(\frac{\mu_b}{m_b^{1S}}\right)^2 + 4\,\phi_{77}^{(1)}(\delta) \ , \tag{386}
$$

$$
K_{78}^{(1)} = \frac{44}{9} - \frac{8}{27}\pi^2 - \frac{1}{2}\gamma_{87}^{(0)\mathrm{eff}} \ln\left(\frac{\mu_b}{m_b^{1S}}\right)^2 + 2\,\phi_{78}^{(1)}(\delta) \ , \tag{387}
$$

$$
K_{ij}^{(1)} = 2(1+\delta_{ij})\,\phi_{ij}^{(1)}(\delta) \ , \qquad \text{for } i,j \neq 7 \ . \tag{388}
$$

The matrix $\hat{\gamma}^{(0)\mathrm{eff}}$ and the quantities $r_i^{(1)}$ are given by [55]:

$$
\begin{aligned}
r_1^{(1)} &= \frac{833}{729} - \frac{1}{3}\Big[a(z)+b(z)\Big] + \frac{40}{243}\, i\pi \ , \\
r_2^{(1)} &= -\frac{1666}{243} + 2\Big[a(z)+b(z)\Big] - \frac{80}{81}\, i\pi \ , \\
r_3^{(1)} &= \frac{2392}{243} + \frac{8\pi}{3\sqrt{3}} + \frac{32}{9}\, X_b - a(1) + 2\, b(1) + \frac{56}{81}\, i\pi \ , \\
r_4^{(1)} &= -\frac{761}{729} - \frac{4\pi}{9\sqrt{3}} - \frac{16}{27}\, X_b + \frac{1}{6}\, a(1) + \frac{5}{3}\, b(1) + 2\, b(z) - \frac{148}{243}\, i\pi \ , \\
r_5^{(1)} &= \frac{56680}{243} + \frac{32\pi}{3\sqrt{3}} + \frac{128}{9}\, X_b - 16\, a(1) + 32\, b(1) + \frac{896}{81}\, i\pi \ , \\
r_6^{(1)} &= \frac{5710}{729} - \frac{16\pi}{9\sqrt{3}} - \frac{64}{27}\, X_b - \frac{10}{3}\, a(1) + \frac{44}{3}\, b(1) + 12a(z) + 20b(z) - \frac{2296}{243}\, i\pi \ , \\
r_8^{(1)} &= \frac{44}{9} - \frac{8}{27}\, \pi^2 + \frac{8}{9}\, i\pi \ ,
\end{aligned}
\tag{389}
$$

where

$$
z = \left(\frac{m_c(\mu_c)}{m_b^{1S}}\right)^2 \ , \tag{390}
$$

and the constant X_b can be written as

$$
X_b = \int_0^1 dx \int_0^1 dy \int_0^1 dv\ xy \ln\Big[v + x(1-x)(1-v)(1-v+vy)\Big] \simeq -0.1684 \ , \tag{391}
$$

and

$$
\hat{\gamma}^{(0)\mathrm{eff}} = \begin{bmatrix}
-4 & \frac{8}{3} & 0 & -\frac{2}{9} & 0 & 0 & -\frac{208}{243} & \frac{173}{162} \\
12 & 0 & 0 & \frac{4}{3} & 0 & 0 & \frac{416}{81} & \frac{70}{27} \\
0 & 0 & 0 & -\frac{52}{3} & 0 & 2 & -\frac{176}{81} & \frac{14}{27} \\
0 & 0 & -\frac{40}{9} & -\frac{100}{9} & \frac{4}{9} & \frac{5}{6} & -\frac{152}{243} & -\frac{587}{162} \\
0 & 0 & 0 & -\frac{256}{3} & 0 & 20 & -\frac{6272}{81} & \frac{6596}{27} \\
0 & 0 & -\frac{256}{9} & \frac{56}{9} & \frac{40}{9} & -\frac{2}{3} & \frac{4624}{243} & \frac{4772}{81} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{32}{3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{32}{9} & \frac{28}{3}
\end{bmatrix} . \tag{392}
$$

The small-m_c expansions of $a(z)$ and $b(z)$ up to $\mathcal{O}(z^4)$ read

$$\begin{aligned}
a(z) \;=\; & \frac{16}{9}\Bigg\{ \left[\frac{5}{2} - \frac{\pi^2}{3} - 3\zeta(3) + \left(\frac{5}{2} - \frac{3\pi^2}{4}\right)\ln z + \frac{1}{4}(\ln z)^2 + \frac{1}{12}(\ln z)^3\right] z \\
& + \left[\frac{7}{4} + \frac{2\pi^2}{3} - \frac{\pi^2}{2}\ln z - \frac{1}{4}(\ln z)^2 + \frac{1}{12}(\ln z)^3\right] z^2 + \left[-\frac{7}{6} - \frac{\pi^2}{4} + 2\ln z - \frac{3}{4}(\ln z)^2\right] z^3 \\
& + \left[\frac{457}{216} - \frac{5\pi^2}{18} - \frac{1}{72}\ln z - \frac{5}{6}(\ln z)^2\right] z^4 + i\pi\left[\left(4 - \frac{\pi^2}{3} + \ln z + (\ln z)^2\right)\frac{z}{2}\right. \\
& \left. + \left(\frac{1}{2} - \frac{\pi^2}{6} - \ln z + \frac{1}{2}(\ln z)^2\right) z^2 + z^3 + \frac{5}{9}z^4\right]\Bigg\} + \mathcal{O}(z^5(\ln z)^2)\,,
\end{aligned} \tag{393}$$

$$\begin{aligned}
b(z) \;=\; & -\frac{8}{9}\Bigg\{ \left[-3 + \frac{\pi^2}{6} - \ln z\right] z - \frac{2\pi^2}{3} z^{3/2} + \left[\frac{1}{2} + \pi^2 - 2\ln z - \frac{1}{2}(\ln z)^2\right] z^2 \\
& + \left[-\frac{25}{12} - \frac{1}{9}\pi^2 - \frac{19}{18}\ln z + 2(\ln z)^2\right] z^3 + \left[-\frac{1376}{225} + \frac{137}{30}\ln z + 2(\ln z)^2 + \frac{2\pi^2}{3}\right] z^4 \\
& + i\pi\left[-z + (1 - 2\ln z) z^2 + \left(-\frac{10}{9} + \frac{4}{3}\ln z\right) z^3 + z^4\right]\Bigg\} + \mathcal{O}(z^5(\ln z)^2)\,,
\end{aligned} \tag{394}$$

where $\zeta(3)$ is the Riemann zeta function given in Eq. (21). Defining

$$\delta \equiv 1 - 2E_0/m_b^{1S}\,, \tag{395}$$

the explicit form of the functions $\phi_{ij}(\delta)$ which originate from the gluon bremsstrahlung $b \to s\gamma g$, can be written as [56]:

$$\begin{aligned}
\phi_{22}(\delta) &= \frac{16z}{27}\left[\delta\int_0^{(1-\delta)/z} dt\,(1-zt)\left|\frac{G(t)}{t} + \frac{1}{2}\right|^2 + \int_{(1-\delta)/z}^{1/z} dt\,(1-zt)^2\left|\frac{G(t)}{t} + \frac{1}{2}\right|^2\right], \\
\phi_{27}(\delta) &= -\frac{8z^2}{9}\left[\delta\int_0^{(1-\delta)/z} dt\,\mathrm{Re}\left(G(t) + \frac{t}{2}\right) + \int_{(1-\delta)/z}^{1/z} dt\,(1-zt)\mathrm{Re}\left(G(t) + \frac{t}{2}\right)\right], \\
\phi_{77}(\delta) &= -\frac{2}{3}\ln^2\delta - \frac{7}{3}\ln\delta - \frac{31}{9} + \frac{10}{3}\delta + \frac{1}{3}\delta^2 - \frac{2}{9}\delta^3 + \frac{1}{3}\delta(\delta-4)\ln\delta\,, \qquad (396) \\
\phi_{78}(\delta) &= \frac{8}{9}\left[\mathrm{Li}_2(1-\delta) - \frac{\pi^2}{6} - \delta\ln\delta + \frac{9}{4}\delta - \frac{1}{4}\delta^2 + \frac{1}{12}\delta^3\right],
\end{aligned}$$

$$\phi_{88}(\delta) = \frac{1}{27}\left\{-2\ln\frac{m_b}{m_s}\left[\delta^2+2\delta+4\ln(1-\delta)\right]\right.$$
$$\left. +4\,\mathrm{Li}_2(1-\delta)-\frac{2\pi^2}{3}-\delta(2+\delta)\ln\delta+8\ln(1-\delta)-\frac{2}{3}\delta^3+3\delta^2+7\delta\right\} ,$$

with

$$G(t)=\begin{cases} -2\arctan^2\sqrt{t/(4-t)}\ , & \text{for } t<4 \\ -\pi^2/2+2\ln^2[(\sqrt{t}+\sqrt{t-4})/2]-2i\pi\ln[(\sqrt{t}+\sqrt{t-4})/2]\ , & \text{for } t\geq 4 \end{cases} \tag{397}$$

and

$$\begin{aligned} \phi_{11} &= \frac{1}{36}\phi_{22}\ , \\ \phi_{12} &= -\frac{1}{3}\phi_{22}\ , \\ \phi_{17} &= -\frac{1}{6}\phi_{27}\ , \\ \phi_{18} &= \frac{1}{18}\phi_{27}\ , \\ \phi_{28} &= -\frac{1}{3}\phi_{27}\ . \end{aligned} \tag{398}$$

The functions ϕ_{47} and ϕ_{48} are given by [53]:

$$\phi_{47}^{(1)}(\delta) = -\frac{1}{54}\delta\left(1-\delta+\frac{1}{3}\delta^2\right) + \frac{1}{12}\lim_{m_c\to m_b}\phi_{27}^{(1)}(\delta)\ , \tag{399}$$
$$\phi_{48}^{(1)}(\delta) = -\frac{1}{3}\phi_{47}^{(1)}(\delta)\ .$$

The remaining NNLO correction to $P(E_0)$ is composed of two parts:

$$P_2^{(2)}(E_0,\mu_b)=P_2^{(2)\beta_0}(E_0,\mu_b)+P_2^{(2)\mathrm{rem}}(E_0,\mu_b)\ , \tag{400}$$

with

$$P_2^{(2)\beta_0}(E_0,\mu_b)\simeq\sum_{i,j=1,2,7,8}C_i^{(0)\mathrm{eff}}(\mu_b)\,C_j^{(0)\mathrm{eff}}(\mu_b)\,K_{ij}^{(2)\beta_0}(E_0,\mu_b)\ +\left(C_i\leftrightarrow C_i'\right)\ , \tag{401}$$

where the contribution for $i,j=3,4,5,6$ are neglected, and

$$K_{27}^{(2)\beta_0} = \beta_0\,\mathrm{Re}\bigg[-\frac{3}{2}\,r_2^{(2)}(z)+2\left(a(z)+b(z)-\frac{290}{81}\right)L_b-\frac{100}{81}\,L_b^2\bigg]+2\,\phi_{27}^{(2)\beta_0}(\delta)\;, \quad (402)$$

$$K_{17}^{(2)\beta_0} = -\frac{1}{6}\,K_{27}^{(2)\beta_0}, \quad (403)$$

$$K_{77}^{(2)\beta_0} = \beta_0\bigg[-\frac{3803}{54}-\frac{46}{27}\,\pi^2+\frac{80}{3}\,\zeta(3)+\left(\frac{8}{9}\pi^2-\frac{98}{3}\right)L_b-\frac{16}{3}\,L_b^2\bigg]+4\,\phi_{77}^{(2)\beta_0}(\delta)\;, \quad (404)$$

$$K_{78}^{(2)\beta_0} = \beta_0\bigg[\frac{1256}{81}-\frac{64}{81}\,\pi^2-\frac{32}{9}\,\zeta(3)+\left(\frac{188}{27}-\frac{8}{27}\pi^2\right)L_b+\frac{8}{9}\,L_b^2\bigg]+2\,\phi_{78}^{(2)\beta_0}(\delta)\;, \quad (405)$$

$$K_{ij}^{(2)\beta_0} = 2(1+\delta_{ij})\,\phi_{ij}^{(2)\beta_0}(\delta)\;, \qquad \text{for } i,j\neq 7\;. \quad (406)$$

The small-m_c expansion of $\mathrm{Re}\,r_2^{(2)}(z)$ up to $\mathcal{O}(z^4)$ leads to

$$\begin{aligned}\mathrm{Re}\,r_2^{(2)}(z) = {} & \frac{67454}{6561}-\frac{124\pi^2}{729}-\frac{4}{1215}\Big[11280-1520\pi^2-171\pi^4-5760\zeta(3) \\ & +6840\ln z-1440\pi^2\ln z-2520\zeta(3)\ln z+120(\ln z)^2+100(\ln z)^3 \\ & -30(\ln z)^4\Big]z-\frac{64\pi^2}{243}\Big[43-12\ln 2-3\ln z\Big]z^{3/2}-\frac{2}{1215}\Big[11475-380\pi^2 \\ & +96\pi^4+7200\zeta(3)-1110\ln z-1560\pi^2\ln z+1440\zeta(3)\ln z+990(\ln z)^2 \\ & +260(\ln z)^3-60(\ln z)^4\Big]z^2+\frac{2240\pi^2}{243}z^{5/2}-\frac{2}{2187}\Big[62471-2424\pi^2 \\ & -33264\zeta(3)-19494\ln z-504\pi^2\ln z-5184(\ln z)^2+2160(\ln z)^3\Big]z^3 \\ & -\frac{2464}{6075}\pi^2z^{7/2}+\Big[-\frac{15103841}{546750}+\frac{7912}{3645}\pi^2+\frac{2368}{81}\zeta(3)+\frac{147038}{6075}\ln z \\ & +\frac{352}{243}\pi^2\ln z+\frac{88}{243}(\ln z)^2-\frac{512}{243}(\ln z)^3\Big]z^4+\mathcal{O}\big(z^{9/2}(\ln z)^4\big)\;,\end{aligned} \quad (407)$$

where

$$L_b = \ln\left(\frac{\mu_b}{m_b^{1S}}\right)^2\;. \quad (408)$$

The functions $\phi_{ij}^{(2)\beta_0}(\delta)$ read [53, 57]:

$$\phi_{22}^{(2)\beta_0}(\delta) = \beta_0 \left[\phi_{22}^{(1)}(\delta) L_b + h_{22}^{(2)}\right] , \tag{409}$$

$$\phi_{11}^{(2)\beta_0}(\delta) = \frac{1}{36}\,\phi_{22}^{(2)\beta_0}(\delta) , \tag{410}$$

$$\phi_{12}^{(2)\beta_0}(\delta) = -\frac{1}{3}\,\phi_{22}^{(2)\beta_0}(\delta) , \tag{411}$$

$$\phi_{27}^{(2)\beta_0}(\delta) = \beta_0 \left[\phi_{27}^{(1)}(\delta) L_b + h_{27}^{(2)}\right] , \tag{412}$$

$$\phi_{28}^{(2)\beta_0}(\delta) = \beta_0 \left[\phi_{28}^{(1)}(\delta) L_b + h_{28}^{(2)}\right] , \tag{413}$$

$$\phi_{18}^{(2)\beta_0}(\delta) = -\frac{1}{6}\,\phi_{28}^{(2)\beta_0}(\delta) , \tag{414}$$

$$\phi_{77}^{(2)\beta_0}(\delta) = \beta_0 \left[\phi_{77}^{(1)}(\delta) L_b + 4\int_0^{1-\delta} dx\; F^{(2,nf)}\right] , \tag{415}$$

$$\phi_{88}^{(2)\beta_0}(\delta) = \beta_0 \left[\phi_{88}^{(1)}(\delta) L_b + h_{88}^{(2)}\right] , \tag{416}$$

where

$$\begin{aligned} h_{22}^{(2)}(\delta) &= 0.01370 + 0.3357\,\delta - 0.08668\,\delta^2 + (0.3575 + 1.825\,\delta - 0.3743\,\delta^2)\,z^{\frac{1}{2}} \\ &+(-2.306 - 5.800\,\delta - 6.226\,\delta^2)\,z + (3.449 - 0.5480\,\delta + 17.27\,\delta^2)\,z^{\frac{3}{2}} , \end{aligned} \tag{417}$$

$$\begin{aligned} h_{27}^{(2)}(\delta) &= -0.1755 - 1.455\,\delta + 1.119\,\delta^2 + (0.7260 - 7.230\,\delta + 5.977\,\delta^2)\,z^{\frac{1}{2}} \\ &+(13.79 + 113.7\,\delta - 100.4\,\delta^2)\,z + (-145.1 - 307.1\,\delta + 388.5\,\delta^2)\,z^{\frac{3}{2}} \\ &+(475.2 + 313.0\,\delta - 775.8\,\delta^2)\,z^2 + (-509.7 - 126.1\,\delta + 646.2\,\delta^2)\,z^{\frac{5}{2}} , \end{aligned} \tag{418}$$

$$\begin{aligned} h_{28}^{(2)}(\delta) &= 0.02605 + 0.1679\,\delta - 0.1970\,\delta^2 + (-0.03801 + 0.6017\,\delta - 0.7558\,\delta^2)\,z^{\frac{1}{2}} \\ &+(2.755 - 10.03\,\delta + 11.27\,\delta^2)\,z + (-27.05 + 68.47\,\delta - 72.51\,\delta^2)\,z^{\frac{3}{2}} \\ &+(85.87 - 289.3\,\delta + 297.7\,\delta^2)\,z^2 + (-91.53 + 399.8\,\delta - 399.9\,\delta^2)\,z^{\frac{5}{2}} , \end{aligned} \tag{419}$$

$$\begin{aligned} h_{88}^{(2)}(\delta) &= \frac{4}{27}\left\{\left[\Big(1+\frac{1}{2}\delta\Big)\delta\ln\delta - 6\ln(1-\delta) - 2\mathrm{Li}_2(1-\delta) + \frac{1}{3}\pi^2 - \frac{16}{3}\delta - \frac{5}{3}\delta^2 + \frac{1}{9}\delta^3\right]\ln\frac{m_b}{m_s}\right. \\ &-2\mathrm{Li}_3(\delta) + (5-2\ln\delta)\left[\mathrm{Li}_2(1-\delta) - \frac{1}{6}\pi^2\right] - \frac{1}{12}\pi^2\delta\,(2+\delta) + \left[\frac{1}{2}\delta + \frac{1}{4}\delta^2 - \ln(1-\delta)\right]\ln^2\delta \\ &\left.+\Big(\frac{151}{18} - \frac{1}{3}\pi^2\Big)\ln(1-\delta) + \Big(-\frac{53}{12} - \frac{19}{12}\delta + \frac{2}{9}\delta^2\Big)\delta\ln\delta + \frac{787}{72}\delta + \frac{227}{72}\delta^2 - \frac{41}{72}\delta^3\right\} , \end{aligned} \tag{420}$$

and $F^{(2,nf)}$ is given by [58]

$$F^{(2,\mathrm{nf})} = S_{\mathrm{nf}}\delta(1-z) - \frac{1}{2}\left[\frac{\ln^2(1-z)}{1-z}\right]_+ - \frac{13}{36}\left[\frac{\ln(1-z)}{1-z}\right]_+ + \left(-\frac{\pi^2}{18}+\frac{85}{72}\right)\left[\frac{1}{1-z}\right]_+$$

$$+\frac{z^2-3}{6(z-1)}\,\mathrm{Li}_2(1-z) + \frac{z^2-3}{6(z-1)}\ln(1-z)\ln(z) - \frac{1+z}{4}\ln^2(1-z) \tag{421}$$

$$-\frac{6z^3-25z^2-z-18}{36z}\ln(1-z) - (1+z)\frac{\pi^2}{36} + \frac{-49+38z^2-55z}{72}\,,$$

where the $[\ln^n(1-x)/(1-x)]_+$ are the plus-distributions defined in the standard way, and $S_{\mathrm{nf}} = 49/24 + \pi^2/8 - 2\zeta(3)/3 \approx 2.474$. The remaining $\phi_{ij}^{(2)\beta_0}(\delta)$ functions are neglected.

The $P_2^{(2)\mathrm{rem}}$ term is more difficult to calculate, as its analytic expression is only known in the limit $m_c \gg m_b/2$. In this limit, we have

$$P_2^{(2)\mathrm{rem}}(E_0,\mu_b) \simeq \sum_{i,j=1,2,7,8} C_i^{(0)\mathrm{eff}}(\mu_b)\, C_j^{(0)\mathrm{eff}}(\mu_b)\, K_{ij}^{(2)\mathrm{rem}}(E_0,\mu_b) \;\; + \left(C_i \leftrightarrow C_i'\right)\,, \tag{422}$$

where

$$K_{22}^{(2)\mathrm{rem}} = 36\,K_{11}^{(2)\mathrm{rem}} + \mathcal{O}\left(\frac{1}{z}\right) = -6\,K_{12}^{(2)\mathrm{rem}} + \mathcal{O}\left(\frac{1}{z}\right) \tag{423}$$

$$= \left(K_{27}^{(1)}\right)^2 + \mathcal{O}\left(\frac{1}{z}\right) = \left(\frac{218}{243} - \frac{208}{81}L_D\right)^2 + \mathcal{O}\left(\frac{1}{z}\right)\,,$$

$$K_{27}^{(2)\mathrm{rem}} = K_{27}^{(1)}K_{77}^{(1)} + \left(\frac{127}{324} - \frac{35}{27}L_D\right)K_{78}^{(1)} + \frac{2}{3}(1-L_D)K_{47}^{(1)\mathrm{rem}} \tag{424}$$

$$-\frac{4736}{729}L_D^2 + \frac{1150}{729}L_D - \frac{1617980}{19683} + \frac{20060}{243}\zeta(3) + \frac{1664}{81}L_c + \mathcal{O}\left(\frac{1}{z}\right)\,,$$

$$K_{28}^{(2)\mathrm{rem}} = K_{27}^{(1)}K_{78}^{(1)} + \left(\frac{127}{324} - \frac{35}{27}L_D\right)K_{88}^{(1)} + \frac{2}{3}(1-L_D)K_{48}^{(1)} + \mathcal{O}\left(\frac{1}{z}\right)\,, \tag{425}$$

$$K_{17}^{(2)\mathrm{rem}} = -\frac{1}{6}K_{27}^{(2)\mathrm{rem}} + \left(\frac{5}{16} - \frac{3}{4}L_D\right)K_{78}^{(1)} - \frac{1237}{729} + \frac{232}{27}\zeta(3) + \frac{70}{27}L_D^2 \tag{426}$$

$$-\frac{20}{27}L_D + \mathcal{O}\left(\frac{1}{z}\right)\,,$$

$$K_{18}^{(2)\mathrm{rem}} = -\frac{1}{6}K_{28}^{(2)\mathrm{rem}} + \left(\frac{5}{16} - \frac{3}{4}L_D\right) K_{88}^{(1)} + \mathcal{O}\left(\frac{1}{z}\right) , \tag{427}$$

$$\begin{aligned} K_{77}^{(2)\mathrm{rem}} = & \left(K_{77}^{(1)} - 4\phi_{77}^{(1)}(\delta) + \frac{2}{3}\ln z\right) K_{77}^{(1)} - \frac{32}{9}L_D^2 + \frac{224}{27}L_D - \frac{628487}{729} \\ & -\frac{628}{405}\pi^4 + \frac{31823}{729}\pi^2 + \frac{428}{27}\pi^2\ln 2 + \frac{26590}{81}\zeta(3) - \frac{160}{3}L_b^2 \\ & -\frac{2720}{9}L_b + \frac{256}{27}\pi^2 L_b + \frac{512}{27}\pi\alpha_\Upsilon \ + \ 4\phi_{77}^{(2)\mathrm{rem}}(\delta) \ + \mathcal{O}\left(\frac{1}{z}\right) , \end{aligned} \tag{428}$$

$$K_{78}^{(2)\mathrm{rem}} = \left(-\frac{50}{3} + \frac{8}{3}\pi^2 - \frac{2}{3}L_D\right) K_{78}^{(1)} + \frac{16}{27}L_D^2 - \frac{112}{81}L_D + \frac{364}{243} + \mathcal{O}\left(\frac{1}{z}\right) , \tag{429}$$

$$K_{88}^{(2)\mathrm{rem}} = \left(-\frac{50}{3} + \frac{8}{3}\pi^2 - \frac{2}{3}L_D\right) K_{88}^{(1)} + \mathcal{O}\left(\frac{1}{z}\right) , \tag{430}$$

with

$$K_{47}^{(1)\mathrm{rem}} = K_{47}^{(1)} - \beta_0\left(\frac{26}{81} - \frac{4}{27}L_b\right), \tag{431}$$

$$L_c = \ln\left(\frac{\mu_c}{m_c(\mu_c)}\right)^2 , \tag{432}$$

and

$$L_D \equiv L_b - \ln z \ = \ln\left(\frac{\mu_b}{m_c(\mu_c)}\right)^2 . \tag{433}$$

The function $\phi_{77}^{(2)\mathrm{rem}}(\delta)$ reads

$$\begin{aligned} \phi_{77}^{(2)\mathrm{rem}}(\delta) = & -4\int_0^{1-\delta} dx \ \left[\frac{16}{9}F^{(2,a)} + 4F^{(2,na)} + \frac{29}{3}F^{(2,nf)}\right] \\ & -\frac{8\pi\,\alpha_\Upsilon}{27\,\delta}\left[2\delta\ln^2\delta + \left(4 + 7\delta - 2\delta^2 + \delta^3\right)\ln\delta + 7 - \frac{8}{3}\delta - 7\delta^2 + 4\delta^3 - \frac{4}{3}\delta^4\right] , \end{aligned} \tag{434}$$

where $F^{(2,a)}$ and $F^{(2,na)}$ are given by [58]:

$$
\begin{aligned}
F^{(2,\mathrm{a})} &= S_{\mathrm{a}}\delta(1-z) + \frac{1}{2}\left[\frac{\ln^3(1-z)}{1-z}\right]_+ + \frac{21}{8}\left[\frac{\ln^2(1-z)}{1-z}\right]_+ + \left(-\frac{\pi^2}{6}+\frac{271}{48}\right)\left[\frac{\ln(1-z)}{1-z}\right]_+ \\
&+ \left(\frac{425}{96}-\frac{\pi^2}{6}-\frac{\zeta(3)}{2}\right)\left[\frac{1}{1-z}\right]_+ + \frac{4z-4z^2+1+z^3}{2(z-1)}\left[\mathrm{Li}_3\left(\frac{z}{2-z}\right)-\mathrm{Li}_3\left(-\frac{z}{2-z}\right)\right. \\
&\left.-2\mathrm{Li}_3\left(\frac{1}{2-z}\right)+\frac{\zeta(3)}{4}\right] - 2(z-1)^2\mathrm{Li}_3(z-1) + \left[\frac{z^3-2z^2+2z-3}{2(z-1)}\ln(1-z)\right. \\
&\left.-\frac{-140z^4+219z^3-124z^2+28z+27z^5+9z^6+z^8-6z^7-6}{12z(z-1)^3}\right]\mathrm{Li}_2(z-1) \\
&+\left[\frac{2z^3-9z^2-2z+11}{4(z-1)}\ln(1-z)-\frac{-27z^2+8z^6-9+21z-3z^3+64z^4-46z^5}{12z(z-1)^3}\right]\mathrm{Li}_2(1-z) \\
&-\frac{-17z^2+4z+4z^3+11}{4(z-1)}\mathrm{Li}_3(1-z)-\frac{2z^3+13-9z^2}{4(z-1)}\mathrm{Li}_3(z)+\frac{4z-4z^2+1+z^3}{6(z-1)}\ln^3(2-z) \\
&+\left[-\frac{-140z^4+219z^3-124z^2+28z+27z^5+9z^6+z^8-6z^7-6}{12z(z-1)^3}\ln(1-z)\right. \\
&\left.-\frac{4z-4z^2+1+z^3}{2(z-1)}\ln^2(1-z)-\frac{4z-4z^2+1+z^3}{z-1}\frac{\pi^2}{12}\right]\ln(2-z) \\
&+\frac{z^3-2z^2+2z+1}{4z}\ln^3(1-z)+\frac{z^5-3z^4+5z^3+7z^2+5z-9}{24z}\ln^2(1-z) \\
&+\left[-\frac{z^2+8z-11}{8(z-1)}\ln^2(1-z)-\frac{-27z^2+8z^6-9+21z-3z^3+64z^4-46z^5}{12z(z-1)^3}\ln(1-z)\right]\ln(z) \\
&+\left[(-z^2+z-3)\frac{\pi^2}{12}-\frac{4z^5+151z+2z^4-48z^2-41z^3-36}{48z(z-1)}\right]\ln(1-z) \\
&+\frac{z^3-11z^2-2z+18}{4(z-1)}\zeta(3)-\frac{8z^4-244z^3+175z^2+598z-569}{96(z-1)} \\
&-\frac{(z-2)(z^4-z^3-11z^2+13z+3)}{z}\frac{\pi^2}{72}\,, \qquad (435)
\end{aligned}
$$

and

$$
\begin{aligned}
F^{(2,\mathrm{na})} &= S_{\mathrm{na}}\delta(1-z) + \frac{11}{8}\left[\frac{\ln^2(1-z)}{1-z}\right]_+ + \left(\frac{\pi^2}{12}+\frac{95}{144}\right)\left[\frac{\ln(1-z)}{1-z}\right]_+ \\
&+ \left(\frac{\zeta(3)}{4} - \frac{905}{288} + \frac{17\pi^2}{72}\right)\left[\frac{1}{1-z}\right]_+ + (z-1)^2\mathrm{Li}_3(z-1) \\
&- \frac{4z-4z^2+1+z^3}{4(z-1)}\left[\mathrm{Li}_3\left(\frac{z}{2-z}\right) - \mathrm{Li}_3\left(-\frac{z}{2-z}\right) - 2\mathrm{Li}_3\left(\frac{1}{2-z}\right) + \frac{\zeta(3)}{4}\right] \\
&+ \left[\frac{-140z^4+219z^3-124z^2+28z+27z^5+9z^6+z^8-6z^7-6}{24z(z-1)^3}\right. \\
&\left. - \frac{z^3-2z^2+2z-3}{4(z-1)}\ln(1-z)\right]\mathrm{Li}_2(z-1) + \left[\frac{(1+z)(2z^4-29z^3+73z^2-57z+15)}{24(z-1)^3}\right. \\
&\left. + \frac{z(3-z)}{4}\ln(1-z)\right]\mathrm{Li}_2(1-z) + \frac{4z-4z^2+1+z^3}{4(z-1)}\mathrm{Li}_3(z) + \frac{(z-3)z}{2}\mathrm{Li}_3(1-z) \\
&- \frac{4z-4z^2+1+z^3}{12(z-1)}\ln^3(2-z) + \left[\frac{4z-4z^2+1+z^3}{4(z-1)}\ln^2(1-z) + \frac{4z-4z^2+1+z^3}{z-1}\frac{\pi^2}{24}\right. \\
&\left. + \frac{-140z^4+219z^3-124z^2+28z+27z^5+9z^6+z^8-6z^7-6}{24z(z-1)^3}\ln(1-z)\right]\ln(2-z) \\
&+ \frac{(1+z)(2z^4-29z^3+73z^2-57z+15)}{24(z-1)^3}\ln(1-z)\ln(z) - \frac{(z-1)^2}{8}\ln^3(1-z) \\
&- \frac{(z+2)(z^3-5z^2+9z-35)}{48}\ln^2(1-z)\frac{z^5-3z^4-3z^3+34z^2-24z+3}{z}\frac{\pi^2}{144} \\
&+ \left[(z^2-z+3)\frac{\pi^2}{24} + \frac{6z^5+72-392z^3+51z^4+219z^2+92z}{144z(z-1)}\right]\ln(1-z) + \\
&- \frac{z^3-10z^2+6z+7}{8(z-1)}\zeta(3) + \frac{12z^4-754z^3+1191z^2+264z-761}{288(z-1)}\,, \qquad (436)
\end{aligned}
$$

where $S_{\mathrm{a}} = 1.216$, $S_{\mathrm{na}} = -4.795$ and

$$
\mathrm{Li}_3(x) = \int_0^x dy\, \frac{\mathrm{Li}_2(y)}{y}\,. \qquad (437)
$$

α_Υ in Eq. (434) is defined as

$$
\alpha_\Upsilon \equiv \alpha_s^{(4)}(\mu = m_b^{1S})\,. \qquad (438)
$$

The missing K_{ij} and ϕ_{ij} are neglected.

To estimate the value of $P_2^{(2)\rm rem}$ for $m_c < m_b/2$, it is necessary to use an interpolation method. We consider the linear combination:

$$P_2^{(2)\rm rem}(z) = x_1 \left[|r_2^{(1)}(z)|^2 - |r_2^{(1)}(0)|^2 \right] + x_2 \,{\rm Re} \left[r_2^{(2)}(z) - r_2^{(2)}(0) \right] \tag{439}$$
$$+ x_3 \,{\rm Re} \left[r_2^{(1)}(z) - r_2^{(2)}(0) \right] + x_4 \, z \frac{d}{dz} {\rm Re}\, r_2^{(1)}(z) + x_5 \; .$$

In this equation, $x_1, \cdots, x_5$ are constants to be determined. Noting that $P_2^{(2)\rm rem}(0) = x_5$, we make two different assumptions:

(a) $x_5 = 0$,
(b) $x_5 = -P_1^{(2)}(z=0) - P_3^{(2)}(z=0)$.

To determine the four other x_i, we impose that for $z \gg 1$, Eqs. (422) and (440) coincide, and by matching the different terms in both equations, the x_i can be worked out. In particular, we can show that

$$x_1 = \big(C_2^{(0)\rm eff}\big)^2 + \frac{1}{36} \big(C_1^{(0)\rm eff}\big)^2 - \frac{1}{3} \big(C_1^{(0)\rm eff}\big)\big(C_2^{(0)\rm eff}\big) \; , \tag{440}$$

and

$$x_2 = C_7^{(0)\rm eff} \left(\frac{4019}{486} C_1^{(0)\rm eff} - \frac{1184}{81} C_2^{(0)\rm eff} - 4 C_7^{(0)\rm eff} + \frac{4}{3} C_8^{(0)\rm eff} \right) \; . \tag{441}$$

Finally, x_3 and x_4 can be determined by choosing two large values for z and requiring matching between Eqs. (422) and (440) for both values.
The two possible determinations of x_5 lead to different results, and we therefore compute the branching ratio in both cases (a) and (b), and then we give the average value as output, as advised in [53].

The non-perturbative correction $N(E_0)$ reads [56]

$$N(E_0) = -\frac{1}{18} \left(K_c^{(0)} + r K_t^{(0)} \right) \left(\eta^{\frac{6}{23}} + \eta^{-\frac{12}{23}} \right) \frac{\lambda_2}{m_c^2} + \ldots \; , \tag{442}$$

where $r = \overline{m}_b(\mu_W)/m_b^{1S}$, $\lambda_2 \approx (m_{B^*}^2 - m_B^2)/4$ is given in Appendix G, and

$$K_c^{(0)} = \sum_{i=1}^{8} d_i \eta^{a_i} \; , \tag{443}$$

with d_i given in Table 15, and a_i in Table 5, and

$$K_t^{(0)} = \left(C_7(\mu_W) + \frac{23}{36} \right) \eta^{\frac{4}{23}} - \frac{8}{3} \left(C_8(\mu_W) + \frac{1}{3} \right) \big(\eta^{\frac{4}{23}} - \eta^{\frac{2}{23}} \big) \; . \tag{444}$$

In Eqs. (443) and (444), $\eta = \alpha_s(\mu_W)/\alpha_s(\mu_b)$.

i	1	2	3	4	5	6	7	8
d_i	1.4107	−0.8380	−0.4286	−0.0714	−0.6494	−0.0380	−0.0185	−0.0057

Table 15: Useful numbers for $K_c^{(0)}$ [56].

The electromagnetic correction ϵ_{em} can be written as [33]:

$$\epsilon_{em} = \frac{\alpha}{\alpha_s(\mu_b)} \left(2 \left[C_7^{(\mathrm{em})}(\mu_b)\, C_7^{(0)}(\mu_b) \right] - k_{\mathrm{SL}}^{(\mathrm{em})}(\mu_b)\, |C_7^{(0)}(\mu_b)|^2 \right) , \tag{445}$$

where

$$k_{\mathrm{SL}}^{(\mathrm{em})}(\mu_b) = \frac{12}{23} \left(\eta^{-1} - 1 \right) = \frac{2\alpha_s(\mu_b)}{\pi} \ln \frac{\mu_W}{\mu_b} , \tag{446}$$

and

$$\begin{aligned} C_7^{(\mathrm{em})}(\mu_b) &= \left(\frac{32}{75}\, \eta^{-\frac{9}{23}} - \frac{40}{69}\, \eta^{-\frac{7}{23}} + \frac{88}{575}\, \eta^{\frac{16}{23}} \right) C_7^{(0)}(\mu_W) \\ &\quad + C_8^{(\mathrm{em})}(\mu_b) C_8^{(0)}(\mu_W) + C_2^{(\mathrm{em})}(\mu_b) , \end{aligned} \tag{447}$$

with

$$C_8^{(\mathrm{em})}(\mu_b) = -\frac{32}{575}\, \eta^{-\frac{9}{23}} + \frac{32}{1449}\, \eta^{-\frac{7}{23}} + \frac{640}{1449}\, \eta^{\frac{14}{23}} - \frac{704}{1725}\, \eta^{\frac{16}{23}} , \tag{448}$$

$$\begin{aligned} C_2^{(\mathrm{em})}(\mu_b) &= -\frac{190}{8073}\, \eta^{-\frac{35}{23}} - \frac{359}{3105}\, \eta^{-\frac{17}{23}} + \frac{4276}{121095}\, \eta^{-\frac{12}{23}} + \frac{350531}{1009125}\, \eta^{-\frac{9}{23}} \\ &\quad + \frac{2}{4347}\, \eta^{-\frac{7}{23}} - \frac{5956}{15525}\, \eta^{\frac{6}{23}} + \frac{38380}{169533}\, \eta^{\frac{14}{23}} - \frac{748}{8625}\, \eta^{\frac{16}{23}} . \end{aligned} \tag{449}$$

Using all the above equations, the inclusive branching ratio of $B \to X_s\gamma$ can be obtained.

E.2 Isospin asymmetry of $B \to K^*\gamma$

The isospin asymmetry Δ_0 in $B \to K^*\gamma$ decays arises when the photon is emitted from the spectator quark. The contribution to the decay width depends therefore on the charge of the spectator quark and is different for charged and neutral B meson decays:

$$\Delta_{0\pm} = \frac{\Gamma(\bar{B}^0 \to \bar{K}^{*0}\gamma) - \Gamma(B^\pm \to K^{*\pm}\gamma)}{\Gamma(\bar{B}^0 \to \bar{K}^{*0}\gamma) + \Gamma(B^\pm \to K^{*\pm}\gamma)} , \tag{450}$$

which can be written as [59]:

$$\Delta_0 = \mathrm{Re}(b_d - b_u) , \tag{451}$$

where the spectator dependent coefficients b_q take the form:

$$b_q = \frac{12\pi^2 f_B\, Q_q}{\overline{m}_b\, T_1^{B\to K^*} a_7^c} \left(\frac{f_{K^*}^\perp}{\overline{m}_b}\, K_1 + \frac{f_{K^*} m_{K^*}}{6\lambda_B m_B}\, K_{2q} \right) . \tag{452}$$

In the same way as for $b \to s\gamma$ branching ratio, the SUSY contributions induced by charged Higgs and chargino loops must be taken into account for the calculation of isospin symmetry breaking.

The functions K_1 and K_{2q} can be written in function of the Wilson coefficients C_i in the traditional basis (see Appendix D.2) at scale μ_b [59]:

$$K_1 = -\left(C_6(\mu_b) + \frac{C_5(\mu_b)}{N}\right) F_\perp + \frac{C_F}{N}\frac{\alpha_s(\mu_b)}{4\pi}\left\{\left(\frac{m_b}{m_B}\right)^2 C_8(\mu_b)\, X_\perp \right. \tag{453}$$
$$\left. -C_2(\mu_b)\left[\left(\frac{4}{3}\ln\frac{m_b}{\mu_b} + \frac{2}{3}\right) F_\perp - G_\perp(x_{cb})\right] + r_1\right\} + \left(C_i \leftrightarrow C_i'\right) ,$$

$$K_{2q} = \frac{V_{us}^* V_{ub}}{V_{cs}^* V_{cb}}\left(C_2(\mu_b) + \frac{C_1(\mu_b)}{N}\right)\delta_{qu} + \left(C_4(\mu_b) + \frac{C_3(\mu_b)}{N}\right) \tag{454}$$
$$+\frac{C_F}{N}\frac{\alpha_s(\mu_b)}{4\pi}\left[C_2(\mu_b)\left(\frac{4}{3}\ln\frac{m_b}{\mu_b} + \frac{2}{3} - H_\perp(x_{cb})\right) + r_2\right] + \left(C_i \leftrightarrow C_i'\right) ,$$

where $x_{cb} = \dfrac{m_c^2}{m_b^2}$ and $N = 3$ and $C_F = 4/3$ are colour factors, and:

$$r_1 = \left[\frac{8}{3}C_3(\mu_b) + \frac{4}{3}n_f\Big(C_4(\mu_b) + C_6(\mu_b)\Big) - 8\Big(NC_6(\mu_b) + C_5(\mu_b)\Big)\right] F_\perp \ln\frac{\mu_b}{\mu_0} + \ldots ,$$
$$r_2 = \left[-\frac{44}{3}C_3(\mu_b) - \frac{4}{3}n_f\Big(C_4(\mu_b) + C_6(\mu_b)\Big)\right]\ln\frac{\mu_b}{\mu_0} + \ldots . \tag{455}$$

Here the number of flavours $n_f = 5$, and $\mu_0 = O(m_b)$ is an arbitrary normalization scale.

The coefficient a_7^c reads [60]:

$$a_7^c(K^*\gamma) = C_7(\mu_b) + \frac{\alpha_s(\mu_b)C_F}{4\pi}\Big[C_2(\mu_b)G_2(x_{cb}) + C_8(\mu_b)G_8\Big] \tag{456}$$
$$+\frac{\alpha_s(\mu_h)C_F}{4\pi}\Big[C_2(\mu_h)H_2(x_{cb}) + C_8(\mu_h)H_8\Big] + \left(C_i \leftrightarrow C_i'\right) ,$$

where $\mu_h = \sqrt{\Lambda_h \mu_b}$ is the spectator scale, and

$$G_2(x_{cb}) = -\frac{104}{27}\ln\frac{\mu_b}{m_b} + g_2(x_{cb}) , \tag{457}$$

$$G_8 = \frac{8}{3}\ln\frac{\mu_b}{m_b} + g_8 , \tag{458}$$

with

$$g_8 = \frac{11}{3} - \frac{2\pi^2}{9} + \frac{2i\pi}{3} \;, \tag{459}$$

$$\begin{aligned} g_2(x) &= \frac{2}{9}\,x\Big[48 + 30i\pi - 5\pi^2 - 2i\pi^3 - 36\zeta(3) + \big(36 + 6i\pi - 9\pi^2\big)\ln x \\ &\quad + (3 + 6i\pi)\ln^2 x + \ln^3 x\Big] \\ &\quad + \frac{2}{9}\,x^2\Big[18 + 2\pi^2 - 2i\pi^3 + \big(12 - 6\pi^2\big)\ln x + 6i\pi\ln^2 x + \ln^3 x\Big] \\ &\quad + \frac{1}{27}\,x^3\Big[-9 + 112i\pi - 14\pi^2 + (182 - 48i\pi)\ln x - 126\ln^2 x\Big] \\ &\quad - \frac{833}{162} - \frac{20i\pi}{27} + \frac{8\pi^2}{9}\,x^{3/2} \;, \end{aligned} \tag{460}$$

where $\zeta(3)$ is given in Eq. (21). The function $H_2(x)$ in Eq. (456) is defined as:

$$H_2(x) = -\frac{2\pi^2}{3N}\frac{f_B f_{K^*}^{\perp}}{T_1^{B\to K^*} m_B^2}\int_0^1 d\xi \frac{\Phi_{B1}(\xi)}{\xi}\int_0^1 dv\, h(\bar{v},x)\Phi_\perp(v) \;, \tag{461}$$

where $h(u,x)$ is the hard-scattering function:

$$h(u,x) = \frac{4x}{u^2}\left[\mathrm{Li}_2\left(\frac{2}{1-\sqrt{\dfrac{u-4x+i\varepsilon}{u}}}\right) + \mathrm{Li}_2\left(\frac{2}{1+\sqrt{\dfrac{u-4x+i\varepsilon}{u}}}\right)\right] - \frac{2}{u} \;, \tag{462}$$

and Li_2 is the usual dilogarithm function given in Eq. (38).
$\Phi_\perp$ is the light-cone wave function with transverse polarization, which can be written in the form [61]:

$$\Phi_\perp(u) = 6u\bar{u}\left[1 + 3a_1^\perp\,\xi + a_2^\perp\,\frac{3}{2}(5\xi^2 - 1)\right] \;, \tag{463}$$

where $\bar{u} = 1 - u$ and $\xi = 2u - 1$, and Φ_{B1} is the distribution amplitude of the B meson involved in the leading-twist projection. Finally:

$$H_8 = \frac{4\pi^2}{3N}\frac{f_B f_{K^*}^{\perp}}{T_1^{B\to K^*} m_B^2}\int_0^1 d\xi \frac{\Phi_{B1}(\xi)}{\xi}\int_0^1 dv \frac{\Phi_\perp(v)}{v} \;. \tag{464}$$

The first negative moment of Φ_{B1} can be parametrized by the quantity λ_B such as

$$\int_0^1 d\xi \frac{\Phi_{B1}(\xi)}{\xi} = \frac{m_B}{\lambda_B} \;. \tag{465}$$

The convolution integrals of the hard-scattering kernels with the meson distribution amplitudes are as follows:

$$
\begin{aligned}
F_\perp &= \int_0^1 dx\, \frac{\phi_\perp(x)}{3\bar{x}} \;, \\
G_\perp(s_c) &= \int_0^1 dx\, \frac{\phi_\perp(x)}{3\bar{x}}\, G(s_c,\bar{x}) \;, \\
H_\perp(s_c) &= \int_0^1 dx \left(g_\perp^{(v)}(x) - \frac{g_\perp^{\prime\,(a)}(x)}{4} \right) G(s_c,\bar{x}) \;, \\
X_\perp &= \int_0^1 dx\, \phi_\perp(x)\, \frac{1+\bar{x}}{3\bar{x}^2} \;,
\end{aligned} \tag{466}
$$

with $s_c = (m_c/m_b)^2$, and

$$
G(s,\bar{x}) = -4 \int_0^1 du\, u\bar{u}\, \ln(s - u\bar{u}\bar{x} - i\epsilon) \;, \tag{467}
$$

and the Gegenbauer momenta read [61]:

$$
\begin{aligned}
g_\perp^{(a)}(u) &= 6u\bar{u} \left\{ 1 + a_1^\parallel \xi + \left[\frac{1}{4} a_2^\parallel + \frac{5}{3}\, \zeta_3^A \left(1 - \frac{3}{16}\, \omega_{1,0}^A \right) + \frac{35}{4} \zeta_3^V \right] (5\xi^2 - 1) \right\} \\
&\quad + 6\, \tilde{\delta}_+ \,(3u\bar{u} + \bar{u} \ln \bar{u} + u \ln u) + 6\, \tilde{\delta}_- \,(\bar{u} \ln \bar{u} - u \ln u) \;,
\end{aligned} \tag{468}
$$

$$
\begin{aligned}
g_\perp^{(v)}(u) &= \frac{3}{4}(1+\xi^2) + a_1^\parallel\, \frac{3}{2}\, \xi^3 + \left(\frac{3}{7}\, a_2^\parallel + 5\zeta_3^A \right) (3\xi^2 - 1) \\
&\quad + \left(\frac{9}{112}\, a_2^\parallel + \frac{105}{16}\, \zeta_3^V - \frac{15}{64}\, \zeta_3^A \omega_{1,0}^A \right) (3 - 30\xi^2 + 35\xi^4) \\
&\quad + \frac{3}{2}\, \tilde{\delta}_+ \,(2 + \ln u + \ln \bar{u}) + \frac{3}{2}\, \tilde{\delta}_- \,(2\xi + \ln \bar{u} - \ln u) \;.
\end{aligned} \tag{469}
$$

To compute $X_\perp$, the parameter $X = \ln(m_B/\Lambda_h)\,(1 + \varrho\, e^{i\varphi})$ is introduced to parametrize the logarithmically divergent integral $\int_0^1 dx/(1-x)$. $\varrho \leq 1$ and the phase φ are arbitrary, and $\Lambda_h \approx 0.5$ GeV is a typical hadronic scale. The remaining parameters are given in Appendix G.

`SuperIso` first computes numerically all the integrals and the Wilson coefficients, and then calculates the isospin asymmetry of $B \to K^*\gamma$ using all the above equations.

E.3 $B \to X_s \ell^+ \ell^-$

The decay $B \to X_s \ell^+ \ell^-$ with $\ell = e$, μ, or τ, is particularly attractive because of kinematic observables such as the dilepton invariant mass spectrum and the forward-backward asymmetry ($A_{\rm FB}$), and plays a complementary role to the inclusive $B \to X_s \gamma$ decay.

E.3.1 Main formulas

The implementation of $B \to X_s \ell^+ \ell^-$ is following [45][5]. The higher order and power corrections are implemented following [62], and the electromagnetic logarithmically enhanced corrections are taken from [63, 64].

The differential decay rate can then be written as:

$$\begin{aligned}
\frac{d\mathcal{B}(B \to X_s \ell^+ \ell^-)}{d\hat{s}} &= \mathcal{B}(B \to X_c l \bar{\nu}) \frac{\alpha^2}{4\pi^2 f(z)\kappa(z)} \frac{|V_{tb}V_{ts}^*|^2}{|V_{cb}|^2} (1-\hat{s})^2 \sqrt{1 - \frac{4\hat{m}_\ell^2}{\hat{s}}} \\
&\times \Bigg\{ |C_9^{new}|^2 (1 + \frac{2\hat{m}_\ell^2}{\hat{s}})(1+2\hat{s}) \left(1 + \frac{\alpha_s}{\pi}\tau_{99}(\hat{s})\right) \\
&+ 4|C_7^{new}|^2 (1 + \frac{2\hat{m}_\ell^2}{\hat{s}})(1 + \frac{2}{\hat{s}}) \left(1 + \frac{\alpha_s}{\pi}\tau_{77}(\hat{s})\right) \\
&+ |C_{10}^{new}|^2 [(1+2\hat{s}) + \frac{2\hat{m}_\ell^2}{\hat{s}}(1-4\hat{s})] \left(1 + \frac{\alpha_s}{\pi}\tau_{99}(\hat{s})\right) \\
&+ 12\mathrm{Re}(C_7^{new} C_9^{new*})(1 + \frac{2\hat{m}_\ell^2}{\hat{s}}) \left(1 + \frac{\alpha_s}{\pi}\tau_{79}(\hat{s})\right) \\
&+ \frac{3}{2}|C_{Q_1}|^2(\hat{s} - 4\hat{m}_\ell^2) + \frac{3}{2}|C_{Q_2}|^2 \hat{s} + 6Re(C_{10}^{new} C_{Q_2}^*)\hat{m}_\ell \Bigg\} \\
&+ \delta_{d\mathcal{B}/d\hat{s}}^{brems,\mathrm{A}} + \delta_{d\mathcal{B}/d\hat{s}}^{brems,\mathrm{B}} + \delta_{d\mathcal{B}/d\hat{s}}^{1/m_b^2} + \delta_{d\mathcal{B}/d\hat{s}}^{1/m_b^3} + \delta_{d\mathcal{B}/d\hat{s}}^{1/m_c^2} + \delta_{d\mathcal{B}/d\hat{s}}^{em} \\
&+ \left(C_i \leftrightarrow C_i'\right) \\
&\equiv \frac{d\mathcal{B}_0}{d\hat{s}} + \delta_{d\mathcal{B}/d\hat{s}} \;,
\end{aligned} \tag{470}$$

where

$$\hat{m}_i \equiv \frac{m_i}{m_{b,pole}} \;, \tag{471}$$

$$\hat{s} \equiv \frac{s}{m_{b,pole}^2} \;, \tag{472}$$

$$z = m_c^2/m_b^2 \;, \tag{473}$$

and

$$f(z) = 1 - 8z + 8z^3 - z^4 - 12z^2 \ln z \;, \tag{474}$$

$$\kappa(z) = 1 - \frac{2\alpha_s(m_b)}{3\pi}\frac{h(z)}{f(z)} \;, \tag{475}$$

[5] Note that O_8 and O_9 in and [45] correspond to O_9 and O_{10} respectively in the recent literature and in this manuscript.

with

$$h(z) = -(1-z^2)\left(\tfrac{25}{4}-\tfrac{239}{3}\,z+\tfrac{25}{4}\,z^2\right)+z\,\ln(z)\left(20+90\,z-\tfrac{4}{3}\,z^2+\tfrac{17}{3}\,z^3\right) \quad (476)$$
$$+z^2\,\ln^2(z)\,(36+z^2)+(1-z^2)\left(\tfrac{17}{3}-\tfrac{64}{3}\,z+\tfrac{17}{3}\,z^2\right)\ln(1-z)$$
$$-4\,(1+30\,z^2+z^4)\,\ln(z)\ln(1-z)-(1+16\,z^2+z^4)\left(6\,\mathrm{Li}(z)-\pi^2\right)$$
$$-32\,z^{3/2}(1+z)\left[\pi^2-4\,\mathrm{Li}(\sqrt{z})+4\,\mathrm{Li}(-\sqrt{z})-2\ln(z)\,\ln\left(\tfrac{1-\sqrt{z}}{1+\sqrt{z}}\right)\right] .$$

We also give the forward-backward asymmetry in $B \to X_s \ell^+ \ell^-$:

$$A_{FB}(\hat{s}) = \int_0^1 dz \frac{d^2\mathcal{B}}{d\hat{s}dz} - \int_{-1}^0 dz \frac{d^2\mathcal{B}}{d\hat{s}dz} \quad (477)$$
$$= -\mathcal{B}(B \to X_c l\bar{\nu}) \frac{3\alpha^2}{4\pi^2 f(z)\kappa(z)} \frac{|V_{tb}V_{ts}^*|^2}{|V_{cb}|^2}(1-\hat{s})^2(1-\frac{4\hat{m}_\ell^2}{\hat{s}})$$
$$\times\left\{\mathrm{Re}(C_9^{new}C_{10}^{new*})\hat{s}\left(1+\frac{\alpha_s}{\pi}\tau_{910}(\hat{s})\right)+2\mathrm{Re}(C_7^{new}C_{10}^{new*})\left(1+\frac{\alpha_s}{\pi}\tau_{710}(\hat{s})\right)\right.$$
$$\left.+\mathrm{Re}(C_9^{new}C_{Q_1}^*)\hat{m}_\ell+2\mathrm{Re}(C_7^{new}C_{Q_1}^*)\hat{m}_\ell\right\}$$
$$+\delta_{A_{FB}}^{1/m_b^2}(\hat{s})+\delta_{A_{FB}}^{1/m_c^2}(\hat{s})+\delta_{A_{FB}}^{brems}(\hat{s})+\delta_{A_{FB}}^{em}(\hat{s})$$
$$+\left(C_i \leftrightarrow C_i'\right)$$
$$\equiv A_{0FB}+\delta_{A_{FB}} ,$$

where [65]:

$$B(B \to X_c l\bar{\nu}) = (10.64 \pm 0.17 \pm 0.06)\% . \quad (478)$$

$f(z)$ and $\kappa(z)$ are given in Eqs. (474) and (475). Also:

$$\tau_{77}(s) = -\frac{2}{9(2+s)}\left[2(1-s)^2\ln(1-s)+\frac{6s(2-2s-s^2)}{(1-s)^2}\ln(s)+\frac{11-7s-10s^2}{(1-s)}\right] \quad (479)$$

$$\tau_{99}(s) = -\frac{4}{9(1+2s)}\left[2(1-s)^2\ln(1-s)+\frac{3s(1+s)(1-2s)}{(1-s)^2}\ln(s)+\frac{3(1-3s^2)}{1-s}\right] \quad (480)$$

$$\tau_{79}(s) = -\frac{4(1-s)^2}{9s}\ln(1-s)-\frac{4s(3-2s)}{9(1-s)^2}\ln(s)-\frac{2(5-3s)}{9(1-s)} \quad (481)$$

$$\tau_{710}(s) = -\frac{5}{2}+\frac{1}{3(1-3s)}-\frac{1}{3}\frac{s(6-7s)\ln(s)}{(1-s)^2}-\frac{1}{9}\frac{(3-7s+4s^2)\ln(1-s)}{s}+\frac{f_7(s)}{3} \quad (482)$$

$$\tau_{910}(s) = -\frac{5}{2}+\frac{1}{3(1-s)}-\frac{1}{3}\frac{s(6-7s)\ln(s)}{((1-s)^2)}-\frac{2}{9}\frac{(3-5s+2s^2)\ln(1-s)}{s}+\frac{f_9(s)}{3} \quad (483)$$

where

$$f_7(s) = \frac{1}{6(s-1)^2}\Big\{24(1+13s-4s^2)\mathrm{Li}_2(\sqrt{s})+12(1-17s+6s^2)\mathrm{Li}_2(s)+6s(6-7s)\ln(s)$$
$$+24(1-s)^2\ln(s)\ln(1-s)+12(-13+16s-3s^2)[\ln(1-\sqrt{s})-\ln(1-s)]$$
$$+39-2\pi^2+252s-26\pi^2 s+21s^2+8\pi^2 s^2-180\sqrt{s}-132s\sqrt{s}\Big\}\ , \qquad (484)$$

$$f_9(s) = -\frac{1}{6(s-1)^2}\Big\{48s(-5+2s)\mathrm{Li}_2(\sqrt{s})+24(-1+7s-3s^2)\mathrm{Li}_2(s)+6s(-6+7s)\ln(s)$$
$$-24(1-s)^2\ln(s)\ln(1-s)+24(5-7s+2s^2)[\ln(1-\sqrt{s})-\ln(1-s)]$$
$$-21-156s+20\pi^2 s+9s^2-8\pi^2 s^2+120\sqrt{s}+48s\sqrt{s}\Big\}\ . \qquad (485)$$

and[6]

$$C_7^{new}(s) = \left(1+\frac{\alpha_s}{\pi}\sigma_7(s)\right)C_7^{\rm eff}-\frac{\alpha_s}{4\,\pi}\left[C_1^{(0)}F_1^{(7)}(s)+C_2^{(0)}F_2^{(7)}(s)+C_8^{\rm eff(0)}F_8^{(7)}(s)\right] \qquad (486)$$

$$C_9^{new}(s) = \left(1+\frac{\alpha_s}{\pi}\sigma_9(s)\right)C_9^{\rm eff}(s)-\frac{\alpha_s}{4\,\pi}\left[C_1^{(0)}F_1^{(9)}(s)+C_2^{(0)}F_2^{(9)}(s)+C_8^{\rm eff(0)}F_8^{(9)}(s)\right] \qquad (487)$$

$$C_{10}^{new}(s) = \left(1+\frac{\alpha_s}{\pi}\sigma_9(s)\right)C_{10}^{\rm eff}\ . \qquad (488)$$

In the above formulas[7] [62]:

$$\sigma_9(s) = \sigma(s)+\frac{3}{2}\ , \qquad (489)$$

$$\sigma_7(s) = \sigma(s)+\frac{1}{6}-\frac{8}{3}\ln\left(\frac{\mu}{m_b}\right)\ , \qquad (490)$$

$$\sigma(s) = -\frac{4}{3}\mathrm{Li}_2(s)-\frac{2}{3}\ln(s)\ln(1-s)-\frac{2}{9}\pi^2-\ln(1-s)-\frac{2}{9}(1-s)\ln(1-s) \qquad (491)$$

[6]The $C_i^{\rm eff}$ are the same as the $\widetilde{C}_i^{\rm eff}$ in [62]. $C_7^{\rm eff}$ is the same as A_7 in [66,67].

[7]The ω_i in [66,67] can be written as $\omega_7=\sigma_7+\tau_{77}/2$, $\omega_9=\sigma_9+\tau_{99}/2$and $\omega_{79}=(\sigma_7+\sigma_9+\tau_{79})/2$.

and

$$C_7^{\rm eff} = C_7(\mu) - \frac{1}{3}C_3(\mu) - \frac{4}{9}C_4(\mu) - \frac{20}{3}C_5(\mu) - \frac{80}{9}C_6(\mu)\ , \tag{492}$$

$$C_8^{\rm eff} = C_8(\mu) + C_3(\mu) - \frac{1}{6}C_4(\mu) + 20C_5(\mu) - \frac{10}{3}C_6(\mu)\ , \tag{493}$$

$$\begin{aligned} C_9^{\rm eff}(s) &= C_9(\mu) + \sum_{i=1}^{6} C_i(\mu)\gamma_{i9}^{(0)} \ln\left(\frac{m_b}{\mu}\right) + \frac{4}{3}C_3(\mu) + \frac{64}{9}C_5(\mu) + \frac{64}{27}C_6(\mu) \\ &+ g\left(\hat{m}_c, s\right)\left(\frac{4}{3}C_1(\mu) + C_2(\mu) + 6C_3(\mu) + 60C_5(\mu)\right) \\ &+ g(1,s)\left(-\frac{7}{2}C_3(\mu) - \frac{2}{3}C_4(\mu) - 38C_5(\mu) - \frac{32}{3}C_6(\mu)\right) \\ &+ g(0,s)\left(-\frac{1}{2}C_3(\mu) - \frac{2}{3}C_4(\mu) - 8C_5(\mu) - \frac{32}{3}C_6(\mu)\right)\ , \end{aligned} \tag{494}$$

$$C_{10}^{\rm eff} = C_{10}(\mu)\ , \tag{495}$$

where

$$g(z,s) = -\frac{4}{9}\ln(z) + \frac{8}{27} + \frac{16}{9}\frac{z}{s} - \frac{2}{9}\left(2 + \frac{4\,z}{s}\right)\sqrt{\left|\frac{4\,z - s}{s}\right|} \times \begin{cases} 2\arctan\sqrt{\dfrac{s}{4\,z-s}} & \text{for } s < 4\,z\,, \\ \ln\left(\dfrac{\sqrt{s}+\sqrt{s-4\,z}}{\sqrt{s}-\sqrt{s-4\,z}}\right) - i\,\pi & \text{for } s > 4\,z\,. \end{cases} \tag{496}$$

The virtual corrections to $O_{1,2}$ and O_8 are embedded in $F_{1,2}^{(7,9)}$ and $F_8^{(7,9)}$. They are given in [66, 67] for small s ($0.05 \le s/m_b^2 \le 0.25$) and in [68] for large s. The small s results are reproduced in the following whereas the large s results are taken from the code provided in [68].

$$\begin{aligned} F_8^{(7)} &= \frac{4\pi^2}{27}\frac{(2+\hat{s})}{(1-\hat{s})^4} - \frac{4}{9}\frac{(11-16\hat{s}+8\hat{s}^2)}{(1-\hat{s})^2} - \frac{8}{9}\frac{\sqrt{\hat{s}}\,\sqrt{4-\hat{s}}}{(1-\hat{s})^3}(9-5\hat{s}+2\hat{s}^2)\arcsin\left(\frac{\sqrt{\hat{s}}}{2}\right) \\ &-\frac{16}{3}\frac{2+\hat{s}}{(1-\hat{s})^4}\arcsin^2\left(\frac{\sqrt{\hat{s}}}{2}\right) - \frac{8\hat{s}}{9(1-\hat{s})}\ln\hat{s} - \frac{32}{9}\ln\frac{\mu}{m_b} - \frac{8}{9}i\pi\ , \end{aligned} \tag{497}$$

$$\begin{aligned} F_8^{(9)} &= -\frac{8\pi^2}{27}\frac{(4-\hat{s})}{(1-\hat{s})^4} + \frac{8}{9}\frac{(5-2\hat{s})}{(1-\hat{s})^2} + \frac{16}{9}\frac{\sqrt{4-\hat{s}}}{\sqrt{\hat{s}}\,(1-\hat{s})^3}(4+3\hat{s}-\hat{s}^2)\arcsin\left(\frac{\sqrt{\hat{s}}}{2}\right) \\ &+\frac{32}{3}\frac{(4-\hat{s})}{(1-\hat{s})^4}\arcsin^2\left(\frac{\sqrt{\hat{s}}}{2}\right) + \frac{16}{9(1-\hat{s})}\ln\hat{s}\ . \end{aligned} \tag{498}$$

Defining $L_s = \ln(\hat{s})$, $L_\mu = \ln\left(\frac{\mu}{m_b}\right)$ and $L_c = \ln\left(\frac{m_c}{m_b}\right) = \ln(\hat{m}_c) = L_z/2$:

$$F_1^{(9)} = \left(-\frac{1424}{729} + \frac{16}{243}\, i\pi + \frac{64}{27}\, L_c\right) L_\mu - \frac{16}{243}\, L_\mu\, L_s + \left(\frac{16}{1215} - \frac{32}{135}\, z^{-1}\right) L_\mu\, \hat{s} \quad (499)$$
$$+ \left(\frac{4}{2835} - \frac{8}{315}\, z^{-2}\right) L_\mu\, \hat{s}^2 + \left(\frac{16}{76545} - \frac{32}{8505}\, z^{-3}\right) L_\mu\, \hat{s}^3 - \frac{256}{243}\, L_\mu^2 + f_1^{(9)}\,,$$

$$F_2^{(9)} = \left(\frac{256}{243} - \frac{32}{81}\, i\pi - \frac{128}{9}\, L_c\right) L_\mu + \frac{32}{81}\, L_\mu\, L_s + \left(-\frac{32}{405} + \frac{64}{45}\, z^{-1}\right) L_\mu\, \hat{s} \quad (500)$$
$$+ \left(-\frac{8}{945} + \frac{16}{105}\, z^{-2}\right) L_\mu\, \hat{s}^2 + \left(-\frac{32}{25515} + \frac{64}{2835}\, z^{-3}\right) L_\mu\, \hat{s}^3 + \frac{512}{81}\, L_\mu^2 + f_2^{(9)}\,,$$

$$F_1^{(7)} = -\frac{208}{243}\, L_\mu + f_1^{(7)}, \quad (501)$$

$$F_2^{(7)} = \frac{416}{81}\, L_\mu + f_2^{(7)}\,. \quad (502)$$

The analytic results for $f_1^{(9)}$, $f_1^{(7)}$, $f_2^{(9)}$, and $f_2^{(7)}$ are rather lengthy. They are decomposed as:

$$f_a^{(b)} = \sum_{i,j,l,m} \kappa_{a,ijlm}^{(b)}\, \hat{s}^i\, L_s^j\, z^l\, L_c^m + \sum_{i,j} \rho_{a,ij}^{(b)}\, \hat{s}^i\, L_s^j\,. \quad (503)$$

The quantities $\rho_{a,ij}^{(b)}$ collect the half-integer powers of $z = m_c^2/m_b^2 = \hat{m}_c^2$. This way, the summation indices in the above equation run over integers only. The coefficients $\kappa_{a,ijlm}^{(b)}$ and $\rho_{a,ij}^{(b)}$ are listed in the appendix of [68].

E.3.2 Λ_{QCD}^2/m_b^2 and Λ_{QCD}^3/m_b^3 corrections

These corrections read [62]

$$\delta_{d\mathcal{B}/d\hat{s}}^{1/m_b^2}(s) = \frac{3\lambda_2}{2m_b^2}\left\{\frac{\alpha^2}{4\pi^2}\left|\frac{V_{ts}}{V_{cb}}\right|^2 \frac{\mathcal{B}(B\to X_c l\bar{\nu})}{f(z)\kappa(z)}\left(-(6+3s-5s^3)\frac{4|C_7^{\rm new}(s)|^2}{s}\right.\right. \quad (504)$$
$$+(1-15s^2+10s^3)\left[|C_9^{\rm new}(s)|^2 + |C_{10}^{\rm new}(s)|^2\right]$$
$$\left.\left. -4(5+6s-7s^2)Re\left[C_7^{\rm new}(s)C_9^{\rm new}(s)^*\right]\right) + \frac{g_\lambda(z)}{f(z)}\frac{d\mathcal{B}_0}{d\hat{s}}\right\},$$

$$\delta_{A_{FB}}^{1/m_b^2}(s) = \frac{3\lambda_2}{2m_b^2}\left\{\frac{\alpha^2}{4\pi^2}\left|\frac{V_{ts}}{V_{cb}}\right|^2 \frac{\mathcal{B}(B\to X_c l\bar{\nu})}{f(z)\kappa(z)}\left(s{\rm Re}\left[C_{10}^{\rm new}(s)^* C_9^{\rm new}(s)\right](9+14s-15s^2)\right.\right.$$
$$\left.\left. +2{\rm Re}\left[C_{10}^{\rm new}(s)^* C_7^{\rm new}(s)\right](7+10s-9s^2)\right) + \frac{g_\lambda(z)}{f(z)}A_{0FB}(s)\right\} \quad (505)$$
$$+\frac{4\lambda_1}{3m_b^2}\frac{s}{(1-s)^2}A_{0FB}(s)\,,$$

with

$$g_\lambda(z) = 3 - 8z + 24z^2 - 24z^3 + 5z^4 + 12z^2 \ln z \; , \tag{506}$$

and

$$\begin{aligned}
\delta^{1/m_b^3}_{d\mathcal{B}/d\hat{s}}(s) &= -\frac{\rho_1}{m_b^3}\Bigg\{\frac{g_\rho(z)}{6f(z)}\frac{d\mathcal{B}_0}{d\hat{s}} + \frac{\alpha^2}{4\pi^2}\left|\frac{V_{ts}}{V_{cb}}\right|^2 \frac{\mathcal{B}(B\to X_c l\bar{\nu})}{f(z)\kappa(z)} \\
&\times\Bigg(\left[\frac{5s^4+19s^3+9s^2-7s+22}{6(1-s)} + 8\Delta(f_1)\delta(1-s)\right]\frac{4|C_7^{\rm new}(s)|^2}{s} \\
&+\left[\frac{10s^4+23s^3-9s^2+13s+11}{6(1-s)} + 8\Delta(f_1)\delta(1-s)\right]\left[|C_9^{\rm new}(s)|^2 + |C_{10}^{\rm new}(s)|^2\right] \\
&+4\left[\frac{-3s^3+17s^2-s+3}{2(1-s)} + 8\Delta(f_1)\delta(1-s)\right] Re\left[C_7^{\rm new}(s)C_9^{\rm new}(s)^*\right]\Bigg)\Bigg\} \; ,
\end{aligned} \tag{507}$$

where

$$g_\rho(z) = 77 - 88z + 24z^2 - 8z^3 + 5z^4 + 48\ln z + 36z^2\ln z \tag{508}$$

arises from the semileptonic normalization and $\Delta(f_1)$ is a local contribution that cures the singularity for $s \to 1$.

E.3.3 Λ^2_{QCD}/m_c^2 correction

These corrections read [62]

$$\begin{aligned}
\delta^{1/m_c^2}_{d\mathcal{B}/d\hat{s}}(s) &= \frac{8\lambda_2}{9m_c^2}\frac{\alpha^2}{4\pi^2}\left|\frac{V_{cs}^*V_{ts}}{V_{cb}^*V_{tb}}\right|\frac{(1-s)^2}{f(z)\kappa(z)}\mathcal{B}(B\to X_c l\bar{\nu}) \\
&\times Re\left[\frac{1+6s-s^2}{s}F\left(\frac{s}{4z}\right)C_2C_7^{\rm new}(s)^* + (2+s)F\left(\frac{s}{4z}\right)C_2C_9^{\rm new}(s)^*\right] \; ,
\end{aligned} \tag{509}$$

$$\begin{aligned}
\delta^{1/m_c^2}_{A_{FB}}(s) &= -\frac{\lambda_2}{3m_c^2}\frac{\alpha^2}{4\pi^2}\left|\frac{V_{cs}^*V_{ts}}{V_{cb}^*V_{tb}}\right|\frac{(1-s)^2}{f(z)\kappa(z)}\mathcal{B}(B\to X_c l\bar{\nu}) \\
&\times Re\left[(1+3s)F\left(\frac{s}{4z}\right)C_2C_{10}^{\rm new}(s)^*\right] \; ,
\end{aligned} \tag{510}$$

where

$$F(r) = \frac{3}{2r}\begin{cases} \dfrac{1}{\sqrt{r(1-r)}}\arctan\sqrt{\dfrac{r}{1-r}} - 1 & 0 < r < 1 \; , \\ \dfrac{1}{2\sqrt{r(r-1)}}\left(\ln\dfrac{1-\sqrt{1-1/r}}{1+\sqrt{1-1/r}} + i\pi\right) - 1 & r > 1 \; . \end{cases} \tag{511}$$

The input values of the HQET parameters are:

$\bar{\Lambda}$	$\lambda_1^{\rm eff}(\mathrm{GeV}^2)$	$\lambda_2^{\rm eff}(\mathrm{GeV}^2)$	$\rho_1(\mathrm{GeV}^3)$
0.40 ± 0.10	-0.15 ± 0.10	0.12 ± 0.02	0.06 ± 0.06

E.3.4 Bremsstrahlung contributions

The sum of the bremsstrahlung contributions from $O_7 - O_8$ and $O_8 - O_9$ interference terms as well as the $O_8 - O_8$ can be written as [69]:

$$\delta^{brems,\mathrm{A}}_{d\mathcal{B}/d\hat{s}} = \left(\frac{\alpha}{4\,\pi}\right)^2 \left(\frac{\alpha_s}{4\,\pi}\right) \frac{m^5_{b,pole}\,|V^*_{ts}\,V_{tb}|^2\,G_F^2}{48\,\pi^3} \Big(2\,\mathrm{Re}\big[c_{78}\,\tau_{78} + c_{89}\,\tau_{89}\big] + c_{88}\,\tau_{88}\Big)\,. \quad (512)$$

Using $\Gamma(B \to X_c l \nu) = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2 f(z)\kappa(z)$ the above expression can be written as

$$\delta^{brems,\mathrm{A}}_{d\mathcal{B}/d\hat{s}} = 4\left(\frac{\alpha}{4\,\pi}\right)^2 \left(\frac{\alpha_s}{4\,\pi}\right) \left|\frac{V^*_{ts}\,V_{tb}}{V_{cb}}\right|^2 \frac{\mathcal{B}(B \to X_c l \bar{\nu})}{f(z)\kappa(z)} \Big(2\,\mathrm{Re}\big[c_{78}\,\tau_{78} + c_{89}\,\tau_{89}\big] + c_{88}\,\tau_{88}\Big)\,. \quad (513)$$

The coefficients c_{ij} are given by

$$c_{78} = C_F \cdot C_7^{(0,eff)} C_8^{(0,eff)*}, \quad c_{89} = C_F \cdot C_8^{(0,eff)} C_9^{(0,eff)*}, \quad c_{88} = C_F \cdot \left|C_8^{(0,eff)}\right|^2\,, \quad (514)$$

while the quantities τ_{ij} read

$$\begin{aligned}
\tau_{78} &= \frac{8}{9\,\hat{s}} \Bigg\{ 25 - 2\,\pi^2 - 27\,\hat{s} + 3\,\hat{s}^2 - \hat{s}^3 + 12\left(\hat{s} + \hat{s}^2\right)\,\ln(\hat{s}) \qquad (515) \\
&+6\left(\frac{\pi}{2} - \arctan\left[\frac{2 - 4\,\hat{s} + \hat{s}^2}{(2-\hat{s})\sqrt{\hat{s}}\,\sqrt{4-\hat{s}}}\right]\right)^2 - 24\,\mathrm{Re}\left(\mathrm{Li}_2\left[\frac{\hat{s} - i\sqrt{\hat{s}}\,\sqrt{4-\hat{s}}}{2}\right]\right) \\
&-12\left((1-\hat{s})\sqrt{\hat{s}}\,\sqrt{4-\hat{s}} - 2\arctan\left[\frac{\sqrt{\hat{s}}\,\sqrt{4-\hat{s}}}{2-\hat{s}}\right]\right) \\
&\times\left(\arctan\left[\sqrt{\frac{4-\hat{s}}{\hat{s}}}\right] - \arctan\left[\frac{\sqrt{\hat{s}}\,\sqrt{4-\hat{s}}}{2-\hat{s}}\right]\right)\Bigg\}\,,
\end{aligned}$$

$$\begin{aligned}
\tau_{88} &= \frac{4}{27\,\hat{s}} \Bigg\{ -8\,\pi^2 + (1-\hat{s})\left(77 - \hat{s} - 4\,\hat{s}^2\right) - 24\,\mathrm{Li}_2(1-\hat{s}) \qquad (516) \\
&+3\left(10 - 4\,\hat{s} - 9\,\hat{s}^2 + 8\ln\left[\frac{\sqrt{\hat{s}}}{1-\hat{s}}\right]\right)\ln(\hat{s}) + 48\,\mathrm{Re}\left(\mathrm{Li}_2\left[\frac{3-\hat{s}}{2} + i\,\frac{(1-\hat{s})\sqrt{4-\hat{s}}}{2\sqrt{\hat{s}}}\right]\right) \\
&-6\left(\frac{20\,\hat{s} + 10\,\hat{s}^2 - 3\,\hat{s}^3}{\sqrt{\hat{s}}\,\sqrt{4-\hat{s}}} - 8\,\pi + 8\arctan\left[\sqrt{\frac{4-\hat{s}}{\hat{s}}}\right]\right) \\
&\times\left(\arctan\left[\sqrt{\frac{4-\hat{s}}{\hat{s}}}\right] - \arctan\left[\frac{\sqrt{\hat{s}}\,\sqrt{4-\hat{s}}}{2-\hat{s}}\right]\right)\Bigg\}\,,
\end{aligned}$$

$$\tau_{89} = \frac{2}{3}\Bigg\{\hat{s}\,(4-\hat{s}) - 3 - 4\,\ln(\hat{s})\big(1-\hat{s}-\hat{s}^2\big) \tag{517}$$
$$-8\,\mathrm{Re}\Bigg(\mathrm{Li}_2\Bigg[\frac{\hat{s}}{2} + i\,\frac{\sqrt{\hat{s}}\,\sqrt{4-\hat{s}}}{2}\Bigg] - \mathrm{Li}_2\Bigg[\frac{-2+\hat{s}(4-\hat{s})}{2} + i\,\frac{(2-\hat{s})\sqrt{\hat{s}}\,\sqrt{4-\hat{s}}}{2}\Bigg]\Bigg)$$
$$+4\left(\hat{s}^2\sqrt{\frac{4-\hat{s}}{\hat{s}}} + 2\,\arctan\Bigg[\frac{\sqrt{\hat{s}}\,\sqrt{4-\hat{s}}}{2-\hat{s}}\Bigg]\right)$$
$$\times\left(\arctan\Bigg[\sqrt{\frac{4-\hat{s}}{\hat{s}}}\Bigg] - \arctan\Bigg[\frac{\sqrt{\hat{s}}\,\sqrt{4-\hat{s}}}{2-\hat{s}}\Bigg]\right)\Bigg\}\,.$$

The bremsstrahlung contributions from O_1 and O_2 and interference terms with O_7, O_8, O_9 and O_{10} is given in the following [69]:

$$\delta^{brems,\mathrm{B}}_{d\mathcal{B}/d\hat{s}} = \left(\frac{\alpha}{4\,\pi}\right)^2\left(\frac{\alpha_s}{4\,\pi}\right)\frac{G_F^2\,m_{b,pole}^5\,|V_{ts}^*\,V_{tb}|^2}{48\,\pi^3} \tag{518}$$
$$\times\int_{\hat{s}}^{1} dw\,\Big[(c_{11}+c_{12}+c_{22})\,\tau_{22} + 2\,\mathrm{Re}\big[(c_{17}+c_{27})\,\tau_{27} + (c_{18}+c_{28})\,\tau_{28} + (c_{19}+c_{29})\,\tau_{29}\big]\Big]$$
$$= 4\left(\frac{\alpha}{4\,\pi}\right)^2\left(\frac{\alpha_s}{4\,\pi}\right)\left|\frac{V_{ts}^*\,V_{tb}}{V_{cb}}\right|^2\frac{\mathcal{B}(B\to X_c l\bar{\nu})}{f(z)\kappa(z)}$$
$$\times\int_{\hat{s}}^{1} dw\,\Big[(c_{11}+c_{12}+c_{22})\,\tau_{22} + 2\,\mathrm{Re}\big[(c_{17}+c_{27})\,\tau_{27} + (c_{18}+c_{28})\,\tau_{28} + (c_{19}+c_{29})\,\tau_{29}\big]\Big]\,.$$

The quantities τ_{ij}, expressed in terms of $\bar{\Delta}i_{23}$ and $\bar{\Delta}i_{27}$, read

$$\tau_{22} = \frac{8}{27}\frac{(w-\hat{s})(1-w)^2}{\hat{s}\,w^3}\Bigg\{\Big[3\,w^2 + 2\,\hat{s}^2(2+w) - \hat{s}\,w\,(5-2\,w)\Big]\,\big|\bar{\Delta}i_{23}\big|^2 \tag{519}$$
$$+\Big[2\,\hat{s}^2\,(2+\,w) + \hat{s}\,w\,(1+2\,w)\Big]\,\big|\bar{\Delta}i_{27}\big|^2 + 4\,\hat{s}\Big[w\,(1-w) - \hat{s}\,(2+w)\Big]\cdot\mathrm{Re}\left[\bar{\Delta}i_{23}\bar{\Delta}i_{27}^*\right]\Bigg\}\,,$$

$$\tau_{27} = \frac{8}{3}\frac{1}{\hat{s}\,w}\times\Bigg\{\Big[(1-w)\left(4\,\hat{s}^2 - \hat{s}\,w + w^2\right) + \hat{s}\,w\,(4+\hat{s}-w)\ln(w)\Big]\bar{\Delta}i_{23} \tag{520}$$
$$-\Big[4\,\hat{s}^2\,(1-w) + \hat{s}\,w\,(4+\hat{s}-w)\,\ln(w)\Big]\bar{\Delta}i_{27}\Bigg\}\,,$$

$$\tau_{28} = \frac{8}{9}\frac{1}{\hat{s}\,w\,(w-\hat{s})}\times\Bigg\{\Big[(w-s)^2(2\,\hat{s}-w)(1-w)\Big]\bar{\Delta}i_{23} - \Big[2\,\hat{s}\,(w-\hat{s})^2(1-w)\Big]\bar{\Delta}i_{27} \tag{521}$$
$$+\hat{s}\,w\Big[(1+2\,\hat{s}-2\,w)\bar{\Delta}i_{23} - 2\,(1+\hat{s}-w)\bar{\Delta}i_{27}\Big]\cdot\ln\Bigg[\frac{\hat{s}}{(1+\hat{s}-w)(w^2+\hat{s}\,(1-w))}\Bigg]\Bigg\}\,,$$

$$\tau_{29} = \frac{4}{3\,w}\Big\{\Big[2\,\hat{s}(1-w)(\hat{s}+w)+4\,\hat{s}\,w\ln(w)\Big]\bar{\Delta}i_{23} \quad (522)$$

$$-\Big[2\,\hat{s}(1-w)(\hat{s}+w)+w(3\,\hat{s}+w)\ln(w)\Big]\bar{\Delta}i_{27}\Big\}\,.$$

The coefficients c_{ij} read

$$c_{11} = C_{\tau_1}\cdot\Big|C_1^{(0)}\Big|^2\,, \quad (523)$$

$$c_{17} = C_{\tau_2}\cdot C_1^{(0)}C_7^{(0,eff)*}\,,$$

$$c_{27} = C_F\cdot C_2^{(0)}C_7^{(0,eff)*}\,,$$

$$c_{12} = C_{\tau_2}\cdot 2\,\mathrm{Re}\Big[C_1^{(0)}C_2^{(0)*}\Big]\,,$$

$$c_{18} = C_{\tau_2}\cdot C_1^{(0)}C_8^{(0,eff)*}\,,$$

$$c_{28} = C_F\cdot C_2^{(0)}C_8^{(0,eff)*}\,,$$

$$c_{22} = C_F\cdot\Big|C_2^{(0)}\Big|^2\,,$$

$$c_{19} = C_{\tau_2}\cdot C_1^{(0)}C_9^{(0,eff)*}\,,$$

$$c_{29} = C_F\cdot C_2^{(0)}C_9^{(0,eff)*}\,,$$

where the colour factors are:

$$C_F = \frac{N_c^2-1}{2\,N_c}\,, \quad (524)$$

$$C_{\tau_1} = \frac{N_c^2-1}{8\,N_c^3}\,, \quad (525)$$

$$C_{\tau_2} = -\frac{N_c^2-1}{4\,N_c^2}\,. \quad (526)$$

Also:

$$\bar{\Delta}i_{23} = -2+\frac{4}{w-\hat{s}}\left[z\,G_{-1}\!\left(\frac{\hat{s}}{z}\right)-z\,G_{-1}\!\left(\frac{w}{z}\right)-\frac{\hat{s}}{2}\,G_0\!\left(\frac{\hat{s}}{z}\right)+\frac{\hat{s}}{2}\,G_0\!\left(\frac{w}{z}\right)\right], \quad (527)$$

$$\bar{\Delta}i_{27} = 2\left[G_0\!\left(\frac{\hat{s}}{z}\right)-G_0\!\left(\frac{w}{z}\right)\right], \quad (528)$$

with

$$G_{-1}(t) = \begin{cases} 2\,\pi\,\arctan\Big(\sqrt{\frac{4-t}{t}}\Big) - \frac{\pi^2}{2} - 2\,\arctan^2\Big(\sqrt{\frac{4-t}{t}}\Big), & t<4 \\ -2\,i\,\pi\,\ln\Big(\frac{\sqrt{t}+\sqrt{t-4}}{2}\Big) - \frac{\pi^2}{2} + 2\,\ln^2\Big(\frac{\sqrt{t}+\sqrt{t-4}}{2}\Big), & t>4 \end{cases}, \tag{529}$$

$$G_0(t) = \begin{cases} \pi\,\sqrt{\frac{4-t}{t}} - 2 - 2\,\sqrt{\frac{4-t}{t}}\,\arctan\Big(\sqrt{\frac{4-t}{t}}\Big), & t<4 \\ -i\,\pi\sqrt{\frac{t-4}{t}} - 2 + 2\,\sqrt{\frac{t-4}{t}}\,\ln\Big(\frac{\sqrt{t}+\sqrt{t-4}}{2}\Big), & t>4 \end{cases}. \tag{530}$$

The terms arising from the interference of the matrix elements of the operators O_1, O_2, O_8 with O_{10} which contribute to the forward-backward asymmetry are taken from [70]:

$$\begin{aligned} \delta^{brems}_{A_{FB}}(\hat{s}) &= \Big(\frac{\alpha}{4\pi}\Big)^2 \frac{G_F^2 m_{b,pole}^5 |V_{ts}^* V_{tb}|^2}{48\pi^3}(1-\hat{s})^2 \\ &\times \frac{2\alpha_s}{3\pi}\left\{ Re\left[C_8^{0,eff} C_{10}^{eff*}\right]\,\tau_{810}(\hat{s}) + Re\left[\Big(C_2^0 - \frac{1}{6}C_1^0\Big) C_{10}^{eff*}\right]\,\tau_{210}(\hat{s})\right\} \\ &= \Big(\frac{\alpha}{4\pi}\Big)^2 \left|\frac{V_{ts}^* V_{tb}}{V_{cb}}\right|^2 \frac{\mathcal{B}(B\to X_c l\bar{\nu})}{f(z)\kappa(z)}(1-\hat{s})^2 \\ &\times \frac{8\alpha_s}{3\pi}\left\{ Re\left[C_8^{0,eff} C_{10}^{eff*}\right]\,\tau_{810}(\hat{s}) + Re\left[\Big(C_2^0 - \frac{1}{6}C_1^0\Big) C_{10}^{eff*}\right]\,\tau_{210}(\hat{s})\right\}, \end{aligned} \tag{531}$$

where

$$\begin{aligned} \tau_{810} &= \frac{1}{6(1-\hat{s})^2}\Bigg\{3\Bigg[(1-\sqrt{\hat{s}})^2(23-6\sqrt{\hat{s}}-\hat{s}) + 4(1-\hat{s})(7+\hat{s})\ln(1+\sqrt{\hat{s}}) \\ &+2s\Big(1+s-\ln(\hat{s})\Big)\ln(\hat{s})\Bigg] + 2\Bigg[-3\pi^2(1+2\hat{s}) + 6(3-\hat{s})\hat{s}\ln(2-\sqrt{\hat{s}}) \\ &-36(1+2\hat{s})Li_2[-\sqrt{\hat{s}}] - 6\sqrt{\frac{\hat{s}}{4-\hat{s}}}\Big[2(-3+\hat{s})\hat{s}\arctan\left(\frac{2+\sqrt{\hat{s}}}{\sqrt{4-\hat{s}}}\right) \\ &+2\pi\ln(2-\sqrt{\hat{s}}) - \arctan\left(\sqrt{\frac{4-\hat{s}}{\hat{s}}}\right)\Big((-3+\hat{s})\hat{s} + 4\ln(2-\sqrt{\hat{s}})\Big) \\ &-\arctan\left(\frac{\sqrt{\hat{s}}\sqrt{4-\hat{s}}}{2-\hat{s}}\right)\Big((-3+\hat{s})\hat{s} - \ln(\hat{s})\Big) + 4Re\left(iLi_2[\frac{(-2+i\sqrt{4-\hat{s}}+\sqrt{\hat{s}})\sqrt{\hat{s}}}{i\sqrt{4-\hat{s}}-\sqrt{\hat{s}}}]\right) \\ &-2Re\left(iLi_2[\frac{i}{2}\sqrt{4-\hat{s}}(1-\hat{s})\sqrt{\hat{s}} + \frac{(3-\hat{s})\hat{s}}{2}]\right)\Big]\Bigg]\Bigg\}, \end{aligned} \tag{532}$$

and

$$
\begin{aligned}
\tau_{210} &= \int_{\hat{s}}^{1} \frac{-dw\,\hat{s}}{(\hat{s}-w)(1-\hat{s})^2}\Bigg\{\Bigg[4(1-\hat{s})(1+w) - \frac{2\sqrt{(\hat{s}-w^2)^2}\Big(w(3+w)-\hat{s}(1-w)\Big)}{w^2} \\
&+\Big(2+5w+2w^2+\hat{s}(3+4w)\Big)\ln\left(\frac{\hat{s}+w^2+\sqrt{(\hat{s}-w^2)^2}}{2w}\right) - \frac{(\hat{s}-w)}{\hat{s}\sqrt{(1+w)^2-4\hat{s}}} \\
&\times\Big(w(2-w)-\hat{s}(6-5w)\Big)\Big[\ln\Big(1+w-\hat{s}(3-w)+(1-\hat{s})\sqrt{(1+w)^2-4\hat{s}}\Big) \\
&-\ln\Big(\hat{s}(1-3w)+w^2(1+w)+\sqrt{(s-w^2)^2}\sqrt{(1+w)^2-4\hat{s}}\Big)\Big]\Bigg]\bar{\Delta}_{23} \\
&-\Bigg[2(1-\hat{s})(1+2w) - \frac{2\sqrt{(\hat{s}-w^2)^2}\Big(w(2+w)-\hat{s}(1-w)\Big)}{w^2} \\
&+2\Big(\hat{s}(1+2w)+w(2+w)\Big)\ln\left(\frac{\hat{s}+w^2+\sqrt{(\hat{s}-w^2)^2}}{2w}\right) \\
&+\frac{4(1-w)(\hat{s}-w)}{\sqrt{(1+w)^2-4\hat{s}}}\Big[\ln\Big(1+w-\hat{s}(3-w)+(1-\hat{s})\sqrt{(1+w)^2-4\hat{s}}\Big) \\
&-\ln\Big(\hat{s}(1-3w)+w^2(1+w)+\sqrt{(\hat{s}-w^2)^2}\sqrt{(1+w)^2-4\hat{s}}\Big)\Big]\Bigg]\bar{\Delta}_{27}\Bigg\}\,, \qquad (533)
\end{aligned}
$$

where $\bar{\Delta}_{i_{23}}$ and $\bar{\Delta}_{i_{27}}$ are given in Eqs. (527) and (528), and the functions $G_{-1}(t)$ and $G_0(t)$ in Eqs. (529) and (530).

E.3.5 Electromagnetic contributions

The electromagnetic logarithmically enhanced contributions read[8] [63, 64]:

$$
\begin{aligned}
\delta^{em}_{d\mathcal{B}/d\hat{s}} &= \frac{4\mathcal{B}(B\to X_c l\bar{\nu})}{f(z)\kappa(z)}\left|\frac{V_{tb}V_{ts}^*}{V_{cb}}\right|^2(1-\hat{s})^2\left(\frac{\alpha}{4\pi}\right)^3\Bigg\{8\,(1+2\hat{s})\Bigg[|C_9|^2\,\omega^{(\mathrm{em})}_{99}(\hat{s})+|C_{10}|^2\,\omega^{(\mathrm{em})}_{1010}(\hat{s}) \\
&+\mathrm{Re}\left[(C_2+C_FC_1)C_9^*\,\omega^{(\mathrm{em})}_{29}(\hat{s})\right]+(C_2+C_FC_1)^2\,\omega^{(\mathrm{em})}_{22}(\hat{s})\Bigg] \\
&+96\left[\mathrm{Re}\,[C_7C_9^*]\;\omega^{(\mathrm{em})}_{79}(\hat{s})+\mathrm{Re}\left[(C_2+C_FC_1)C_7^*\,\omega^{(\mathrm{em})}_{27}(\hat{s})\right]\right] \\
&+8\,(4+\frac{8}{\hat{s}})\;|C_7|^2\,\omega^{(\mathrm{em})}_{77}(\hat{s})\Bigg\}\,, \qquad (534)
\end{aligned}
$$

[8] Note that the operators O_9 and O_{10} here include a prefactor $e^2/(4\pi)^2$ contrary to the convention in [63, 64].

$$\delta^{em}_{A_{FB}}(\hat{s}) = \frac{4\mathcal{B}(B\to X_c l\bar{\nu})}{f(z)\kappa(z)} \left|\frac{V_{tb}V_{ts}^*}{V_{cb}}\right|^2 (1-\hat{s})^2 \left(\frac{\alpha}{4\pi}\right)^3 \Bigg\{ -48\,\mathrm{Re}\,[C_7 C_{10}^*]\ \omega^{(\mathrm{em})}_{710}(\hat{s}) \quad (535)$$

$$-24\,\hat{s}\left[\mathrm{Re}\,[C_9 C_{10}^*]\ \omega^{(\mathrm{em})}_{910}(\hat{s}) + \mathrm{Re}\left[(C_2 + C_F C_1) C_{10}^*\, \omega^{(\mathrm{em})}_{210}(\hat{s})\right]\right]\Bigg\},$$

where

$$\omega^{(\mathrm{em})}_{99}(s) = \ln\left(\frac{m_b^2}{m_\ell^2}\right)\left[-\frac{1+4\,s-8\,s^2}{6\,(1-s)\,(1+2\,s)} + \ln(1-s) - \frac{\left(1-6\,s^2+4\,s^3\right)\,\ln s}{2\,(1-s)^2\,(1+2\,s)}\right] \quad (536)$$

$$-\frac{1}{9}\,Li_2(s) + \frac{4}{27}\pi^2 - \frac{121-27\,s-30\,s^2}{72\,(1-s)\,(1+2\,s)} - \frac{(41+76\,s)\,\ln(1-s)}{36(1+2\,s)}$$

$$+\left(\frac{-3-10\,s-17\,s^2+14\,s^3}{18\,(1-s)^2\,(1+2\,s)} + \frac{17\,\ln(1-s)}{18}\right)\ln s - \frac{\left(1-6\,s^2+4\,s^3\right)\,\ln^2 s}{2\,(1-s)^2\,(1+2\,s)},$$

$$\omega^{(\mathrm{em})}_{1010}(s) = \ln\left(\frac{m_b^2}{m_\ell^2}\right)\left[-\frac{1+4\,s-8\,s^2}{6\,(1-s)\,(1+2\,s)} + \ln(1-s) - \frac{\left(1-6\,s^2+4\,s^3\right)\,\ln s}{2\,(1-s)^2\,(1+2\,s)}\right], \quad (537)$$

$$\omega^{(\mathrm{em})}_{77}(s) = \ln\left(\frac{m_b^2}{m_\ell^2}\right)\left[\frac{s}{2\,(1-s)\,(2+s)} + \ln(1-s) - \frac{s\,\left(-3+2\,s^2\right)}{2\,(1-s)^2\,(2+s)}\,\ln(s)\right], \quad (538)$$

$$\omega^{(\mathrm{em})}_{79}(s) = \ln\left(\frac{m_b^2}{m_\ell^2}\right)\left[-\frac{1}{2\,(1-s)} + \ln(1-s) + \frac{\left(-1+2\,s-2\,s^2\right)}{2\,(1-s)^2}\,\ln(s)\right], \quad (539)$$

$$\omega^{(\mathrm{em})}_{29}(s) = \ln\left(\frac{m_b^2}{m_\ell^2}\right)\left[\frac{\Sigma_1(s)+i\,\Sigma_1^I(s)}{8(1-s)^2(1+2s)}\right] + \frac{16}{9}\,\omega^{(\mathrm{em})}_{1010}(s)\,\ln\left(\frac{\mu_b}{5\ \mathrm{GeV}}\right), \quad (540)$$

$$\omega^{(\mathrm{em})}_{22}(s) = \ln\left(\frac{m_b^2}{m_\ell^2}\right)\left[\frac{\Sigma_2(s)}{8(1-s)^2(1+2s)} + \frac{\Sigma_1(s)}{9(1-s)^2(1+2s)}\ln\left(\frac{\mu_b}{5\ \mathrm{GeV}}\right)\right] \quad (541)$$

$$+\frac{64}{81}\,\omega^{(\mathrm{em})}_{1010}(s)\,\ln^2\left(\frac{\mu_b}{5\ \mathrm{GeV}}\right),$$

$$\omega^{(\mathrm{em})}_{27}(s) = \ln\left(\frac{m_b^2}{m_\ell^2}\right)\left[\frac{\Sigma_3(s)+i\,\Sigma_3^I(s)}{96(1-s)^2}\right] + \frac{8}{9}\,\omega^{(\mathrm{em})}_{79}(s)\,\ln\left(\frac{\mu_b}{5\ \mathrm{GeV}}\right), \quad (542)$$

$$\omega^{(\mathrm{em})}_{710}(s) = \ln\left(\frac{m_b^2}{m_\ell^2}\right)\left[\frac{7-16\,\sqrt{s}+9\,s}{4\,(1-s)} + \ln(1-\sqrt{s}) + \frac{1+3\,s}{1-s}\,\ln\left(\frac{1+\sqrt{s}}{2}\right) - \frac{s\,\ln s}{(1-s)}\right] \quad (543)$$

$$\omega^{(\mathrm{em})}_{910}(s) = \ln\left(\frac{m_b^2}{m_\ell^2}\right)\left[-\frac{5-16\,\sqrt{s}+11\,s}{4\,(1-s)} + \ln(1-\sqrt{s})\right.$$

$$\left.+\frac{1-5\,s}{1-s}\,\ln\left(\frac{1+\sqrt{s}}{2}\right) - \frac{(1-3\,s)\,\ln s}{(1-s)}\right], \quad (544)$$

$$\omega^{(\mathrm{em})}_{210}(s) = \ln\left(\frac{m_b^2}{m_\ell^2}\right)\left[-\frac{\Sigma_7(s)+i\,\Sigma_7^I(s)}{24\,s\,(1-s)^2}\right] + \frac{8}{9}\,\omega^{(\mathrm{em})}_{910}(s)\,\ln\left(\frac{\mu_b}{5\ \mathrm{GeV}}\right). \quad (545)$$

The functions Σ_i have been evaluated numerically in the low q^2 region [63]:

$$\Sigma_1(s) = 23.787 - 120.948\, s + 365.373\, s^2 - 584.206\, s^3 , \quad (546)$$

$$\Sigma_1^I(s) = 1.653 + 6.009\, s - 17.080\, s^2 + 115.880\, s^3 , \quad (547)$$

$$\Sigma_2(s) = 11.488 - 36.987\, s + 255.330\, s^2 - 812.388\, s^3 + 1011.791\, s^4 , \quad (548)$$

$$\Sigma_3(s) = 109.311 - 846.039\, s + 2890.115\, s^2 - 4179.072\, s^3 , \quad (549)$$

$$\Sigma_3^I(s) = 4.606 + 17.650\, s - 53.244\, s^2 + 348.069\, s^3 , \quad (550)$$

$$\Sigma_7(s) = -0.259023 - 28.424\, s + 205.533\, s^2 - 603.219\, s^3 + 722.031\, s^4 , \quad (551)$$

$$\Sigma_7^I(s) = [-12.20658 - 215.8208\,(s-a) + 412.1207\,(s-a)^2]\,(s-a)^2\,\theta(s-a) , \quad (552)$$

with $a = (4m_c^2/m_b^2)^2$, while in the high q^2 region they become [64]

$$\Sigma_1(s) = -148.061\,\delta^2 + 492.539\,\delta^3 - 1163.847\,\delta^4 + 1189.528\,\delta^5 , \quad (553)$$

$$\Sigma_1^I(s) = -261.287\,\delta^2 + 1170.856\,\delta^3 - 2546.948\,\delta^4 + 2540.023\,\delta^5 , \quad (554)$$

$$\Sigma_2(s) = -221.904\,\delta^2 + 900.822\,\delta^3 - 2031.620\,\delta^4 + 1984.303\,\delta^5 , \quad (555)$$

$$\Sigma_3(s) = -298.730\,\delta^2 + 828.0675\,\delta^3 - 2217.6355\,\delta^4 + 2241.792\,\delta^5 , \quad (556)$$

$$\Sigma_3^I(s) = -528.759\,\delta^2 + 2095.723\,\delta^3 - 4681.843\,\delta^4 + 5036.677\,\delta^5 , \quad (557)$$

$$\Sigma_7(s) = 77.0256\,\delta^2 - 264.705\,\delta^3 + 595.814\,\delta^4 - 610.1637\,\delta^5 , \quad (558)$$

$$\Sigma_7^I(s) = 135.858\,\delta^2 - 618.990\,\delta^3 + 1325.040\,\delta^4 - 1277.170\,\delta^5 , \quad (559)$$

with $\delta = (1-s)$.

E.3.6 Long distance contributions

The long distance contributions are parametrized using the replacement [62]

$$g(z,\hat{s}) \longrightarrow g(z,0) + \frac{\hat{s}}{3} P \int_{\hat{s}_c}^{\infty} d\hat{s}' \frac{R_{\rm had}^{c\bar{c}}(\hat{s}')}{\hat{s}'(\hat{s}'-\hat{s})} + i\frac{\pi}{3} R_{\rm had}^{c\bar{c}}(\hat{s}) , \quad (560)$$

where P denotes the principal value, and [71]

$$R^{c\bar{c}} = \sigma_{\rm tot}(e^+e^- \to {\rm hadrons})/\sigma(e^+e^- \to \mu^+\mu^-) = R_{\rm cont}^{c\bar{c}} + R_{\rm res}^{c\bar{c}} , \quad (561)$$

where $\hat{s}_c = 4\hat{m}_D$ and $R_{\rm cont}^{c\bar{c}}$ and $R_{\rm res}^{c\bar{c}}$ denote the contributions from the continuum and the narrow resonances, respectively.
The latter is given by the Breit-Wigner formula

$$R_{\rm res}^{c\bar{c}} = \sum_{V=J/\psi,\psi',\ldots} \frac{9\hat{s}}{\alpha_{em}^2} \frac{BR(V \to l^+l^-)\hat{\Gamma}_{\rm total}^V \hat{\Gamma}_{\rm had}^V}{(\hat{s}-\hat{m}_V^2)^2 + \hat{m}_V^2 \hat{\Gamma}_{\rm total}^{V2}} , \quad (562)$$

where the meson parameters are given in Table 16, whereas $R_{\rm cont}^{c\bar{c}}$ can be determined using

Meson	Mass (GeV)	$BR(V \to \mu^+\mu^-)$	$\Gamma_{\rm total}$ (MeV)	$\Gamma_{\rm had}$ (MeV)
$J/\Psi(1S)$	3.096916	5.93×10^{-2}	0.0929	0.08147
$\Psi(2S)$	3.68609	7.7×10^{-3}	0.304	0.29746
$\Psi(3770)$	3.77292	1.1×10^{-5}	27.3	23.6
$\Psi(4040)$	4.039	1.4×10^{-5}	80	52
$\Psi(4160)$	4.153	1.0×10^{-5}	103	78
$\Psi(4415)$	4.421	1.1×10^{-5}	62	43

Table 16: Meson masses and decay properties [15].

the experimental data:

$$R^{c\bar{c}}_{\rm cont} = \begin{cases} 0 \quad \text{for} \quad 0 \le \hat{s} \le 0.60 \ , \\ -6.80 + 11.33\hat{s} \quad \text{for} \quad 0.60 \le \hat{s} \le 0.69 \ , \\ 1.02 \quad \text{for} \quad 0.69 \le \hat{s} \le 1 \ . \end{cases} \tag{563}$$

E.4 $B \to \bar{K}^*\ell^+\ell^-$

The full angular distribution of the decay $\bar{B}^0 \to \bar{K}^{*0}(\to K^-\pi^+)\ell^+\ell^-$ with $\bar{K}^{*0} \to K^-\pi^+$ on the mass shell, is completely described by four independent kinematic variables, the lepton-pair invariant mass, q^2, and the three angles, θ_{K^*}, θ_ℓ and ϕ. Summing over the lepton spins, the differential decay distribution can be written as [72, 73, 80]:

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_l\, d\cos\theta_{K^*}\, d\phi} = \frac{9}{32\pi} J(q^2, \theta_l, \theta_{K^*}, \phi) \ , \tag{564}$$

where q^2 is the dilepton invariant mass squared, θ_ℓ is defined as the angle between ℓ^- and the $\bar{B}$ in the dilepton frame, θ_{K^*} is the angle between K^- and $\bar{B}$ in the $K^-\pi^+$ frame and ϕ is given by the angle between the normals of the $K^-\pi^+$ and the dilepton planes.

The full kinematically accessible phase space is bounded by

$$4m_\ell^2 \leqslant q^2 \leqslant (M_B - m_{K^*})^2, \quad -1 \leqslant \cos\theta_\ell \leqslant 1, \quad -1 \leqslant \cos\theta_{K^*} \leqslant 1, \quad 0 \leqslant \phi \leqslant 2\pi. \tag{565}$$

The dependence of the decay distribution on the three angles can be made more explicit as

$$\begin{aligned} J(q^2, \theta_\ell, \theta_{K^*}, \phi) = {} & J_1^s \sin^2\theta_{K^*} + J_1^c \cos^2\theta_{K^*} + (J_2^s \sin^2\theta_{K^*} + J_2^c \cos^2\theta_{K^*}) \cos 2\theta_\ell \\ & + J_3 \sin^2\theta_{K^*} \sin^2\theta_\ell \cos 2\phi + J_4 \sin 2\theta_{K^*} \sin 2\theta_\ell \cos\phi \\ & + J_5 \sin 2\theta_{K^*} \sin\theta_\ell \cos\phi + J_6 \sin^2\theta_{K^*} \cos\theta_\ell + J_7 \sin 2\theta_{K^*} \sin\theta_\ell \sin\phi \\ & + J_8 \sin 2\theta_{K^*} \sin 2\theta_\ell \sin\phi + J_9 \sin^2\theta_{K^*} \sin^2\theta_\ell \sin 2\phi \ , \end{aligned} \tag{566}$$

where the coefficients $J_i^{(a)} = J_i^{(a)}(q^2)$ for $i = 1, \ldots, 9$ and $a = s, c$ are functions of the dilepton mass. The angular coefficients can be written both in terms of transversity amplitudes, $A_0, A_\parallel, A_\perp$ and A_S as well as in terms of helicity amplitudes $H_V(\lambda), H_A(\lambda), H_P$, and H_S as given in sections E.4.1 and E.4.2, respectively.

E.4.1 Angular coefficients in terms of transversity amplitudes

The functions J_{1-9} in terms of the transversity amplitudes are given by [79, 80]:

$$J_1^s = \frac{(2+\beta_\ell^2)}{4}\left[|A_\perp^L|^2 + |A_\parallel^L|^2 + (L\to R)\right] + \frac{4m_\ell^2}{q^2}\mathrm{Re}\left(A_\perp^L A_\perp^{R^*} + A_\parallel^L A_\parallel^{R^*}\right) , \tag{567a}$$

$$J_1^c = |A_0^L|^2 + |A_0^R|^2 + \frac{4m_\ell^2}{q^2}\left[|A_t|^2 + 2\mathrm{Re}(A_0^L A_0^{R^*})\right] + \beta_\ell^2 |A_S|^2 , \tag{567b}$$

$$J_2^s = \frac{\beta_\ell^2}{4}\left[|A_\perp^L|^2 + |A_\parallel^L|^2 + (L\to R)\right] , \tag{567c}$$

$$J_2^c = -\beta_\ell^2\left[|A_0^L|^2 + (L\to R)\right] , \tag{567d}$$

$$J_3 = \frac{1}{2}\beta_\ell^2\left[|A_\perp^L|^2 - |A_\parallel^L|^2 + (L\to R)\right] , \tag{567e}$$

$$J_4 = \frac{1}{\sqrt{2}}\beta_\ell^2\left[\mathrm{Re}(A_0^L A_\parallel^{L^*}) + (L\to R)\right] , \tag{567f}$$

$$J_5 = \sqrt{2}\beta_\ell\left[\mathrm{Re}(A_0^L A_\perp^{L^*}) - (L\to R) - \frac{m_\ell}{\sqrt{q^2}}\mathrm{Re}(A_\parallel^L A_S^* + A_\parallel^R A_S^*)\right] , \tag{567g}$$

$$J_6^s = 2\beta_\ell\left[\mathrm{Re}(A_\parallel^L A_\perp^{L^*}) - (L\to R)\right] , \tag{567h}$$

$$J_6^c = 4\beta_\ell\frac{m_\ell}{\sqrt{q^2}}\,\mathrm{Re}\left[A_0^L A_S^* + (L\to R)\right] , \tag{567i}$$

$$J_7 = \sqrt{2}\beta_\ell\left[\mathrm{Im}(A_0^L A_\parallel^{L^*}) - (L\to R) + \frac{m_\ell}{\sqrt{q^2}}\mathrm{Im}(A_\perp^L A_S^* + A_\perp^R A_S^*)\right] , \tag{567j}$$

$$J_8 = \frac{1}{\sqrt{2}}\beta_\ell^2\left[\mathrm{Im}(A_0^L A_\perp^{L^*}) + (L\to R)\right], \tag{567k}$$

$$J_9 = \beta_\ell^2\left[\mathrm{Im}(A_\parallel^{L^*} A_\perp^L) + (L\to R)\right] , \tag{567l}$$

where

$$\beta_\ell = \sqrt{1 - \frac{4m_\ell^2}{q^2}} . \tag{568}$$

The transversity amplitudes at leading order are described in terms of the short-distance Wilson coefficients $C_{7,9,10}$ and the matrix elements (form factors) of their corresponding operators. There are two common strategies for treating the long-distance form factors; the *full form factor* (full FF) and the *soft form factor* (soft FF) approaches. While in the former all seven independent form factors $V, A_{0,1,2}, T_{1,2,3}$ are considered in the latter they are reduced to two universal soft form factors $\xi_\perp, \xi_\parallel$ (see e.g. [119] for further details). For the $V(q^2), A_{0,1,2,}(q^2), T_{1,2,3}$ we use the combined LCSR + lattice fit of [120].

Transversity amplitudes, in the full FF approach at NLO

The transversity amplitudes at NLO in the full FF approach, within QCDF in the large recoil limit ($q^2 \lesssim 7$ GeV^2), are given by

$$A_\perp^{L,R} = N\sqrt{2\lambda}\bigg[\left[(C_9 + Y(q^2) + C_9') \mp (C_{10} + C_{10}')\right] \frac{V(q^2)}{M_B + M_{K^*}} + \frac{2m_b}{q^2}(C_7^{\rm eff} + C_7')T_1(q^2)\bigg] + \delta A_\perp^{L,R}\,, \tag{569}$$

$$A_\parallel^{L,R} = -N\sqrt{2}(M_B^2 - M_{K^*}^2)\bigg[\left[(C_9 + Y(q^2) - C_9') \mp (C_{10} - C_{10}')\right] \frac{A_1(q^2)}{M_B - M_{K^*}} + \frac{2m_b}{q^2}(C_7^{\rm eff} - C_7')T_2(q^2)\bigg] + \delta A_\parallel^{L,R}\,, \tag{570}$$

$$A_0^{L,R} = -\frac{N}{2M_{K^*}\sqrt{q^2}}\bigg\{ \left[(C_9 + Y(q^2) - C_9') \mp (C_{10} - C_{10}')\right] \times \bigg[(M_B^2 - M_{K^*}^2 - q^2)(M_B + M_{K^*})A_1(q^2) - \lambda\frac{A_2(q^2)}{M_B + M_{K^*}}\bigg] + 2m_b(C_7^{\rm eff} - C_7')\bigg[(M_B^2 + 3M_{K^*}^2 - q^2)T_2(q^2) - \frac{\lambda}{M_B^2 - M_{K^*}^2}T_3(q^2)\bigg]\bigg\} + \delta A_0^{L,R}\,, \tag{571}$$

$$A_t = \frac{N}{\sqrt{q^2}}\sqrt{\lambda}\left[2(C_{10} - C_{10}') + \frac{q^2}{m_\ell(m_b + m_q)}(C_{Q_2} - C_{Q_2}')\right] A_0(q^2)\,, \tag{572}$$

$$A_S = -\frac{2N}{m_b + m_q}\sqrt{\lambda}(C_{Q_1} - C_{Q_1}')A_0(q^2)\,, \tag{573}$$

where m_q is the spectator quark mass, and $Y(q^2)$ is defined in eq. 641 and

$$N = V_{tb}V_{ts}^*\left[\frac{G_F^2\alpha^2}{3\cdot 2^{10}\pi^5 M_B^3}q^2\beta_\ell\sqrt{\lambda(M_B^2, M_{K^*}^2, q^2)}\right]^{1/2}\,, \tag{574}$$

with λ the Källén function

$$\lambda(x,y,z) \equiv x^2 + y^2 + z^2 - 2(xy + yz + xz)\,. \tag{575}$$

In the above transversity amplitudes, besides the leading order contributions from the the semileptonic part the Hamiltonian (corresponding to $O_{7,9,10}$), the sub-leading non-local contributions from the the hadronic part of the Hamiltonian (corresponding to $O_{1-6,8}$) are contained in

$$\delta A_\perp^{L,R} = \frac{32\pi^2 N M_B^3}{\sqrt{2}\,q^2}\left(\mathcal{N}_+(q^2) - \mathcal{N}_-(q^2)\right)\,, \tag{576}$$

$$\delta A_\parallel^{L,R} = \frac{32\pi^2 N M_B^3}{\sqrt{2}\,q^2}\left(\mathcal{N}_+(q^2) + \mathcal{N}_-(q^2)\right)\,, \tag{577}$$

$$\delta A_0^{L,R} = \frac{32\pi^2 N M_B^3}{q^2}\left(\mathcal{N}_0(q^2)\right)\,. \tag{578}$$

At leading order in Λ/m_b (only reliable at low $q^2 \lesssim 7$ GeV [84]), the sub-leading contributions have been calculated in QCDf [81, 84]. These effects are more clearly described in terms of the helicity amplitudes (as $\mathcal{N}_\lambda$ in eq. 616) since they contribute through the emission of a virtual photon decaying into a lepton pair, and due to the vectorial coupling of the photon to the dilepton pair, they enter the vectorial helicity amplitude. The QCDf calculated non-local contributions $\mathcal{N}_{\pm,0}$ are given by

$$\mathcal{N}_{\pm}^{\text{QCDf}} = -\frac{1}{16\pi^2}\frac{m_b}{M_B}\left[(M_B^2 - M_{K^*}^2)\frac{2E_{K^*}}{M_B^3}\left(\mathcal{T}_{\perp}^{-(t),\text{nf+WA+hsa}} + \hat{\lambda}_u \mathcal{T}_{\perp}^{-(u)}\right) \mp \frac{\sqrt{\lambda}}{M_B^2}\left(\mathcal{T}_{\perp}^{+(t),\text{nf+WA+hsa}} + \hat{\lambda}_u \mathcal{T}_{\perp}^{+(u)}\right)\right], \tag{579}$$

$$\mathcal{N}_{0}^{\text{QCDf}} = -\frac{1}{16\pi^2}\frac{m_b}{M_B}\frac{\sqrt{q^2}}{2M_{K^*}}\left\{\left[(M_B^2 + 3M_{K^*}^2 - q^2)\frac{2E_{K^*}}{M_B^3} - \frac{\lambda}{(M_B^2 - M_{K^*}^2)M_B^2}\right] \times \left(\mathcal{T}_{\perp}^{-(t),\text{nf+WA}} + \hat{\lambda}_u \mathcal{T}_{\perp}^{-(u)}\right) - \frac{\lambda}{(M_B^2 - M_{K^*}^2)M_B^2}\left(\mathcal{T}_{\parallel}^{-(t),\text{nf+WA}} + \hat{\lambda}_u \mathcal{T}_{\parallel}^{-(u)}\right)\right\}, \tag{580}$$

where

$$\hat{\lambda}_u \equiv \frac{V_{ub}V_{us}^*}{V_{tb}V_{ts}^*}, \tag{581}$$

and E_{K^*} is the energy of the final vector meson in the B rest frame

$$E_{K^*} = \frac{M_B^2 + M_{K^*}^2 - q^2}{2M_B}, \tag{582}$$

where "WA+nf+hsa" on $\mathcal{T}_{\perp,\parallel}$ indicates that only weak annihilation, non-factorisable contributions and hard spectator scattering corrections are to be considered. The $\mathcal{T}_{\perp,\parallel}$ expressions are given in section E.4.4.

Transversity amplitudes, in the soft FF approach at NLO

The transversity amplitudes at NLO in the soft FF approach, within QCDF in the large recoil limit ($q^2 \lesssim 7$ GeV^2), are[9] [10] [73, 79]:

$$A_\perp^{L,R} = N\sqrt{2}\sqrt{\lambda}\Bigg[\left[(C_9 + C_9') \mp (C_{10} + C_{10}')\right]\frac{V(q^2)}{M_B + M_{K^*}} + \frac{2m_b}{q^2}\mathcal{T}_\perp^+\Bigg], \tag{584}$$

$$A_\parallel^{L,R} = -N\sqrt{2}(M_B^2 - M_{K^*}^2)\Bigg[\left[(C_9 - C_9') \mp (C_{10} - C_{10}')\right]\frac{A_1(q^2)}{M_B - M_{K^*}} \tag{585}$$
$$+ \frac{4m_b}{M_B}\frac{E_{K^*}}{q^2}\mathcal{T}_\perp^-\Bigg],$$

$$A_0^{L,R} = -\frac{N}{2M_{K^*}\sqrt{q^2}}\Bigg\{\left[(C_9 - C_9') \mp (C_{10} - C_{10}')\right] \tag{586}$$
$$\times\left[(M_B^2 - M_{K^*}^2 - q^2)(M_B + M_{K^*})A_1(q^2) - \lambda\frac{A_2(q^2)}{M_B + M_{K^*}}\right]$$
$$+ 2m_b\left[\frac{2E_{K^*}}{M_B}(M_B^2 + 3M_{K^*}^2 - q^2)\mathcal{T}_\perp^- - \frac{\lambda}{M_B^2 - M_{K^*}^2}\left(\mathcal{T}_\perp^- + \mathcal{T}_\parallel^-\right)\right]\Bigg\},$$

$$A_t = \frac{N}{\sqrt{q^2}}\sqrt{\lambda}\left[2(C_{10} - C_{10}') + \frac{q^2}{m_\ell(m_b + m_s)}(C_{Q_2} - C_{Q_2}')\right]\frac{E_{K^*}}{M_{K^*}}\frac{\xi_\parallel}{\Delta_\parallel}, \tag{587}$$

$$A_S = -\frac{2N}{m_b + m_s}\sqrt{\lambda}(C_{Q_1} - C_{Q_1}')\frac{E_{K^*}}{M_{K^*}}\frac{\xi_\parallel}{\Delta_\parallel}, \tag{588}$$

where

$$\Delta_\parallel(q^2) = 1 + \frac{\alpha_s C_F}{4\pi}\big(-2 + 2L\big) \tag{589}$$
$$-\frac{\alpha_s C_F}{4\pi}\frac{2q^2}{E_{K^*}^2}\frac{\pi^2 f_B f_{K^*\,\parallel}\,\lambda_{B+}^{-1}}{N_c M_B (E_{K^*}/m_{K^*})\xi_\parallel(q^2)}\int_0^1 \frac{du}{\bar{u}}\,\Phi_{K^*,\parallel}$$

with

$$L \equiv -\frac{m_b^2 - q^2}{q^2}\ln\left(1 - \frac{q^2}{m_b^2}\right), \tag{590}$$

[9]In QCD factorization to include NLO corrections in α_s to the transversity amplitudes at large recoil, the following replacements should be made in the leading order relations [73, 75]:

$$(C_7^{\rm eff} + C_7')T_i(q^2) \to \mathcal{T}_i^+, \qquad (C_7^{\rm eff} - C_7')T_i(q^2) \to \mathcal{T}_i^-, \qquad C_9^{\rm eff}(q^2) \to C_9,$$

where $\mathcal{T}_1^\pm = \mathcal{T}_\perp^\pm$, $\mathcal{T}_2^- = \frac{2E}{M_B}\mathcal{T}_\perp^-$, $\mathcal{T}_3^- = \mathcal{T}_\perp^- + \mathcal{T}_\parallel^-$.

The functions $\mathcal{T}_{\perp,\parallel}^-$ can be obtained from the $\mathcal{T}_{\perp,\parallel}$ by substituting $C_7^{\rm eff}$ with $C_7^{\rm eff} - C_7'$ whereas $\mathcal{T}_\perp^+$ is obtained from $\mathcal{T}_\perp$ by replacing $C_7^{\rm eff}$ with $C_7^{\rm eff} + C_7'$.

[10]In the following, unless stated otherwise, m_b denotes the potential subtracted (PS) bottom mass at the factorization scale $\mu_f \sim \sqrt{\Lambda_{QCD} M_B}$, which is related to the pole mass by:

$$m_b^{pole} = m_b^{PS}(\mu_f) + 4\frac{\alpha_s}{3\pi}\mu_f. \tag{583}$$

and $\Phi_{K^*,\parallel}$ will be given in Eq. (630). For the universal soft form factors $\xi_\perp$ and $\xi_\parallel$ we use [73, 81]:

$$\xi_\perp = \frac{M_B}{M_B + M_{K^*}} V \,, \qquad \xi_\parallel = \frac{M_B + M_{K^*}}{2E_{K^*}} A_1 - \frac{M_B - M_{K^*}}{M_B} A_2 \,. \tag{591}$$

The form factors A_1 and A_2 are obtained by:

$$\begin{aligned} A_1 &= \frac{2m_B\, E_{K^*}}{(m_B + m_{K^*})^2}\, V \,, \\ A_2 &= \left(\frac{m_B + m_{K^*}}{2\, E_{K^*}}\, A_1 - \frac{m_{K^*}}{E_{K^*}}\, A_0 \right) \frac{m_B}{m_B + m_{K^*}} \,. \end{aligned} \tag{592}$$

The functions $\mathcal{T}_{\perp,\parallel}$ are known at NLO in the framework of QCDf, and are given in section E.4.4.

Low recoil region

The predictions for $\bar{B}^0 \to \bar{K}^{*0}\ell^+\ell^-$ are worked out in [77] using the heavy quark effective theory (HQET) framework by Grinstein and Pirjol [88]. The low recoil transversity amplitudes to leading order in $1/m_b$ are given as [77]:

$$A_\perp^{L,R} = +i\Big\{(C_9^{\rm eff} \mp C_{10}) + \kappa\frac{2\hat{m}_b}{\hat{s}}\,C_7^{\rm eff}\Big\}f_\perp\ , \tag{593}$$

$$A_\parallel^{L,R} = -i\Big\{(C_9^{\rm eff} \mp C_{10}) + \kappa\frac{2\hat{m}_b}{\hat{s}}\,C_7^{\rm eff}\Big\}f_\parallel\ , \tag{594}$$

$$A_0^{L,R} = -i\Big\{(C_9^{\rm eff} \mp C_{10}) + \kappa\frac{2\hat{m}_b}{\hat{s}}\,C_7^{\rm eff}\Big\}f_0\ , \tag{595}$$

where

$$\kappa = 1 - 2\,\frac{\alpha_s}{3\pi}\ln\left(\frac{\mu}{m_b}\right)\ , \tag{596}$$

and the form factors read

$$f_\perp = N m_B\frac{\sqrt{2\hat{\lambda}}}{1+\hat{m}_{K^*}}V\ , \tag{597}$$

$$f_\parallel = N m_B\sqrt{2}\,(1+\hat{m}_{K^*})\,A_1\ , \tag{598}$$

$$f_0 = N m_B\frac{(1-\hat{s}-\hat{m}_{K^*}^2)(1+\hat{m}_{K^*})^2A_1 - \hat{\lambda}\,A_2}{2\,\hat{m}_{K^*}(1+\hat{m}_{K^*})\sqrt{\hat{s}}}\ , \tag{599}$$

with the normalization factor:

$$N = V_{tb}V_{ts}^*\left[\frac{G_F^2\alpha_{em}^2\,m_B\,\hat{s}\sqrt{\hat{\lambda}}}{3\cdot 2^{10}\,\pi^5}\right]^{1/2}\ . \tag{600}$$

In the above equation $\hat{s} = q^2/m_B^2$, $\hat{m}_i = m_i/m_B$ and $\hat{\lambda} = 1+\hat{s}^2+\hat{m}_{K^*}^4 - 2\,(\hat{s}+\hat{s}\hat{m}_{K^*}^2+\hat{m}_{K^*}^2)$ are dimensionless variables.

The effective coefficients C_9 and C_7 in the high-q^2 region take the form:

$$\begin{aligned} C_9^{\rm eff} &= C_9 + h(0,q^2)\Big[\frac{4}{3}\,C_1 + C_2 + \frac{11}{2}\,C_3 - \frac{2}{3}\,C_4 + 52\,C_5 - \frac{32}{3}\,C_6\Big] \\ &\quad -\frac{1}{2}\,h(m_b,q^2)\Big[7\,C_3 + \frac{4}{3}\,C_4 + 76\,C_5 + \frac{64}{3}\,C_6\Big] + \frac{4}{3}\Big[C_3 + \frac{16}{3}\,C_5 + \frac{16}{9}\,C_6\Big] \\ &\quad +\frac{\alpha_s}{4\pi}\Big[C_1\,\big(B(q^2) + 4\,C(q^2)\big) - 3\,C_2\,\big(2\,B(q^2) - C(q^2)\big) - C_8F_8^{(9)}(q^2)\Big] \\ &\quad +\frac{8\,m_c^2}{q^2}\Big[\left(\frac{4}{9}\,C_1 + \frac{1}{3}\,C_2\right)(1+\hat{\lambda}_u) + 2\,C_3 + 20\,C_5\Big]\ , \end{aligned} \tag{601}$$

$$\begin{aligned} C_7^{\rm eff} &= C_7 - \frac{1}{3}\Big[C_3 + \frac{4}{3}\,C_4 + 20\,C_5 + \frac{80}{3}C_6\Big] \\ &\quad +\frac{\alpha_s}{4\pi}\Big[(C_1 - 6\,C_2)\,A(q^2) - C_8F_8^{(7)}(q^2)\Big]\ , \end{aligned} \tag{602}$$

with $\hat{\lambda}_u$ given in Eq.(581) and the lowest order charm loop function reads

$$h(0,q^2) = \frac{8}{27} + \frac{4}{9}\left(\ln\frac{\mu^2}{q^2} + i\pi\right) , \qquad (603)$$

while the b quarks loops from penguin operators are taken into account by the function

$$h(m_b,q^2) = \frac{4}{9}\left(\ln\frac{\mu^2}{m_b^2} + \frac{2}{3} + z\right) - \frac{4}{9}(2+z)\sqrt{z-1}\arctan\frac{1}{\sqrt{z-1}} , \qquad (604)$$

with $z = 4m_b^2/q^2$. The functions $F_8^{(7)}, F_8^{(9)}$ are given in Eqs.(497)-(498) and the functions $A(s), B(s), C(s)$ are as follows [87]

$$\begin{aligned} A(s) &= -\frac{104}{243}\ln\left(\frac{m_b^2}{\mu^2}\right) + \frac{4\hat{s}}{27(1-\hat{s})}\Big[\mathrm{Li}_2(\hat{s}) + \ln(\hat{s})\ln(1-\hat{s})\Big] \qquad (605)\\ &+\frac{1}{729(1-\hat{s})^2}\Big[6\hat{s}(29-47\hat{s})\ln(\hat{s}) + 785 - 1600\hat{s} + 833\hat{s}^2 + 6\pi i(20-49\hat{s}+47\hat{s}^2)\Big]\\ &-\frac{2}{243(1-\hat{s})^3}\Big[2\sqrt{z-1}(-4+9\hat{s}-15\hat{s}^2+4\hat{s}^3)\mathrm{arccot}(\sqrt{z-1}) + 9\hat{s}^3\ln^2(\hat{s})\\ &+18\pi i\hat{s}(1-2\hat{s})\ln(\hat{s})\Big]\\ &+\frac{2\hat{s}}{243(1-\hat{s})^4}\Big[36\,\mathrm{arccot}^2(\sqrt{z-1}) + \pi^2(-4+9\hat{s}-9\hat{s}^2+3\hat{s}^3)\Big] , \end{aligned}$$

$$\begin{aligned} B(s) &= \frac{8}{243\hat{s}}\left[(4-34\hat{s}-17\pi i\hat{s})\ln\left(\frac{m_b^2}{\mu^2}\right) + 8\hat{s}\ln^2\left(\frac{m_b^2}{\mu^2}\right) + 17\hat{s}\ln(\hat{s})\ln\left(\frac{m_b^2}{\mu^2}\right)\right] \qquad (606)\\ &+\frac{(2+\hat{s})\sqrt{z-1}}{729\hat{s}}\Bigg\{-48\ln\left(\frac{m_b^2}{\mu^2}\right)\mathrm{arccot}(\sqrt{z-1}) - 18\pi\ln(z-1) + 3i\ln^2(z-1)\\ &-24i\,\mathrm{Li}_2\left(\frac{-x_2}{x_1}\right) - 5\pi^2 i - 12\pi\Big[2\ln(x_1)+\ln(x_3)+\ln(x_4)\Big]\\ &+6i\Big[-9\ln^2(x_1)+\ln^2(x_2)-2\ln^2(x_4)+6\ln(x_1)\ln(x_2)-4\ln(x_1)\ln(x_3)+8\ln(x_1)\ln(x_4)\Big]\Bigg\}\\ &-\frac{2}{243\hat{s}(1-\hat{s})}\Bigg\{4\hat{s}(-8+17\hat{s})\Big[\mathrm{Li}_2(\hat{s})+\ln(\hat{s})\ln(1-\hat{s})\Big]\\ &+3(2+\hat{s})(3-\hat{s})\ln^2\left(\frac{x_2}{x_1}\right) + 12\pi(-6-\hat{s}+\hat{s}^2)\mathrm{arccot}(\sqrt{z-1})\Bigg\}\\ &+\frac{2}{2187\hat{s}(1-\hat{s})^2}\Big[-18\hat{s}(120-211\hat{s}+73\hat{s}^2)\ln(\hat{s})\\ &-288-8\hat{s}+934\hat{s}^2-692\hat{s}^3+18\pi i\hat{s}(82-173\hat{s}+73\hat{s}^2)\Big] \end{aligned}$$

$$-\frac{4}{243\hat{s}(1-\hat{s})^3}\Big[-2\sqrt{z-1}\big(4-3\hat{s}-18\hat{s}^2+16\hat{s}^3-5\hat{s}^4\big)\mathrm{arccot}(\sqrt{z-1})$$

$$-9\hat{s}^3\ln^2(\hat{s})+2\pi i\hat{s}\big(8-33\hat{s}+51\hat{s}^2-17\hat{s}^3\big)\ln(\hat{s})\Big]$$

$$+\frac{2}{729\hat{s}(1-\hat{s})^4}\Big[72\big(3-8\hat{s}+2\hat{s}^2\big)\mathrm{arccot}^2(\sqrt{z-1})$$

$$-\pi^2\big(54-53\hat{s}-286\hat{s}^2+612\hat{s}^3-446\hat{s}^4+113\hat{s}^5\big)\Big]\,,$$

$$C(s)=-\frac{16}{81}\ln(\frac{s}{\mu^2})+\frac{428}{243}-\frac{64}{27}\zeta(3)+\frac{16}{81}\pi i\,, \tag{607}$$

where the following definitions are used in the above formulae:

$$s=q^2\,,\qquad \hat{s}=\frac{s}{m_b^2}\,,\qquad z=\frac{4m_b^2}{s}\,, \tag{608}$$

$$x_1=\frac{1}{2}+\frac{i}{2}\sqrt{z-1}\,, \tag{609}$$

$$x_2=\frac{1}{2}-\frac{i}{2}\sqrt{z-1}\,, \tag{610}$$

$$x_3=\frac{1}{2}+\frac{i}{2\sqrt{z-1}}\,, \tag{611}$$

$$x_4=\frac{1}{2}-\frac{i}{2\sqrt{z-1}}\,. \tag{612}$$

$\mu\sim m_b$ denotes the renormalization scale, ζ the Riemannian Zeta function and

$$\mathrm{Li}_2(x)=-\int_0^x dt\,\frac{\ln(1-t)}{t}\,, \tag{613}$$

is the Dilogarithm.

E.4.2 Angular coefficients in terms of helicity amplitudes

The functions J_{1-9} written in terms of helicity amplitudes are given by [121][11]

$$J_1^c=F\left\{\frac{1}{2}\left(|H_V^0|^2+|H_A^0|^2\right)+|H_P|^2+\frac{2m_\ell^2}{q^2}\left(|H_V^0|^2-|H_A^0|^2\right)+\beta_\ell^2|H_S|^2\right\}, \tag{614a}$$

$$J_1^s=F\left\{\frac{\beta_\ell^2+2}{8}\left(|H_V^+|^2+|H_V^-|^2+(V\to A)\right)+\frac{m_\ell^2}{q^2}\left(|H_V^+|^2+|H_V^-|^2-(V\to A)\right)\right\}, \tag{614b}$$

$$J_2^c=-F\,\frac{\beta_\ell^2}{2}\left(|H_V^0|^2+|H_A^0|^2\right), \tag{614c}$$

[11]There is a β_ℓ^2 factor missing in eq. 39 of [121].

$$J_2^s = F\,\frac{\beta_\ell^2}{8}\left(|H_V^+|^2 + |H_V^-|^2\right) + (V \to A), \tag{614d}$$

$$J_3 = -F\frac{\beta_\ell^2}{2}\text{Re}\left[H_V^+(H_V^-)^*\right] + (V \to A),$$

$$J_4 = F\,\frac{\beta_\ell^2}{4}\text{Re}\left[\left(H_V^- + H_V^+\right)\left(H_V^0\right)^*\right] + (V \to A), \tag{614e}$$

$$J_5 = F\left\{\frac{\beta_\ell}{2}\text{Re}\left[\left(H_V^- - H_V^+\right)\left(H_A^0\right)^*\right] + (V \leftrightarrow A) - \frac{\beta_\ell\, m_\ell}{\sqrt{q^2}}\text{Re}\left[H_S^*(H_V^+ + H_V^-)\right]\right\}, \tag{614f}$$

$$J_6^s = F\beta_\ell\,\text{Re}\left[H_V^-(H_A^-)^* - H_V^+(H_A^+)^*\right], \tag{614g}$$

$$J_6^c = 2F\frac{\beta_\ell\, m_\ell}{\sqrt{q^2}}\text{Re}\left[H_S^* H_V^0\right], \tag{614h}$$

$$J_7 = F\left\{\frac{\beta_\ell}{2}\,\text{Im}\left[\left(H_A^+ + H_A^-\right)(H_V^0)^* + (V \leftrightarrow A)\right] - \frac{\beta_\ell\, m_\ell}{\sqrt{q^2}}\,\text{Im}\left[H_S^*(H_V^- - H_V^+)\right]\right\}, \tag{614i}$$

$$J_8 = F\,\frac{\beta_\ell^2}{4}\text{Im}\left[(H_V^- - H_V^+)(H_V^0)^*\right] + (V \to A), \tag{614j}$$

$$J_9 = F\,\frac{\beta_\ell^2}{2}\text{Im}\left[H_V^+(H_V^-)^*\right] + (V \to A), \tag{614k}$$

with

$$F = \frac{\lambda^{1/2}\beta_\ell\, q^2}{3\times 2^5\,\pi^3\, M_B^3}. \tag{615}$$

Helicity amplitudes, in the full FF approach at NLO

The helicity amplitudes at NLO in the full FF approach, within QCDF in the large recoil limit ($q^2 \lesssim 7$ GeV^2), are given by

$$H_V^\lambda = -i\,N'\left\{(C_9 + Y(q^2))\tilde{V}_\lambda - C_9'\tilde{V}_{-\lambda} + \frac{M_B^2}{q^2}\left[\frac{2\,\hat{m}_b}{M_B}(C_7^{\text{eff}}\tilde{T}_\lambda - C_7'\tilde{T}_{-\lambda}) - 16\pi^2\mathcal{N}_\lambda\right]\right\}, \tag{616}$$

$$H_A^\lambda = -i\,N'(C_{10}\tilde{V}_\lambda - C_{10}'\tilde{V}_{-\lambda}), \tag{617}$$

$$H_P = i\,N'\left\{(C_{Q_2} - C_{Q_2}') + \frac{2\,m_\ell\hat{m}_b}{q^2}\left(1 + \frac{m_s}{m_b}\right)(C_{10} - C_{10}')\right\}\tilde{S}, \tag{618}$$

$$H_S = i\,N'(C_{Q_1} - C_{Q_1}')\tilde{S}, \tag{619}$$

where $\lambda = \pm, 0$ corresponds to the helicity of the K^*-meson and

$$N' = -\frac{4G_F M_B}{\sqrt{2}}\frac{e^2}{16\pi^2}V_{tb}V_{ts}^*. \tag{620}$$

The sub-leading non-local contributions $\mathcal{N}_{\pm,0}$ calculated in QCDf are given in 579 and the helicity form factor $\tilde{V}_\lambda, \tilde{T}_\lambda, \tilde{S}$ can be described in terms of the form factors $V(q^2), A_{0,1,2,}(q^2)$ and $T_{1,2,3}$ [121]

$$\tilde{V}_\pm\left(q^2\right) = \frac{1}{2}\left[\left(1+\frac{M_{K^*}}{M_B}\right)A_1\left(q^2\right) \mp \frac{\sqrt{\lambda(q^2)}}{M_B(M_B+M_{K^*})}V\left(q^2\right)\right],$$

$$\tilde{T}_\pm\left(q^2\right) = \frac{M_B^2-M_{K^*}^2}{2M_B^2}T_2\left(q^2\right) \mp \frac{\sqrt{\lambda(q^2)}}{2M_B^2}T_1\left(q^2\right),$$

$$\tilde{V}_0(q^2) = \frac{4M_{K^*}}{\sqrt{q^2}}A_{12}(q^2), \quad \tilde{T}_0(q^2) = \frac{2\sqrt{q^2}M_{K^*}}{M_B(M_B+M_{K^*})}T_{23}(q^2), \quad \tilde{S} = -\frac{\sqrt{\lambda(q^2)}}{2M_B(m_b+m_s)}A_0,$$

where

$$A_{12} = \frac{(M_B+M_{K^*})^2\left(M_B^2-M_{K^*}^2-q^2\right)A_1-\lambda(q^2)A_2}{16M_BM_{K^*}^2\left(M_B+M_{K^*}\right)}$$

$$T_{23} = \frac{\left(M_B^2-M_{K^*}^2\right)\left(M_B^2+3M_{K^*}^2-q^2\right)T_2-\lambda(q^2)T_3}{8M_BM_{K^*}^2\left(M_B-M_{K^*}\right)}.$$

E.4.3 Translation between transversity and helicity amplitudes

The relations between transversity amplitudes ($A^{L,R}_{\perp,\parallel,0,S}$) and helicity amplitudes ($H^\lambda_{V,A,S,P}$) are:

$$A_\perp^{L/R} = \frac{i\sqrt{F}}{2\sqrt{2}}\left[\left(H_V^+-H_V^-\right)\mp\left(H_A^+-H_A^-\right)\right], \quad A_0^{L/R} = \frac{i\sqrt{F}}{2}\left[H_V^0\mp H_A^0\right], \tag{621}$$

$$A_\parallel^{L/R} = \frac{i\sqrt{F}}{2\sqrt{2}}\left[\left(H_V^++H_V^-\right)\mp\left(H_A^++H_A^-\right)\right], \quad A_t = -\frac{i\sqrt{F}}{2m_\ell}\sqrt{q^2}H_P^0, \qquad A_S = i\sqrt{F}H_S^0,$$

and

$$H_V^\pm = \frac{\sqrt{q^2}}{i2\sqrt{F}}\left[(A_\parallel^R+A_\parallel^L)\pm(A_\perp^R+A_\perp^L)\right], \qquad H_V^0 = \frac{1}{i\sqrt{F}}\left(A_0^R+A_0^L\right), \tag{622}$$

$$H_A^\pm = \frac{\sqrt{q^2}}{i2\sqrt{F}}\left[(A_\parallel^R-A_\parallel^L)\pm(A_\perp^R-A_\perp^L)\right], \qquad H_A^0 = \frac{1}{i\sqrt{F}}\left(A_0^R-A_0^L\right),$$

$$H_P = -\frac{2m_\ell}{i\sqrt{F}\sqrt{q^2}}A_t, \qquad H_S = \frac{1}{i\sqrt{F}}A_S,$$

where

$$N'\sqrt{F} = -4m_BN. \tag{623}$$

E.4.4 Calculation of $\mathcal{T}_a^\pm$

In the heavy quark limit the matrix elements of $B \to K^*$ depend only on four independent functions $\mathcal{T}_a^\pm$ corresponding to a transversely ($a =\perp$) and longitudinally ($a =\parallel$) polarized K^*. At next-to-leading order we have [84]:

$$\mathcal{T}_a = \xi_a C_a + \frac{\pi^2}{N_c}\frac{f_B f_{K^*,a}}{M_B}\Xi_a \sum_\pm \int \frac{d\omega}{\omega}\,\Phi_{B,\pm}(\omega)\int_0^1 du \Phi_{K^*,a}(u) T_{a,\pm}(u,\omega)\ , \tag{624}$$

where $C_F = 4/3$, $N_c = 3$, $\Xi_\perp \equiv 1$, $\Xi_\parallel \equiv m_{K^*}/E_{K^*}$ and μ_f is the scale at which the typical virtualities of the hard scattering terms are ($\mu_f = \sqrt{\mu \times \Lambda_{\rm QCD}}$) [81,84].

In practice, we need $\mathcal{T}^\pm_{\perp,\parallel}$ which can be obtained by:

$$\begin{aligned}
\mathcal{T}^+_{\perp,\parallel} &= (\mathcal{T}_{\perp,\parallel},\ \text{in which } C_7^{\rm eff} \to C_7^{\rm eff} + C_7^{\rm eff'})\ ,\\
\mathcal{T}^-_{\perp,\parallel} &= (\mathcal{T}_{\perp,\parallel},\ \text{in which } C_7^{\rm eff} \to C_7^{\rm eff} - C_7^{\rm eff'})\ .
\end{aligned} \tag{625}$$

These replacements lead to:

$$\mathcal{T}_\perp^+ = \xi_\perp C_\perp^+ + \frac{\pi^2}{N_c}\frac{f_B f_{K^*,\perp}}{M_B}\sum_\pm \int \frac{d\omega}{\omega}\,\Phi_{B,\pm}(\omega)\int_0^1 du \Phi_{K^*,\perp}(u) T^+_{\perp,\pm}(u,\omega)\ , \tag{626}$$

$$\mathcal{T}_\perp^- = \xi_\perp C_\perp^- + \frac{\pi^2}{N_c}\frac{f_B f_{K^*,\perp}}{M_B}\sum_\pm \int \frac{d\omega}{\omega}\,\Phi_{B,\pm}(\omega)\int_0^1 du \Phi_{K^*,\perp}(u) T^-_{\perp,\pm}(u,\omega)\ , \tag{627}$$

$$\mathcal{T}_\parallel^+ = \xi_\parallel C_\parallel^+ + \frac{\pi^2}{N_c}\frac{f_B f_{K^*,\parallel}}{M_B}\frac{m_{K^*}}{E_{K^*}}\sum_\pm \int \frac{d\omega}{\omega}\,\Phi_{B,\pm}(\omega)\int_0^1 du \Phi_{K^*,\parallel}(u) T^+_{\parallel,\pm}(u,\omega)\ , \tag{628}$$

$$\mathcal{T}_\parallel^- = \xi_\parallel C_\parallel^- + \frac{\pi^2}{N_c}\frac{f_B f_{K^*,\parallel}}{M_B}\frac{m_{K^*}}{E_{K^*}}\sum_\pm \int \frac{d\omega}{\omega}\,\Phi_{B,\pm}(\omega)\int_0^1 du \Phi_{K^*,\parallel}(u) T^-_{\parallel,\pm}(u,\omega)\ . \tag{629}$$

$f_{K^*,\perp}$, $f_{K^*,\parallel}$ and f_B can all be found in Table 17. $f_{K^*,\parallel}$ is scale dependent, but as its variation has a negligible effect therefore its scale dependency is usually ignored. However, $f_\perp$ is evolved using $f_\perp(\mu) = f_\perp(\mu_0)\left(\alpha_s(\mu)/\alpha_s(\mu_0)\right)^{4/23}$, μ_0 being the scale at which it has been given (usually 1 GeV) and $\mu \simeq m_b$.

Light-cone-distribution amplitudes Φ

To compute the integrals, it is necessary to know the the light-cone-distribution amplitudes Φ. The K^* light-cone distribution amplitude can be written in terms of the Gegenbauer coefficients [84,85]:

$$\Phi_{\bar{K}^*,a}(u) = 6u(1-u)\left\{1 + a_1(\bar{K}^*)_a\, C_1^{(3/2)}(2u-1) + a_2(\bar{K}^*)_a\, C_2^{(3/2)}(2u-1)\right\}\ . \tag{630}$$

The Gegenbauer polynomials are

$$C_1^{(3/2)}(x) = 3x \qquad C_2^{(3/2)}(x) = -\frac{3}{2} + \frac{15}{2}x^2\ , \tag{631}$$

and the Gegenbauer coefficients $(a_1(\bar{K}^*)_a, a_2(\bar{K}^*)_a)$ are given in Table 17. They are scale dependent [85]:

$$a_n(\mu) = a_n(\mu_0) \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{(\gamma_{(n)}-\gamma_{(0)})/\beta_0} , \tag{632}$$

with $\beta_0 = 11 - (2/3)n_f$. The one-loop anomalous dimensions are

$$\begin{aligned} \gamma_{(n)} = \gamma^{\parallel}_{(n)} &= C_F \left(1 - \frac{2}{(n+1)(n+2)} + 4\sum_{j=2}^{n+1} 1/j\right) , \\ \gamma^{\perp}_{(n)} &= C_F \left(1 + 4\sum_{j=2}^{n+1} 1/j\right) . \end{aligned} \tag{633}$$

$\gamma_{(0)}$ is the anomalous dimension of the local current and vanishes for vector and axial vector currents.

The two B light-cone distribution amplitudes $(\Phi_{B,+}, \Phi_{B,-})$ are not used directly as they appear as moments [84]. When calculating $\mathcal{T}_a$ where we have $T^{(1)}_{\perp,+}$, the moment

$$\lambda^{-1}_{B,+} = \int_0^\infty d\omega \frac{\Phi_{B,+}(\omega)}{\omega} \tag{634}$$

is needed. $\lambda^{-1}_{B,+}$ is given in Table 17 and it evolves using the following evolution relation [86]:

$$\lambda^{-1}_B(\mu) = \lambda^{-1}_B(\mu_0) \left\{1 + \frac{\alpha_s}{3\pi} \ln\frac{\mu^2}{\mu_0^2} (1 - 2\sigma_B(\mu_0))\right\} , \tag{635}$$

where $\sigma_B(1\,\mathrm{GeV}) = 1.4 \pm 0.4$.
When computing $\mathcal{T}_a$ where we have $T^{(1)}_{\parallel,-}$ we will also need the moment

$$\lambda^{-1}_{B,-}(q^2) = \int_0^\infty d\omega \frac{\Phi_{B,-}(\omega)}{\omega - q^2/M_B - i\epsilon} , \tag{636}$$

which can be expressed as:

$$\lambda^{-1}_{B,-}(q^2) = \frac{e^{-q^2/(M_B\omega_0)}}{\omega_0} \left[-\mathrm{Ei}(q^2/M_B\omega_0) + i\pi\right] , \tag{637}$$

where $\mathrm{Ei}(z)$ is the exponential integral function, and $\omega_0 = 2\bar{\Lambda}_{\mathrm{HQET}}/3$ and $\bar{\Lambda}_{\mathrm{HQET}} = M_B - m_b$.

Form factor correction $C^{\pm}_a$

The following formulas[12] are taken from [84], in which we applied (625).

$$C^{\pm}_a = C^{\pm(0)}_a + \frac{\alpha_s(\mu_b)C_F}{4\pi} C^{\pm(1)}_a . \tag{638}$$

[12]When employing the full form factor approach, the factorisable corrections which are indicated with an asterisk $(*)$ below the equation numbers, should be neglected.

At leading-order for $C_a^{\pm(0)}$ we have:

$$C_\perp^{\pm(0)} = (C_7^{\rm eff} \pm C_7^{\rm eff\prime}) + \frac{q^2}{2m_b M_B} Y(q^2) \,, \tag{639*}$$

$$C_\parallel^{\pm(0)} = -(C_7^{\rm eff} \pm C_7^{\rm eff\prime}) - \frac{M_B}{2m_b} Y(q^2) \,, \tag{640*}$$

with

$$\begin{aligned} Y(q^2) \;\; = \;\; & h(q^2, m_c)\left(\frac{4}{3}C_1 + C_2 + 6C_3 + 60C_5\right) \\ & -\frac{1}{2}h(q^2, m_b^{\rm pole})\left(7C_3 + \frac{4}{3}C_4 + 76C_5 + \frac{64}{3}C_6\right) \\ & -\frac{1}{2}h(q^2, 0)\left(C_3 + \frac{4}{3}C_4 + 16C_5 + \frac{64}{3}C_6\right) \\ & +\frac{4}{3}C_3 + \frac{64}{9}C_5 + \frac{64}{27}C_6 \,. \end{aligned} \tag{641}$$

The function

$$h(q^2, m_q) = -\frac{4}{9}\left(\ln\frac{m_q^2}{\mu^2} - \frac{2}{3} - z\right) - \frac{4}{9}(2+z)\sqrt{|z-1|} \times \begin{cases} \arctan\dfrac{1}{\sqrt{z-1}} & z > 1 \\ \ln\dfrac{1+\sqrt{1-z}}{\sqrt{z}} - \dfrac{i\pi}{2} & z \le 1 \end{cases} \tag{642}$$

with $z = 4m_q^2/q^2$, is related to the basic fermion loop.

The next-to-leading order coefficients $C_a^{\pm(1)}$ contain a factorisable as well as non-factorisable part:

$$C_a^{\pm(1)} = C_a^{\pm({\rm f})} + C_a^{\pm({\rm nf})} \,, \tag{643}$$

By “non-factorisable it's meant, all those corrections that are not contained in the definition of the QCD form factors for heavy-to-light transitions.

The factorisable corrections are [81, 84]

$$C_\perp^{\pm({\rm f})} = (C_7^{\rm eff} \pm C_7^{\rm eff\prime})\left(\ln\frac{m_b^2}{\mu^2} - L + \Delta M\right) \,, \tag{644*}$$

$$C_\parallel^{\pm({\rm f})} = -(C_7^{\rm eff} \pm C_7^{\rm eff\prime})\left(\ln\frac{m_b^2}{\mu^2} + 2L + \Delta M\right) \,, \tag{645*}$$

where L is defined in Eq. (589) and ΔM depends on the mass renormalization convention for m_b:

$$\begin{cases} \Delta M = 0 \,, & \overline{MS} \text{ scheme} \\ \Delta M = 3\ln(m_b^2/\mu^2) - 4(1-\mu_f/m_b) \,, & \text{Potential Subtracted scheme} \\ \Delta M = 3\ln(m_b^2/\mu^2) - 4 \,, & \text{Pole Mass scheme} \end{cases} \tag{646}$$

The non-factorisable correction is obtained by computing matrix elements of four-quark operators and the chromomagnetic dipole operator [84].

$$C_F\, C_\perp^{\pm(\mathrm{nf})} = -\bar{C}_2 F_2^{(7)} - C_8^{\mathrm{eff}} F_8^{(7)} - \frac{q^2}{2m_b M_B}\left[\bar{C}_2 F_2^{(9)} + 2\bar{C}_1\left(F_1^{(9)} + \frac{1}{6}F_2^{(9)}\right) + C_8^{\mathrm{eff}} F_8^{(9)}\right] \quad (647)$$

$$C_F\, C_\parallel^{\pm(\mathrm{nf})} = \bar{C}_2 F_2^{(7)} + C_8^{\mathrm{eff}} F_8^{(7)} + \frac{M_B}{2m_b}\left[\bar{C}_2 F_2^{(9)} + 2\bar{C}_1\left(F_1^{(9)} + \frac{1}{6}\, F_2^{(9)}\right) + C_8^{\mathrm{eff}} F_8^{(9)}\right] . \quad (648)$$

The quantities $F_{1,2}^{(7,9)}$ and $F_8^{(7,9)}$ are those given in Eqs. (497)-(502).

The barred coefficients are related to the Wilson coefficients in our usual Standard Basis as [84]:

$$\begin{aligned}
\bar{C}_1 &= \frac{1}{2}\, C_1 , \quad (649)\\
\bar{C}_2 &= C_2 - \frac{1}{6}\, C_1 ,\\
\bar{C}_3 &= C_3 - \frac{1}{6}\, C_4 + 16\, C_5 - \frac{8}{3}\, C_6 ,\\
\bar{C}_4 &= \frac{1}{2}\, C_4 + 8\, C_6 ,\\
\bar{C}_5 &= C_3 - \frac{1}{6}\, C_4 + 4\, C_5 - \frac{2}{3}\, C_6 ,\\
\bar{C}_6 &= \frac{1}{2}\, C_4 + 2\, C_6 ,
\end{aligned}$$

and the effective Wilson coefficients are

$$\begin{aligned}
C_7^{\mathrm{eff}} &= C_7 - \frac{1}{3} C_3 - \frac{4}{9} C_4 - \frac{20}{3} C_5 - \frac{80}{9} C_6 , \quad (650)\\
C_8^{\mathrm{eff}} &= C_8 + C_3 - \frac{1}{6}\, C_4 + 20 C_5 - \frac{10}{3} C_6 ,\\
C_9^{\mathrm{eff}} &= C_9 + Y(q^2) ,\\
C_{10}^{\mathrm{eff}} &= C_{10} .
\end{aligned}$$

The relevant expressions for $C_a^{(0,u)}$ are given by the following replacements [81]

$$C_7^{\mathrm{eff}} \to 0$$

$$Y(s) \to Y^{(u)}(s) \equiv \left(\frac{4}{3} C_1 + C_2\right)\left[h(s, m_c) - h(s, 0)\right] , \quad (651)$$

where $h(s, m_q)$ is given in Eq. 642 and with this prescription, the factorisable corrections $C_{\perp,\parallel}^{(f,u)} = 0$. The non-factorisable corrections $C_a^{(nf,u)}$ are obtained by the following replacements [81]

$$F_8^{(7,9)} \to 0$$

$$F_{1,2}^{(7,9)} \to F_{1,2}^{(7,9)} + F_{1,2,u}^{(7,9)} \,, \tag{652}$$

with $F_{1,2,u}^{(7,9)}$ from [87]

$$F_{1,u}^{(7)} = A(s) \,, \tag{653}$$

$$F_{2,u}^{(7)} = -6A(s) \,, \tag{654}$$

$$F_{1,u}^{(9)} = B(s) + 4C(s) \,, \tag{655}$$

$$F_{2,u}^{(9)} = -6B(s) + 3C(s) \,, \tag{656}$$

with the functions $A(s)$, $B(s)$ and $C(s)$ given in Eqs(605)-(607).

Spectator scattering

For the hard scattering kernel

$$T_{a,\pm}(u,\omega) = T_{a,\pm}^{(0)}(u,\omega) + \frac{\alpha_s(\mu_f)C_F}{4\pi} T_{a,\pm}^{(1)}(u,\omega) \,. \tag{657}$$

At leading-order we have the weak annihilation amplitude which has no analogue in the inclusive decay and generates the hard-(spectator)scattering term $T_{a,\pm}^{(0)}(u,\omega)$ [84]:

$$T_{\perp,+}^{(0)\pm}(u,\omega) = T_{\perp,-}^{(0)\pm}(u,\omega) = T_{\parallel,+}^{(0)\pm}(u,\omega) = 0 \,, \tag{658}$$

$$T_{\parallel,-}^{(0)\pm}(u,\omega) = -e_q \frac{M_B\omega}{M_B\omega - q^2 - i\epsilon} \frac{4M_B}{m_b} (\bar{C}_3 + 3\bar{C}_4) \,. \tag{659}$$

The next-to-leading order coefficients $T_a^{(1)}$ contain a factorisable as well as non-factorisable part:

$$T_a^{(1)} = T_a^{(\mathrm{f})} + T_a^{(\mathrm{nf})} \,. \tag{660}$$

The hard scattering functions $T_{a,\pm}^{(1)}$ contain a factorisable term from expressing the full QCD form factors in terms of ξ_a, related to the α_s-correction [81, 84]:

$$T_{\perp,+}^{(\mathrm{f})\pm}(u,\omega) = (C_7^{\mathrm{eff}} \pm C_7^{\mathrm{eff}\prime}) \frac{2M_B}{\bar{u}E_{K^*}} \,, \tag{661*}$$

$$T_{\perp,-}^{(\mathrm{f})\pm}(u,\omega) = T_{\parallel,-}^{(\mathrm{f})\pm}(u,\omega) = 0 \,, \tag{662*}$$

$$T_{\parallel,+}^{(\mathrm{f})\pm}(u,\omega) = (C_7^{\mathrm{eff}} \pm C_7^{\mathrm{eff}\prime}) \frac{4M_B}{\bar{u}E_{K^*}} \,, \tag{663*}$$

where $\bar{u} = 1 - u$. The non-factorisable correction is obtained by computing matrix elements of four-quark operators and the chromomagnetic dipole operator [84].

$$T^{(\mathrm{nf})\pm}_{\perp,+}(u,\omega) = -\frac{4e_d C_8^{\mathrm{eff}}}{u + \bar{u}q^2/M_B^2} + \frac{M_B}{2m_b}\Big[e_u t_\perp(u,m_c)(\bar{C}_2 + \bar{C}_4 - \bar{C}_6) \tag{664}$$

$$+ e_d t_\perp(u,m_b)(\bar{C}_3 + \bar{C}_4 - \bar{C}_6 - 4m_b/M_B \bar{C}_5) + e_d t_\perp(u,0)\bar{C}_3\Big] ,$$

$$T^{(\mathrm{nf})\pm}_{\perp,-}(u,\omega) = 0 , \tag{665}$$

$$T^{(\mathrm{nf})\pm}_{\parallel,+}(u,\omega) = \frac{M_B}{m_b}\Big[e_u t_\parallel(u,m_c)(\bar{C}_2 + \bar{C}_4 - \bar{C}_6) \tag{666}$$

$$+ e_d t_\parallel(u,m_b)(\bar{C}_3 + \bar{C}_4 - \bar{C}_6) + e_d t_\parallel(u,0)\bar{C}_3\Big] ,$$

$$T^{(\mathrm{nf})\pm}_{\parallel,-}(u,\omega) = e_q \frac{M_B\omega}{M_B\omega - q^2 - i\epsilon}\Bigg[\frac{8C_8^{\mathrm{eff}}}{\bar{u} + uq^2/M_B^2} \tag{667}$$

$$+ \frac{6M_B}{m_b}\Big(h(\bar{u}M_B^2 + uq^2, m_c)(\bar{C}_2 + \bar{C}_4 + \bar{C}_6)$$

$$+ h(\bar{u}M_B^2 + uq^2, m_b^{\mathrm{pole}})(\bar{C}_3 + \bar{C}_4 + \bar{C}_6)$$

$$+ h(\bar{u}M_B^2 + uq^2, 0)(\bar{C}_3 + 3\bar{C}_4 + 3\bar{C}_6) - \frac{8}{27}(\bar{C}_3 - \bar{C}_5 - 15\bar{C}_6)\Big)\Bigg] .$$

Here $e_u = 2/3$, $e_d = -1/3$ and e_q is the electric charge of the spectator quark in the B meson. The functions $t_a(u, m_q)$ are given below

$$t_\perp(u,m_q) = \frac{2M_B}{\bar{u}E_{K^*}} I_1(m_q) + \frac{q^2}{\bar{u}^2 E_{K^*}^2}\left(B_0(\bar{u}M_B^2 + uq^2, m_q) - B_0(q^2, m_q)\right) , \tag{668}$$

$$t_\parallel(u,m_q) = \frac{2M_B}{\bar{u}E_{K^*}} I_1(m_q) + \frac{\bar{u}M_B^2 + uq^2}{\bar{u}^2 E_{K^*}^2}(B_0(\bar{u}M_B^2 + uq^2, m_q) - B_0(q^2, m_q)) , \tag{669}$$

where B_0 and I_1 are defined as

$$B_0(s, m_q) = -2\sqrt{4m_q^2/s - 1}\,\arctan\frac{1}{\sqrt{4m_q^2/s - 1}} , \tag{670}$$

$$I_1(m_q) = 1 + \frac{2m_q^2}{\bar{u}(M_B^2 - q^2)}\Big[L_1(x_+) + L_1(x_-) - L_1(y_+) - L_1(y_-)\Big] , \tag{671}$$

and

$$x_\pm = \frac{1}{2} \pm \left(\frac{1}{4} - \frac{m_q^2}{\bar{u}M_B^2 + uq^2}\right)^{1/2} ,\; y_\pm = \frac{1}{2} \pm \left(\frac{1}{4} - \frac{m_q^2}{q^2}\right)^{1/2} , \tag{672}$$

$$L_1(x) = \ln\frac{x-1}{x}\ln(1-x) - \frac{\pi^2}{6} + \mathrm{Li}_2\left(\frac{x}{x-1}\right) . \tag{673}$$

It should be noted that m_q^2 can be treated as $m_q^2 - i\epsilon$ when imaginary parts are involved. The barred coefficients are given in Eq. (650).

The spectator scattering expressions relevant for $\mathcal{T}_a^u$ are given by

$$T_{\|,-}^{(0,u)}(u,\omega) = e_q \frac{M_B\omega}{M_B\omega - q^2 - i\epsilon} \frac{12M_B}{m_b} \delta_{qu} C_2 \,, \tag{674}$$

where q denotes the spectator quark.

$$T_{\perp,+}^{(nf,u)}(u,\omega) = e_u \frac{M_B}{2m_b} \left(C_2 - \frac{1}{6}C_1\right) \Big[t_\perp(u,m_c) - t_\perp(u,0)\Big] \,, \tag{675}$$

$$T_{\perp,-}^{(nf,u)}(u,\omega) = 0 \tag{676}$$

$$T_{\|,+}^{(nf,u)}(u,\omega) = e_u \frac{M_B}{m_b} \left(C_2 - \frac{1}{6}C_1\right) \Big[t_\|(u,m_c) - t_\|(u,0)\Big] \,, \tag{677}$$

$$\begin{aligned} T_{\|,-}^{(nf,u)}(u,\omega) &= e_q \frac{M_B\omega}{M_B\omega - q^2 - i\epsilon} \frac{6M_B}{m_b} \\ &\times \left(C_2 - \frac{1}{6}C_1\right) \Big[h(\bar{u}M_B^2 + uq^2, m_c) - h(\bar{u}M_B^2 + uq^2, 0)\Big] \,, \end{aligned} \tag{678}$$

where the functions $t_{\perp,\|}(u,m)$ are defined in Eqs. (668-669).

Weak annihilation: The power-suppressed weak annihilation corrections $\Delta\mathcal{T}_\perp^{(i)}$ to $\mathcal{T}_\perp^{(i)}$ defined in (626-627) are given by:

$$\begin{aligned} \Delta\mathcal{T}_\perp^{(t)}\Big|_{\text{ann}} &= -e_q \frac{4\pi^2}{3} \frac{f_B f_\perp}{m_b M_B} \Big[C_3 + \frac{4}{3}(C_4 + 3C_5 + 4C_6)\Big] \int_0^1 du \, \frac{\phi_\perp(u)}{\bar{u} + u\, q^2/M_B^2} \\ &+ e_q \frac{2\pi^2}{3} \frac{f_B f_\|}{m_b M_B} \frac{m_V}{(1 - q^2/M_B^2)\, \lambda_{B,+}(q^2)} (\bar{C}_3 + 3\bar{C}_4) \,, \end{aligned} \tag{679}$$

$$\Delta\mathcal{T}_\perp^{(u)}\Big|_{\text{ann}} = -e_q \, 2\pi^2 \frac{f_B f_\|}{m_b M_B} \frac{m_V}{(1 - q^2/M_B^2)\, \lambda_{B,+}(q^2)} \delta_{qu} C_2 \,, \tag{680}$$

Hard spectator scattering: The power-suppressed hard scattering corrections are [59, 76]:

$$\begin{aligned} \Delta\mathcal{T}_\perp^{(t)}\Big|_{\text{hsa}} = e_q \frac{\alpha_s C_F}{4\pi} \frac{\pi^2 f_B}{N_c m_b M_B} &\Bigg[12 C_8^{\text{eff}} \frac{m_b}{M_B} f_\perp X_\perp(q^2/M_B^2) \\ &+ 8 f_\perp \int_0^1 du \, \frac{\phi_\perp(u)}{\bar{u} + u\, q^2/M_B^2} F_V^{(t)}(\bar{u}M_B^2 + uq^2) \\ &- \frac{4 m_V f_\|}{(1 - q^2/M_B^2)\, \lambda_{B,+}(q^2)} \int_0^1 du \int_0^u dv \, \frac{\phi_\|(v)}{\bar{v}} F_V^{(t)}(\bar{u}M_B^2 + uq^2)\Bigg] \,. \end{aligned} \tag{681}$$

The quark-loop function $F_V^{(t)}(s)$

$$F_V^{(u)}(s) = \frac{3}{4}\left(C_2 - \frac{1}{6}C_1\right)\Big[h(s,m_c) - h(s,0)\Big]\ . \tag{682}$$

$$\lambda(q^2, m_V^2) = \left[\left(1 - \frac{q^2}{M_B^2}\right)^2 - \frac{2m_V^2}{M_B^2}\left(1 + \frac{q^2}{M_B^2}\right) + \frac{m_V^4}{M_B^4}\right]^{1/2}\ , \tag{683}$$

$$\frac{(\mathcal{C}_7^{(u)})^{\rho^0}}{(\mathcal{C}_7^{(t)})^{\rho^0}} \equiv \epsilon_0\, e^{i\theta_0} \simeq -0.06 - 0.11i, \qquad \frac{(\mathcal{C}_7^{(u)})^{\rho^+}}{(\mathcal{C}_7^{(t)})^{\rho^+}} \equiv \epsilon_+\, e^{i\theta_+} \simeq 0.24 - 0.12i\ . \tag{684}$$

E.4.5 Observables

The dilepton invariant mass spectrum for $B \to K^* l^+ l^-$ can be obtained after integrating the 4-differential distribution over all angles [73]:

$$\frac{d\Gamma}{dq^2} = \frac{3}{4}\left(J_1 - \frac{J_2}{3}\right), \qquad \text{and} \qquad \frac{d\bar{\Gamma}}{dq^2} = \frac{3}{4}\left(\bar{J}_1 - \frac{\bar{J}_2}{3}\right), \tag{685}$$

where the functions $\bar{J}_{1-9}$ are obtained from J_{1-9} by replacing the weak phases by their conjugates. The (normalized) forward-backward asymmetry $A_{\rm FB}$ is given, after full ϕ and θ_{K^*} integration as [73]

$$\begin{aligned} A_{\rm FB}(q^2) &\equiv \left[\int_{-1}^{0} - \int_{0}^{1}\right] d\cos\theta_l\, \frac{d^2\Gamma}{dq^2\, d\cos\theta_l} \Bigg/ \frac{d\Gamma}{dq^2} \\ &= -\frac{3}{8}\, \frac{J_6 + \bar{J}_6}{d\Gamma/dq^2 + d\bar{\Gamma}/dq^2}\ . \end{aligned} \tag{686}$$

where $J_i \equiv 2J_i^s + J_i^c$. A particularly interesting observable is the zero–crossing of the forward-backward asymmetry (q_0^2), which is calculated numerically in SuperIso.

The fractions of the K^* are [74]

$$F_L(s) = \frac{3\,(J_1^c + \bar{J}_1^c) - (J_2^c + \bar{J}_2^c)}{4\,(d\Gamma/dq^2 + d\bar{\Gamma}/dq^2)}\ , \tag{687}$$

$$F_T(s) = \frac{4\,(J_2^s + \bar{J}_2^s)}{d\Gamma/dq^2 + d\bar{\Gamma}/dq^2}\ . \tag{688}$$

The transverse amplitudes can be written as [75]

$$A_T^{(1)}(s) = \frac{-2\,\mathrm{Re}(A_\parallel A_\perp^*)}{|A_\perp|^2 + |A_\parallel|^2}\ , \tag{689}$$

$$A_T^{(2)}(s) = \frac{J_3 + \bar{J}_3}{2\,(J_2^s + \bar{J}_2^s)}\ , \tag{690}$$

$$A_T^{(3)}(s) = \left(\frac{4(J_4+\bar{J}_4)^2 + \beta_\ell^2 (J_7+\bar{J}_7)^2}{-2\,(J_2^c+\bar{J}_2^c)\,(2(J_2^s+\bar{J}_2^s)+J_3+\bar{J}_3)} \right)^{1/2} , \tag{691}$$

$$A_T^{(4)}(s) = \left(\frac{\beta_\ell^2 (J_5+\bar{J}_5)^2 + 4(J_8+\bar{J}_8)^2}{4(J_4+\bar{J}_4)^2 + \beta_\ell^2 (J_7+\bar{J}_7)^2} \right)^{1/2} , \tag{692}$$

$$A_T^{(5)}(s) = \frac{|A_\perp^L A_\parallel^{R*} + A_\parallel^L A_\perp^{R*}|}{|A_\perp|^2 + |A_\parallel|^2} , \tag{693}$$

$$A_{Im}(s) = \frac{J_9+\bar{J}_9}{d\Gamma/dq^2 + d\bar{\Gamma}/dq^2} , \tag{694}$$

where

$$A_i A_j^* \equiv A_{iL}(q^2) A_{jL}^*(q^2) + A_{iR}(q^2) A_{jR}^*(q^2) \quad (i,j = 0, \parallel, \perp) . \tag{695}$$

For the K^* polarisation parameter we have [75]

$$\alpha_{K^*}(s) = \frac{2F_L}{F_T} - 1 = -\frac{J_2+\bar{J}_2}{2\,(J_2^s+\bar{J}_2^s)} . \tag{696}$$

Another observable which is rather independent of hadronic input parameters is the isospin asymmetry arising from non-factorisable effects which depend on the charge of the spectator quark. Hence, depending on whether the decaying B meson is charged or neutral, there will be a difference in the contribution of these effects to the decay width which can cause an isospin asymmetry. The (CP-averaged) isospin asymmetry is defined as [76]

$$\frac{dA_I}{dq^2} = \frac{d\Gamma[B^0 \to K^{*0}\ell^+\ell^-]/dq^2 - d\Gamma[B^\pm \to K^{*\pm}\ell^+\ell^-]/dq^2}{d\Gamma[B^0 \to K^{*0}\ell^+\ell^-]/dq^2 + d\Gamma[B^\pm \to K^{*\pm}\ell^+\ell^-]/dq^2} . \tag{697}$$

The following transversity observables are also defined for the high q^2 region [77]:

$$H_T^{(1)}(s) = \frac{\sqrt{2}\,(J_4+\bar{J}_4)}{\sqrt{-(J_2^c+\bar{J}_2^c)\left[2\,(J_2^s+\bar{J}_2^s) - (J_3+\bar{J}_3)\right]}} , \tag{698}$$

$$H_T^{(2)}(s) = \frac{\beta_l\,(J_5+\bar{J}_5)}{\sqrt{-2(J_2^c+\bar{J}_2^c)\left[2\,(J_2^s+\bar{J}_2^s) + (J_3+\bar{J}_3)\right]}} , \tag{699}$$

$$H_T^{(3)}(s) = \frac{J_6+\bar{J}_6}{2\sqrt{4(J_2^s+\bar{J}_2^s)^2 - (J_3+\bar{J}_3)^2}} . \tag{700}$$

The $H_T^{(i)}$ are designed to have very small hadronic uncertainties at low recoil.

In addition, a set of *primary* (or *optimised*) observables have also been suggested in [78] which are appropriate ratios of angular coefficients, designed to cancel most of the dependence on the form factors (when employing the soft form factor approach). They read:

$$P_1(s) = \frac{J_3+\bar{J}_3}{2\,(J_2^s+\bar{J}_2^s)} = A_T^{(2)} , \tag{701}$$

$$P_2(s) = \frac{J_6^s + \bar{J}_6^s}{8\,(J_2^s + \bar{J}_2^s)}\ , \tag{702}$$

$$P_3(s) = -\frac{J_9 + \bar{J}_9}{4\,(J_2^s + \bar{J}_2^s)}\ , \tag{703}$$

$$P_4(s) = \frac{\sqrt{2}\,(J_4 + \bar{J}_4)}{\sqrt{-(J_2^c + \bar{J}_2^c)\left[2\,(J_2^s + \bar{J}_2^s) - (J_3 + \bar{J}_3)\right]}} = H_T^{(1)}\ , \tag{704}$$

$$P_5(s) = \frac{\beta_l\,(J_5 + \bar{J}_5)}{\sqrt{-2(J_2^c + \bar{J}_2^c)\left[2\,(J_2^s + \bar{J}_2^s) + (J_3 + \bar{J}_3)\right]}} = H_T^{(2)}\ , \tag{705}$$

$$P_6(s) = -\frac{\beta_l\,(J_7 + \bar{J}_7)}{\sqrt{-2(J_2^c + \bar{J}_2^c)\left[2\,(J_2^s + \bar{J}_2^s) - (J_3 + \bar{J}_3)\right]}}\ , \tag{706}$$

$$P_8(s) = -\frac{\sqrt{2}\,(J_8 + \bar{J}_8)}{\sqrt{-(J_2^c + \bar{J}_2^c)\left[2\,(J_2^s + \bar{J}_2^s) + (J_3 + \bar{J}_3)\right]}}\ . \tag{707}$$

And the primed observables are given by:

$$P_4'(s) = \frac{J_4 + \bar{J}_4}{\sqrt{-(J_2^c + \bar{J}_2^c)\,(J_2^s + \bar{J}_2^s)}}\ , \tag{708}$$

$$P_5'(s) = \frac{J_5 + \bar{J}_5}{2\,\sqrt{-(J_2^c + \bar{J}_2^c)\,(J_2^s + \bar{J}_2^s)}}\ , \tag{709}$$

$$P_6'(s) = -\frac{J_7 + \bar{J}_7}{2\,\sqrt{-(J_2^c + \bar{J}_2^c)\,(J_2^s + \bar{J}_2^s)}}\ , \tag{710}$$

$$P_8'(s) = -\frac{J_8 + \bar{J}_8}{\sqrt{-(J_2^c + \bar{J}_2^c)\,(J_2^s + \bar{J}_2^s)}}\ . \tag{711}$$

Another set of $B \to K^*\ell^+\ell^-$ observables are based on the CP-averaged angular coefficients [79]

$$S_i = \frac{J_i + \bar{J}_i}{d(\Gamma + \bar{\Gamma})/dq^2}, \tag{712}$$

with $i = 3, 4, 5, 7, 8, 9$ which together with $A_{\rm FB}$ and F_L form a complete set of observables.

E.5 $B \to K^*\gamma$

The radiative $B \to K^*\gamma$ decay can be described with a subset of the amplitudes of the $B \to K^*\ell^+\ell^-$ decay in term of the vectorial helicity amplitude $H_V(\lambda = \pm)$ [121]

$$\begin{aligned}\mathcal{A}_\lambda(\bar{B} \to \bar{K}^*\gamma) &= \lim_{q^2 \to 0} \frac{q^2}{e} H_V(q^2 = 0; \lambda) \\ &= \frac{iN' m_B^2}{e}\left[\frac{2\hat{m}_b}{m_B}(C_7\tilde{T}_\lambda(0) - C_7'\tilde{T}_{-\lambda}(0) - 16\pi^2\mathcal{N}_\lambda(q^2 = 0)\right],\end{aligned} \tag{713}$$

with the QCDf corrections ($\mathcal{N}_\lambda$) as described in 579. The decay rate of the $B \to K^*\gamma$ decay is given by

$$\Gamma(\bar{B} \to \bar{K}^*\gamma) = \frac{m_B^2 - m_{K^*}^2}{16\pi m_B^3} \left[|\mathcal{A}_+|^2 + |\mathcal{A}_-|^2 \right] . \tag{714}$$

E.6 $B_s \to \phi\ell^+\ell^-$

The $B_s \to \phi\ell^+\ell^-$ decay, similar to $B \to K^*\ell^+\ell^-$ is a $b \to s\ell^+\ell^-$ process a with a vector meson in the final state. And similarly both the full and soft form factor approaches are applicable. For the $B_s \to \phi\ell^+\ell^-$ decay, the $B_s \to \phi$ form factors are used (we use LCSR+lattice result of [120]). Moreover, in this decay, the spectator quark is a strange quark and contrary to the $B \to K^*\ell^+\ell^-$ decay it is not self-tagging, hence the untagged average over the $\bar{B}_s$ and B_s decay distributions is required. In order to take into account the CP-conjugated decay, the $\widetilde{J}_i$ angular coefficients [122] can be employed

$$\widetilde{J}_i = \zeta_i \bar{J}_i \ , \tag{715}$$

with

$$\zeta_i = 1 \quad \text{for} \quad i = 1s, 1c, 2s, 2c, 3, 4, 7\,; \qquad \zeta_i = -1 \quad \text{for} \quad i = 5, 6s, 6c, 8, 9\,, \tag{716}$$

and

$$x = \frac{\Delta M}{\Gamma}\,, \qquad y = \frac{\Delta\Gamma}{2\Gamma}\,, \tag{717}$$

where the angular coefficients $\bar{J}_i$ involves amplitudes denoted by $\bar{A}_X$ which are obtained from A_X by conjugating all weak phases.

E.6.1 Observables

The time-integrated measurements by hadronic machines[13] such as the LHCb is given by

$$\langle J_i + \widetilde{J}_i \rangle_{\text{Hadronic}} = \frac{1}{\Gamma} \left[\frac{J_i + \widetilde{J}_i}{1 - y^2} - \frac{y\, h_i}{1 - y^2} \right] \tag{718}$$

The time-integrated decay rate is then given by

$$\left\langle \frac{d\Gamma}{dq^2} \right\rangle = \frac{\langle \mathcal{I} \rangle}{\Gamma(1 - y^2)} \ , \tag{719}$$

$$\begin{aligned} \langle \mathcal{I} \rangle_{\text{Hadronic}} = {} & \frac{3}{4} \Big[2(J_{1s} + \bar{J}_{1s} - y\, h_{1s}) + (J_{1c} + \bar{J}_{1c} - y\, h_{1c}) \Big] \\ & - \frac{1}{4} \Big[2(J_{2s} + \bar{J}_{2s} - y\, h_{2s}) + (J_{2c} + \bar{J}_{2c} - y\, h_{2c}) \Big] \end{aligned} \tag{720}$$

[13] See [122] for the case of B-factories.

where $\mathcal{I}$ corresponds to the normalisation in the analyses of the angular observables

$$S_i \equiv \langle \Sigma_i \rangle_{\text{Hadronic}} \equiv \frac{\langle J_i + \widetilde{J}_i \rangle_{\text{Hadronic}}}{\langle d\Gamma/dq^2 \rangle_{\text{Hadronic}}} = \frac{(J_i + \widetilde{J}_i) - y \times h_i}{\langle \mathcal{I} \rangle_{\text{Hadronic}}}. \tag{721}$$

Since $B_s \to \phi\ell^+\ell^-$ is not self-tagging, the only measurable CP-averaged combinations are for $i = 1s, 1c, 2s, 2c, 3, 4, 7$, where the relevant h_i coefficients are given by

$$h_{1s} = \frac{2+\beta_\ell^2}{2}\text{Re}[e^{i\phi}\{\widetilde{A}_\perp^L A_\perp^{L*} + \widetilde{A}_{||}^L A_{||}^{L*} + \widetilde{A}_\perp^R A_\perp^{R*} + \widetilde{A}_{||}^R A_{||}^{R*}\}] \tag{722a}$$
$$+ \frac{4m_\ell^2}{q^2}\text{Re}[e^{i\phi}\{\widetilde{A}_\perp^L A_\perp^{R*} + \widetilde{A}_{||}^L A_{||}^{R*}\} + e^{-i\phi}\{A_\perp^L \widetilde{A}_\perp^{R*} + A_{||}^L \widetilde{A}_{||}^{R*}\}]$$

$$h_{1c} = 2\text{Re}[e^{i\phi}\{\widetilde{A}_0^L A_0^{L*} + \widetilde{A}_0^R A_0^{R*}\}] \tag{722b}$$
$$+ \frac{8m_\ell^2}{q^2}[\text{Re}[e^{i\phi}\widetilde{A}_t A_t^*] + \text{Re}\{e^{i\phi}\widetilde{A}_0^L A_0^{R*} + e^{-i\phi}A_0^L \widetilde{A}_0^{R*}\}] + 2\beta_\ell^2\text{Re}[e^{i\phi}\{\widetilde{A}_S A_S^*\}]$$

$$h_{2s} = \frac{\beta_\ell^2}{2}\text{Re}[e^{i\phi}\{\widetilde{A}_\perp^L A_\perp^{L*} + \widetilde{A}_{||}^L A_{||}^{L*} + \widetilde{A}_\perp^R A_\perp^{R*} + \widetilde{A}_{||}^R A_{||}^{R*}\}] \tag{722c}$$

$$h_{2c} = -2\beta_\ell^2\text{Re}[e^{i\phi}\{\widetilde{A}_0^L A_0^{L*} + \widetilde{A}_0^R A_0^{R*}\}] \tag{722d}$$

$$h_3 = \beta_\ell^2\text{Re}[e^{i\phi}\{\widetilde{A}_\perp^L A_\perp^{L*} - \widetilde{A}_{||}^L A_{||}^{L*} + \widetilde{A}_\perp^R A_\perp^{R*} - \widetilde{A}_{||}^R A_{||}^{R*}\}] \tag{722e}$$

$$h_4 = \frac{1}{\sqrt{2}}\beta_\ell^2\text{Re}[e^{i\phi}\{\widetilde{A}_0^L A_{||}^{L*} + \widetilde{A}_0^R A_{||}^{R*}\} + e^{-i\phi}\{A_0^L \widetilde{A}_{||}^{L*} + A_0^R \widetilde{A}_{||}^{R*}\}] \tag{722f}$$

$$h_7 = \sqrt{2}\beta_\ell\Big[\text{Im}[e^{i\phi}\{\widetilde{A}_0^L A_{||}^{L*} - \widetilde{A}_0^R A_{||}^{R*}\} + e^{-i\phi}\{A_0^L \widetilde{A}_{||}^{L*} - A_0^R \widetilde{A}_{||}^{R*}\}] \tag{722g}$$

with $\phi = 2\beta_s$, $\sin\phi = 0.0369 \pm 0.001, x = 26.81 \pm 0.08$ and $y_s = 0.068 \pm 0.004$ [123].

E.7 $B \to K\ell^+\ell^-$

Another $b \to s\ell^+\ell^-$ semileptonic B-decay is $B \to K\ell^+\ell^-$ which unlike the $B \to K^*\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$ decays has a pseudoscalar meson in the final state. The double differential distribution of the $B \to K\ell^+\ell^-$ decay can be written as [125, 129]

$$\frac{d^2\,\Gamma(B \to K\ell^+\ell^-)}{dq^2\,d\cos\theta} = a_\ell(q^2) + b_\ell(q^2)\cos\theta + c_\ell(q^2)\cos^2\theta, \tag{723}$$

where q^2 is the dilepton invariant mass-squared, and θ is the angle between ℓ^- and the flight direction of $\bar{B}$ in the dilepton rest frame. The kinematically accessible phase space is

$$4m_\ell^2 \leq q^2 \leq (M_B - M_K)^2, \qquad -1 \leq \cos\theta \leq 1. \tag{724}$$

In (723), a_ℓ, b_ℓ and c_ℓ are defined as

$$a_\ell(q^2) = \mathcal{C}(q^2)\Big[q^2\left(\beta_\ell^2|F_S|^2 + |F_P|^2\right) + \frac{\lambda}{4}(|F_A|^2 + |F_V|^2) \\ + 2m_\ell(M_B^2 - M_K^2 + q^2)Re(F_P F_A^*) + 4m_\ell^2 M_B^2|F_A|^2\Big], \tag{725}$$

$$b_\ell(q^2) = 2\,\mathcal{C}(q^2)\Bigg\{\Big\{q^2\left[\beta_\ell^2 Re(F_S F_T^*) + Re(F_P F_{T5}^*)\right] \\ + m_\ell\left[\sqrt{\lambda}\beta_\ell Re(F_S F_V^*) + (M_B^2 - M_K^2 + q^2)Re(F_{T5}F_A^*)\right]\Big\}\Bigg\}, \tag{726}$$

$$c_\ell(q^2) = \mathcal{C}(q^2)\Big[q^2\left(\beta_\ell^2|F_T|^2 + |F_{T5}|^2\right) \\ - \frac{\lambda}{4}\beta_\ell^2(|F_A|^2 + |F_V|^2) + 2m_\ell\sqrt{\lambda}\beta_\ell Re(F_T F_V^*)\Big], \tag{727}$$

where

$$\mathcal{C}(q^2) = \Gamma_0\,\beta_\ell\,\sqrt{\lambda}, \tag{728}$$

and

$$\Gamma_0 = \frac{G_F^2\alpha_e^2|V_{tb}V_{ts}^*|^2}{512\pi^5 M_B^3}, \qquad \beta_\ell = \sqrt{1 - 4\frac{m_\ell^2}{q^2}}, \tag{729}$$

$$\lambda = M_B^4 + M_K^4 + q^4 - 2(M_B^2 M_K^2 + M_B^2 q^2 + M_K^2 q^2).$$

Similar to the $B \to V\ell^+, \ell^-$ decays, the $B \to K\ell^+\ell^-$ can be described both within the *full form factor* and the *soft form factor* approaches. For the $B \to K\ell^+\ell^-$ decay, in the low q^2 region ($q^2 \ll M_B^2$ and $\Lambda_{QCD} \ll E_K$), the three form factors f_0, f_+ and f_T reduce to one soft form factor ξ_P [126, 127].

Full FF approach at NLO

In the full FF approach, the F_i functions representing specific Lorentz structures ($i = S, P, V, A, T, T5$) are given by [125]

$$F_V(q^2) = (C_9^{\text{eff}} + C_9')f_+(q^2) + \frac{2m_b}{M_B + M_K}\left(C_7^{\text{eff}} + C_7' + \frac{4m_\ell}{m_b}C_T\right)f_T(q^2) + \delta F_V, \tag{730a}$$

$$F_A(q^2) = (C_{10} + C_{10}')f_+(q^2), \tag{730b}$$

$$F_S(q^2) = \frac{M_B^2 - M_K^2}{2(m_b - m_s)}(C_{Q_1} + C_{Q_1}')f_0(q^2), \tag{730c}$$

$$F_P(q^2) = \frac{M_B^2 - M_K^2}{2(m_b - m_s)}(C_{Q_2} + C'_{Q_2})f_0(q^2) \qquad (730d)$$

$$- m_\ell(C_{10} + C'_{10})\left[f_+(q^2) - \frac{M_B^2 - M_K^2}{q^2}\left(f_0(q^2) - f_+(q^2)\right)\right]$$

$$F_T(q^2) = \frac{2\sqrt{\lambda}\,\beta_\ell}{M_B + M_K}C_T f_T(q^2), \qquad (730e)$$

$$F_{T5}(q^2) = \frac{2\sqrt{\lambda}\,\beta_\ell}{M_B + M_K}C_{T5} f_T(q^2). \qquad (730f)$$

where $C_{T,T5}$ are tensor Wilson coefficients and f_+, f_-, f_0 and f_T are the $B \to K$ form factors which we take from [128]. In the SM, $F_S = F_T = F_{T5} = 0$ since $C_{Q_1}^{(\prime)} = C_{Q_2}^{(\prime)} = C_T = C_{T5} = 0$ and if we further consider $m_\ell = 0$ then we also have $F_P = 0$. The non-local sub-leading contributions are calculable within QCDf at leading order in Λ/m_b in the low-q^2 region. These effects contribute to F_V via

$$\delta F_V = \frac{2m_b}{M_B + M_K}\mathcal{T}_P^{\text{nf+WA}}\,, \qquad (731)$$

where $\mathcal{T}_P$ is given in section E.4.4.

Soft FF approach at NLO

In the soft FF approach, considering the factorisation scheme of [127] ($f_+(q^2) \equiv \xi_P(q^2)$), the F_i functions can be written as [129][14]

$$F_V(q^2) = \xi_P(q^2)\left\{(C_9 + C'_9) + \frac{2m_b}{M_B + M_K}\frac{\mathcal{T}_P}{\xi_P(q^2)} + \frac{8m_\ell}{M_B + M_K}\frac{f_T(q^2)}{f_+(q^2)}C_T\right\}, \qquad (733)$$

$$F_A(q^2) = \xi_P(q^2)(C_{10} + C'_{10}), \qquad (734)$$

$$F_S(q^2) = \xi_P(q^2)\left\{\frac{M_B^2 - M_K^2}{2(m_b - m_s)}\frac{f_0(q^2)}{f_+(q^2)}(C_S + C'_S)\right\}, \qquad (735)$$

$$F_P(q^2) = \xi_P(q^2)\Bigg\{\frac{M_B^2 - M_K^2}{2(m_b - m_s)}\frac{f_0(q^2)}{f_+(q^2)}(C_P + C'_P) \qquad (736)$$

$$- m_\ell\left[1 - \frac{M_B^2 - M_K^2}{q^2}\left(\frac{f_0(q^2)}{f_+(q^2)} - 1\right)\right](C_{10} + C'_{10})\Bigg\},$$

[14] In QCD factorization to include NLO corrections in α_s to the transversity amplitudes at large recoil, the following replacements should be made

$$(C_7^{eff} + C'_7)f_T(q^2) \longrightarrow \mathcal{T}_P(q^2), \qquad C_9^{eff} \longrightarrow C_9. \qquad (732)$$

$$F_T(q^2) = \xi_P(q^2) \left\{ \frac{2\sqrt{\lambda}\,\beta_\ell}{M_B + M_K} \frac{f_+(q^2)}{f_T(q^2)} C_T \right\}, \tag{737}$$

$$F_{T5}(q^2) = \xi_P(q^2) \left\{ \frac{2\sqrt{\lambda}\,\beta_\ell}{M_B + M_K} \frac{f_+(q^2)}{f_T(q^2)} C_{T5} \right\}, \tag{738}$$

where the form factor ratios f_+/f_T and f_0/f_+ are eliminated by using the symmetry relations between the f_+ and $f_{0,T}$ form factors [127]

$$\frac{f_0}{f_+} = \frac{2E_K}{M_B} \left(1 + \frac{\alpha_s C_F}{4\pi} [2 - 2L] + \frac{\alpha_s C_F}{4\pi} \frac{M_B(M_B - 2E_K)}{(2E_K)^2} \frac{\Delta F_P}{\xi_P} \right), \tag{739}$$

$$\frac{f_T}{f_+} = \frac{M_K + M_B}{M_B} \left(1 + \frac{\alpha_s C_F}{4\pi} \left[\ln \frac{m_b^2}{\mu^2} + 2L \right] - \frac{\alpha_s C_F}{4\pi} \frac{M_B}{2E_K} \frac{\Delta F_P}{\xi_P} \right) \tag{740}$$

where

$$\Delta F_P = \frac{8\pi^2 f_B f_P}{N_C M_B} \int \frac{d\omega}{\omega} \, \Phi_{B,+}(\omega) \int_0^1 du \frac{\Phi_K(u)}{\bar{u}}. \tag{741}$$

and

$$L \equiv -\frac{m_b^2 - q^2}{q^2} \ln \left(1 - \frac{q^2}{m_b^2} \right) \tag{742}$$

Low recoil region

In the high-q^2 region, the effective Wilson coefficients $C_9^{\rm eff}$ and $C_7^{\rm eff}$ in the full FF approach should be replaced by those given in Eq.(601) and Eq.(602), respectively.

In the high-q^2 region also symmetry relations among the form factors can be explored with the improved Isgur-Wise relation between f_T and f_+ given by [130]

$$f_T(q^2, \mu) = \frac{m_B\,(m_B + m_K)}{q^2} \left[\kappa(\mu) f_+(q^2) + \frac{2\,\delta_+^{(0)}(q^2)}{m_B} \right] + \mathcal{O}\left(\alpha_s \frac{\Lambda}{m_b}, \frac{\Lambda^2}{m_b^2} \right) \tag{743}$$

$$= \frac{m_B\,(m_B + m_K)}{q^2} \, \kappa(\mu)\, f_+(q^2) + \mathcal{O}\left(\frac{\Lambda}{m_b} \right), \tag{744}$$

where in the second line the subleading $1/m_b$ HQET form factor $\delta_+^{(0)}$ has been neglected, and the coefficient κ is given in Eq.(596).

E.7.1 Calculation of $\mathcal{T}_\mathbf{P}$

The amplitude $\mathcal{T}_P$ can be written from [84, 129]

$$\mathcal{T}_P = \xi_P \left[C_P^{(0)} + \frac{\alpha_s C_F}{4\pi} \left(C_P^{(\rm f)} + C_P^{(\rm nf)} \right) \right] \tag{745}$$

$$+\frac{\pi^2}{N_C}\frac{f_B f_K}{M_B}\sum_{\pm}\int\frac{d\omega}{\omega}\Phi_{B,\pm}(\omega)\int_0^1 du\,\Phi_K(u)\left[T^{(0)}_{P,\pm}+\frac{\alpha_s C_F}{4\pi}\left(T^{(\mathrm{f})}_{P,\pm}+T^{(\mathrm{nf})}_{P,\pm}\right)\right],$$

where

$$C^{(0)}_P=-C^{+(0)}_{\parallel},\qquad C^{(\mathrm{f})}_P=-C^{+(\mathrm{f})}_{\parallel},\qquad C^{(\mathrm{nf})}_P=-C^{+(\mathrm{nf})}_{\parallel},\tag{746}$$

$$T^{(0)}_{P,\pm}=-T^{(0)+}_{\parallel,\pm},\qquad T^{(\mathrm{f})}_{P,+}=-T^{(\mathrm{f})+}_{\parallel,+},\qquad T^{(\mathrm{f})}_{P,-}=-T^{(\mathrm{f})+}_{\parallel,-}=0\qquad T^{(\mathrm{nf})}_{P,\pm}=-T^{(\mathrm{nf})+}_{\parallel,\pm}.$$

where for the full FF approach, $C^{(0)}_P, C^{(\mathrm{f})}_P$ and $T^{(\mathrm{f})}_P$ should be neglected. The $C_\parallel$ and $T_\parallel$ expressions are given in section E.4.4.

E.7.2 Observables

From eq. 723, the angular distribution is given by

$$\frac{d\Gamma(B\to K\ell^+\ell^-)}{d\cos\theta}=A_\ell+B_\ell\cos\theta+C_\ell\cos^2\theta,\tag{747}$$

where

$$A_\ell=\int_{q^2_{\min}}^{q^2_{\max}}dq^2\,a_\ell(q^2),\qquad B_\ell=\int_{q^2_{\min}}^{q^2_{\max}}dq^2\,b_\ell(q^2),\qquad C_\ell=\int_{q^2_{\min}}^{q^2_{\max}}dq^2\,c_\ell(q^2).\tag{748}$$

The observables can be described in terms of these q^2-integrated coefficients, where for decay rate

$$\Gamma(B\to K\ell^+\ell^-)=2\left(A_\ell+\frac{1}{3}C_\ell\right),\tag{749}$$

and the normalised forward-backward

$$A_{FB}=\frac{B_\ell}{\Gamma_\ell}\tag{750}$$

and the so-called flat term

$$F^\ell_H\equiv\frac{2}{\Gamma_\ell}(A_\ell+C_\ell)=\int_{q^2_{\min}}^{q^2_{\max}}dq^2\left[a_\ell(q^2)+c_\ell(q^2)\right]\Big/\int_{q^2_{\min}}^{q^2_{\max}}dq^2\left[a_\ell(q^2)+\frac{1}{3}c_\ell(q^2)\right].\tag{751}$$

E.8 Branching ratio of $B_{s,d}\to\mu^+\mu^-$

E.8.1 CP-averaged branching ratio

The rare decay $B_s\to\mu^+\mu^-$ proceeds via Z^0 penguin and box diagrams in the SM, and the branching ratio is therefore highly suppressed. In supersymmetry, for large values of $\tan\beta$ this decay can receive large contributions from neutral Higgs bosons in chargino, charged Higgs and W-mediated penguins.

The branching fraction for $\text{BR}(B_s \to \mu^+\mu^-)$ is given by [89, 90]

$$\text{BR}(B_s \to \mu^+\mu^-) = \frac{G_F^2\alpha^2}{64\pi^3} f_{B_s}^2 \tau_{B_s} m_{B_s}^3 |V_{tb}V_{ts}^*|^2 \sqrt{1 - \frac{4m_\mu^2}{m_{B_s}^2}} \tag{752}$$

$$\times \left\{ \left(1 - \frac{4m_\mu^2}{m_{B_s}^2}\right) \left| \left(\frac{m_{B_s}}{m_b + m_s}\right) (C_{Q_1} - C'_{Q_1}) \right|^2 + \left| \left(\frac{m_{B_s}}{m_b + m_s}\right) (C_{Q_2} - C'_{Q_2}) + 2\,(C_{10} - C'_{10}) \frac{m_\mu}{m_{B_s}} \right|^2 \right\} ,$$

where f_{B_s} is the B_s decay constant, m_{B_s} is the B_s meson mass and τ_{B_s} is the B_s mean life, all given in Appendix G. The involved Wilson coefficients can be found in Appendix C.

Similarly, the branching fraction for $\text{BR}(B_d \to \mu^+\mu^-)$ can be obtained from:

$$\begin{aligned} \text{BR}(B_d \to \mu^+\mu^-) &= \frac{G_F^2\alpha^2}{64\pi^3} f_B^2 \tau_{B_d} m_{B_d}^3 |V_{tb}V_{td}^*|^2 \sqrt{1 - \frac{4m_\mu^2}{m_{B_d}^2}} \\ &\times \left\{ \left(1 - \frac{4m_\mu^2}{m_{B_d}^2}\right) | \left(\frac{m_{B_d}}{m_b + m_d}\right) C_{Q_1} |^2 + \left| \left(\frac{m_{B_d}}{m_b + m_d}\right) C_{Q_2} + 2\,C_{10} \frac{m_\mu}{m_{B_d}} \right|^2 \right\} , \end{aligned} \tag{753}$$

In `SuperIso`, first all the Wilson coefficients are calculated numerically, and then the branching ratios of $B_s \to \mu^+\mu^-$ and $B_d \to \mu^+\mu^-$ are evaluated.

E.8.2 Untagged Branching ratio

The branching ratio of $B_s \to \mu^+\mu^-$ described in the previous section is CP-averaged, while the experimental value is untagged. The untagged branching ratio is related to the CP-averaged one by [91]:

$$\text{BR}^{\text{untag}}(B_s \to \mu^+\mu^-) = \left[\frac{1 + \mathcal{A}_{\Delta\Gamma}\, y_s}{1 - y_s^2}\right] \text{BR}(B_s \to \mu^+\mu^-) , \tag{754}$$

where

$$y_s \equiv \frac{1}{2}\tau_{B_s}\Delta\Gamma_s = 0.088 \pm 0.014 , \tag{755}$$

and

$$\mathcal{A}_{\Delta\Gamma} = \frac{|P|^2 \cos(2\varphi_P) - |S|^2 \cos(2\varphi_S)}{|P|^2 + |S|^2} , \tag{756}$$

with

$$S \equiv \sqrt{1 - 4\frac{m_\mu^2}{M_{B_s}^2}} \frac{M_{B_s}^2}{2m_\mu} \frac{1}{m_b + m_s} \frac{C_{Q_1} - C'_{Q_1}}{C_{10}^{SM}} , \tag{757}$$

$$P \equiv \frac{C_{10} - C'_{10}}{C_{10}^{SM}} + \frac{M_{B_s}^2}{2m_\mu} \frac{1}{m_b + m_s} \frac{C_{Q_2} - C'_{Q_2}}{C_{10}^{SM}} , \tag{758}$$

and

$$\varphi_S = \arg(S) , \qquad \varphi_P = \arg(P) . \tag{759}$$

The obtained value can then be directly compared to the experimental one.

E.9 Branching ratio of $K \to \pi\nu\nu$

Similar to the rare B-decays with $\Delta F = 1$ (Eq. (24)), the $s \to d$ transitions can be described with an effective Hamiltonian

$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}}\lambda_t^{sd}\frac{\alpha_e}{4\pi}\sum_k \left(C_k^{sd,\ell}O_k^{sd,\ell} + C_{Q_k}^{sd,\ell}Q_k^{sd,\ell}\right) \tag{760}$$

with $\lambda_U^{sd} \equiv V_{Us}^* V_{Ud}$ and $U = u, c, t$. The relevant effective operators are given by

$$O_9^{sd,\ell} = (\bar{s}\gamma_\mu P_L d)\,(\bar{\ell}\gamma^\mu \ell), \qquad O_{10}^{sd,\ell} = (\bar{s}\gamma_\mu P_L d)\,(\bar{\ell}\gamma^\mu\gamma_5 \ell), \tag{761}$$

$$Q_1^{sd,\ell} = (\bar{s}P_R d)\,(\bar{\ell}\ell), \qquad Q_2^{sd,\ell} = (\bar{s}P_R d)\,(\bar{\ell}\gamma_5\ell), \tag{762}$$

$$O_L^{sd,\ell} = (\bar{s}\gamma_\mu P_L d)\,(\bar{\nu}\gamma^\mu(1-\gamma_5)\nu), \tag{763}$$

where ℓ corresponds to the lepton flavour. In general, the are can also be contributions from the primed version of the above operators where the chirality of the quark currents are reversed ($P_L \leftrightarrow P_R$). The Wilson coefficients $C_k^{sd,\ell}$ contain the short-distance effects which can be parametrised as

$$C_k^{sd,\ell} = C_{k,\rm SM}^{sd,\ell} + C_{k,\rm NP}^{sd,\ell}. \tag{764}$$

where the SM and NP subscripts indicate the Standard Model and New Physics contributions, respectively.

The branching fractions of the $K_L \to \pi^0\nu\bar{\nu}$ and $K^+ \to \pi^+\nu\bar{\nu}$ decays with a sum over all neutrino flavours adopting to our notation are [131, 132]

$$\mathrm{BR}(K_L \to \pi^0\nu\bar{\nu}) = \frac{\kappa_L}{\lambda^{10}}\frac{1}{3}s_W^4\sum_{\nu_\ell}\mathrm{Im}^2\left[\lambda_t C_L^{\nu_\ell}\right] \tag{765}$$

$$\mathrm{BR}(K^+ \to \pi^+\nu\bar{\nu}) = \frac{\kappa_+(1+\Delta_{\rm EM})}{\lambda^{10}}\frac{1}{3}s_W^4\sum_{\nu_\ell}\left[\mathrm{Im}^2\Big(\lambda_t C_L^{\nu_\ell}\Big) + \mathrm{Re}^2\Big(-\frac{\lambda_c X_c}{s_W^2} + \lambda_t^{sd}C_L^{\nu_\ell}\Big)\right] \tag{766}$$

where the electromagnetic radiative correction due to photon exchanges is given by $\Delta_{\rm EM} = -0.003$ for $E_{\rm max}^\gamma \approx 20$ MeV, and for the factors $\kappa_{+,L}$ we have [133]

$$\kappa_L = (2.231 \pm 0.013)\cdot 10^{-10}\left[\frac{\lambda}{0.225}\right]^8, \tag{767}$$

$$\kappa_+ = (0.5173 \pm 0.0025)\cdot 10^{-10}\left[\frac{\lambda}{0.225}\right]^8. \tag{768}$$

The short-distance SM contribution is given by $C_{L,\rm SM}^{\nu_\ell} = C_{L,\rm SM} = -X(x_t)/s_W^2$ with $X(x_t)$ [138] extracted from the original papers [134–137]

$$X(x_t) = X_0(x_t) + \frac{\alpha_s(\mu_t)}{4\pi}X_1(x_t) + \frac{\alpha}{4\pi}X_{\rm EW}(x_t), \tag{769}$$

where X_0 is the leading order result, and X_1, $X_{\rm EW}$ are the NLO QCD and EW corrections, respectively. The coupling constants α_s and α, as well as the parameter $x_t = m_t^2/m_W^2$ have to be evaluated at scale $\mu \sim \mathcal{O}(M_t)$. The LO expression is the gauge-independent linear combination $X_0(x_t) \equiv C(x_t) - 4B(x_t)$ [139, 140]

$$X_0(x_t) = \frac{x_t}{8}\left[\frac{x_t+2}{x_t-1} + \frac{3x_t-6}{(x_t-1)^2}\log x_t\right]. \tag{770}$$

The NLO QCD correction [134–136], in the $\overline{\rm MS}$ scheme reads,

$$\begin{aligned} X_1(x_t) = &-\frac{29x_t - x_t^2 - 4x_t^3}{3(1-x_t)^2} - \frac{x_t + 9x_t^2 - x_t^3 - x_t^4}{(1-x_t)^3}\log x_t \\ &+ \frac{8x_t + 4x_t^2 + x_t^3 - x_t^4}{2(1-x_t)^3}\log^2 x_t - \frac{4x_t - x_t^3}{(1-x_t)^2}{\rm Li}_2(1-x_t) \\ &+ 8x_t\frac{\partial X_0}{\partial x_t}\log\frac{\mu^2}{M_W^2}, \end{aligned} \tag{771}$$

where μ is the renormalisation scale. The 2-loop EW correction $X_{\rm EW}$ has been calculated in [137].

The charm contributions $X_c^\nu (\equiv \lambda^4 P_c(X))$ are described via

$$P_c(X) = P_c^{\rm SD}(X) + \delta P_{c,u} \tag{772}$$

where $\delta P_{c,u} = 0.04 \pm 0.02$ corresponds to the long-distance contributions as calculated in Ref. [141]. The short-distance contribution of the charm quark $P_c(X)$ including NNLO correction is calculated in Ref. [142] but the explicit analytical expression is not given. However, an approximate formula is given by

$$\begin{aligned} P_c^{\rm SD}(X) = 0.38049 &\left(\frac{m_c(m_c)}{1.30{\rm GeV}}\right)^{0.5081}\left(\frac{\alpha_s(M_Z)}{0.1176}\right)^{1.0192}\left(1 + \sum_{i,j}\kappa_{ij}L_{m_c}^i L_{\alpha_s}^j\right)\left(\frac{0.2255}{\lambda}\right)^4 \\ \pm 0.008707 &\left(\frac{m_c(m_c)}{1.30{\rm GeV}}\right)^{0.5276}\left(\frac{\alpha_s(M_Z)}{0.1176}\right)^{1.8970}\left(1 + \sum_{i,j}\epsilon_{ij}L_{m_c}^i L_{\alpha_s}^j\right)\left(\frac{0.2255}{\lambda}\right)^4, \end{aligned} \tag{773}$$

where

$$L_{m_c} = \ln\left(\frac{m_c(m_c)}{1.30{\rm GeV}}\right), \qquad L_{\alpha_s} = \ln\left(\frac{\alpha_s(M_Z)}{0.1176}\right), \tag{774}$$

and

$$\begin{aligned} &\kappa_{10} = 1.6624, \quad \kappa_{01} = -2.3537, \quad \kappa_{11} = -1.5862, \quad \kappa_{20} = 1.5036, \quad \kappa_{02} = -4.3477, \\ &\epsilon_{10} = -0.3537, \quad \epsilon_{01} = 0.6003, \quad \epsilon_{11} = -4.7652, \quad \epsilon_{20} = 1.0253, \quad \epsilon_{02} = 0.8866. \end{aligned} \tag{775}$$

In general, NP effects can be neutrino-flavour dependent, and besides the NP×NP terms, there are also contributions via SM×NP interference terms. This is more visible with

Eqs. 765 and 766 in their expanded form

$$\mathrm{BR}(K_L \to \pi^0 \nu\bar{\nu}) = \frac{\kappa_L}{\lambda^{10}} \frac{1}{3} s_W^4 \sum_{\nu_\ell} \mathrm{Im}^2 \left[\lambda_t C_L^{\nu_\ell}\right] \tag{776}$$

$$= \mathrm{BR}(K_L \to \pi^0 \nu\bar{\nu})_{\mathrm{SM}} + \frac{\kappa_L}{\lambda^{10}} \frac{1}{3} s_W^4 \left[\sum_{\nu_\ell} \mathrm{Im}^2 \left(\lambda_t\, C_{L,\mathrm{NP}}^{\nu_\ell}\right) + 2\,\mathrm{Im}\left(\lambda_t\, C_{L,\mathrm{SM}}\right) \sum_{\nu_\ell} \mathrm{Im}\left(\lambda_t\, C_{L,\mathrm{NP}}^{\nu_\ell}\right) \right]$$

$$\mathrm{BR}(K^+ \to \pi^+ \nu\bar{\nu}) = \frac{\kappa_+(1+\Delta_{\mathrm{EM}})}{\lambda^{10}} \frac{1}{3} s_W^4 \sum_{\nu_\ell} \left[\mathrm{Im}^2\Big(\lambda_t C_L^{\nu_\ell}\Big) + \mathrm{Re}^2\Big(-\frac{\lambda_c X_c}{s_W^2} + \lambda_t C_L^{\nu_\ell}\Big)\right] \tag{777}$$

$$= \mathrm{BR}(K^+ \to \pi^+ \nu\bar{\nu})_{\mathrm{SM}} + \frac{\kappa_+(1+\Delta_{\mathrm{EM}})}{\lambda^{10}} \frac{1}{3} s_W^4 \Bigg[\sum_{\nu_\ell} \mathrm{Im}^2 \left(\lambda_t C_{L,\mathrm{NP}}^{\nu_\ell}\right) + 2\,\mathrm{Im}\left(\lambda_t C_{L,\mathrm{SM}}\right) \sum_{\nu_\ell} \mathrm{Im}\left(\lambda_t C_{L,\mathrm{NP}}^{\nu_\ell}\right)$$
$$+ \sum_{\nu_\ell} \mathrm{Re}^2 \left(\lambda_t C_{L,\mathrm{NP}}^{\nu_\ell}\right) + 2\,\mathrm{Re}\left(\lambda_t C_{L,\mathrm{SM}}\right) \sum_{\nu_\ell} \mathrm{Re}\left(\lambda_t C_{L,\mathrm{NP}}^{\nu_\ell}\right) - 2 \sum_{\nu_\ell} \mathrm{Re}\left(\frac{\lambda_c X_c^\nu}{s_W^2}\right) \mathrm{Re}\left(\lambda_t C_{L,\mathrm{NP}}^{\nu_\ell}\right) \Bigg]$$

For the interference terms of the $\mathrm{BR}(K^+ \to \pi^+ \nu\bar{\nu})$ decays, the NNLO charm contributions for the different neutrino flavours are needed in separated form for which we use the NLO results $X_c^{e/\mu} = 10.05 \times 10^{-4}$ and $X_c^\tau = 6.64 \times 10^{-4}$ at $\mu_c = 1.3$ GeV [132].

E.10 Branching ratio of $K_{L,S} \to \mu^+\mu^-$

The branching ratios of the $K_L \to \mu^+\mu^-$ and $K_S \to \mu^+\mu^-$ decays, adopted to our notation is given by [141, 143]

$$\mathrm{BR}(K_L^0 \to \mu^+\mu^-) = \tau_L \frac{f_K^2 m_K^3 \beta_{\mu,K}}{16\pi} \left(\frac{G_F \alpha_e}{\sqrt{2}\pi}\right)^2 \times \Bigg\{ \beta_{\mu,K}^2 \left| \frac{m_K}{m_s + m_d} \mathrm{Im}\big(\lambda_t C_{Q_1}\big)\right|^2$$
$$+ \left| \frac{\sqrt{2}\pi}{G_F \alpha_e} N_L^{\mathrm{LD}} - \frac{2 m_\mu}{m_K} \mathrm{Re}\Big(-\lambda_c \frac{Y_c}{s_W^2} + \lambda_t C_{10}\Big) - \frac{m_K}{m_s + m_d} \mathrm{Re}\big(\lambda_t C_{Q_2}\big) \right|^2 \Bigg\}, \tag{778}$$

$$\mathrm{BR}(K_S^0 \to \mu^+\mu^-) = \tau_S \frac{f_K^2 m_K^3 \beta_{\mu,K}}{16\pi} \left(\frac{G_F \alpha_e}{\sqrt{2}\pi}\right)^2 \times \Bigg\{ \beta_{\mu,K}^2 \left| \frac{\sqrt{2}\pi}{G_F \alpha_e} N_S^{\mathrm{LD}} - \frac{m_K}{m_s + m_d} \mathrm{Re}\big(\lambda_t C_{Q_1}\big)\right|^2$$
$$+ \left| \frac{2 m_\mu}{m_K} \mathrm{Im}\Big(-\lambda_c \frac{Y_c}{s_W^2} + \lambda_t C_{10}\Big) + \frac{m_K}{m_s + m_d} \mathrm{Im}\big(\lambda_t C_{Q_2}\big) \right|^2 \Bigg\}, \tag{779}$$

with α_e at the M_Z scale, $\beta_{\mu,K} \equiv \sqrt{1 - 4 m_\mu / M_K^2}$ and in order to include chirality flipped contributions, the Wilson coefficients should be replaced by $C_i \to C_i - C_i'$. The short-distance SM contribution is given by $C_{10,\mathrm{SM}}^\ell$ and the charm contributions $Y_c\,\big(\equiv \lambda^4 P_c(Y)\big)$, and the long-distance contributions are denoted as N_L^{LD} and N_S^{LD} where the former has an unknown sign.

$P_c(Y)$ is known at NNLO in QCD [156] and while the analytic expression for $P_c(Y)$ is not given in [156], an approximate formula with λ-dependence is offered

$$P_c(Y) = 0.115 \pm 0.008_{\text{theor}} \pm 0.008_{m_c} \pm 0.001_{\alpha_s} = (0.115 \pm 0.018) \left(\frac{0.225}{\lambda}\right)^4 , \qquad (780)$$

Another approximate formula with an accuracy of better than $\pm 1.0\%$ in the ranges $1.15\,\text{GeV} \leq m_c(m_c) \leq 1.45\,\text{GeV}$, $0.1150 \leq \alpha_s(M_Z) \leq 0.1230$, $1.0\,\text{GeV} \leq \mu_c \leq 3.0\,\text{GeV}$ and $2.5\,\text{GeV} \leq \mu_b \leq 10.0\,\text{GeV}$ is also given in Ref. [156]

$$\begin{aligned} P_c(Y) = 0.1198 & \left(\frac{m_c(m_c)}{1.30\,\text{GeV}}\right)^{2.3595} \left(\frac{\alpha_s(M_Z)}{0.1187}\right)^{6.6055} \\ & \times \left(1 + \sum_{i,j,k,l} \kappa_{ijlm} L^i_{m_c} L^j_{\alpha_s} L^k_{\mu_c} L^l_{\mu_b}\right) \left(\frac{0.225}{\lambda}\right)^4 , \end{aligned} \qquad (781)$$

where

$$\begin{aligned} L_{m_c} &= \ln\left(\frac{m_c(m_c)}{1.30\,\text{GeV}}\right) , & L_{\alpha_s} &= \ln\left(\frac{\alpha_s(M_Z)}{0.1187}\right) , \\ L_{\mu_c} &= \ln\left(\frac{\mu_c}{1.5\,\text{GeV}}\right) , & L_{\mu_b} &= \ln\left(\frac{\mu_b}{5.0\,\text{GeV}}\right) , \end{aligned} \qquad (782)$$

with

$$\begin{aligned} \kappa_{1000} &= -0.5373, & \kappa_{0100} &= -6.0472, & \kappa_{0010} &= -0.0956, \\ \kappa_{0001} &= 0.0114, & \kappa_{1100} &= 3.9957, & \kappa_{1010} &= 0.3604, \\ \kappa_{0110} &= 0.0516, & \kappa_{0101} &= -0.0658, & \kappa_{2000} &= -0.1767, \\ \kappa_{0200} &= 16.4465, & \kappa_{0020} &= -0.1294, & \kappa_{0030} &= 0.0725. \end{aligned} \qquad (783)$$

The long-distance contributions as extracted in [143,145] from [141,144,146] is given by

$$N_L^{\text{LD}} = \frac{\pm 4\, \alpha_0\, m_\mu}{\pi\, f_K\, M_K^2} \sqrt{\frac{2\pi}{M_K} \frac{\text{Br}(K_L^0 \to \gamma\gamma)^{\text{EXP}}}{\tau_L}} \times (\chi_{\text{disp}} + i\chi_{\text{abs}}) \qquad (784)$$

$$N_S^{\text{LD}} = \frac{2\, \alpha_0\, m_\mu}{\pi\, f_K\, M_K^2\, |H(0)|} \sqrt{\frac{2\pi}{M_K} \frac{\text{Br}(K_S^0 \to \gamma\gamma)^{\text{EXP}}}{\tau_S}} \times (\mathcal{I}_{\text{disp}} + i\mathcal{I}_{\text{abs}}). \qquad (785)$$

with $\alpha_0 = 1/137.04$.

For the $K_L \to \mu^+\mu^-$ decay, $\text{Br}(K_L^0 \to \gamma\gamma)^{\text{EXP}} = (5.47 \pm 0.04) \times 10^{-4}$ [150] and the 2γ intermediate state [152,153] is given by $\chi_{\text{disp}} + i\chi_{\text{abs}} = (0.71 \pm 0.15 \pm 1.0) + i(-5.21)$ where the absorptive contribution is calculated via $\chi_{\text{abs}} = \frac{\pi}{2\beta_{\mu,K}} \ln\left(\frac{1-\beta_{\mu,K}}{1+\beta_{\mu,K}}\right)$ and the dispersive part is taken from Ref. [146] as extracted from Ref. [141].

For the $K_S \to \mu^+\mu^-$ decay, $H(0)$ corresponds to the one-loop pion contribution with two external on-shell photons [144] with the general formula for $H(z)$ given below. $\mathcal{I}_{\text{disp}} + i\mathcal{I}_{\text{abs}} = \mathcal{I}(m_\mu^2/M_K^2, m_{\pi^\pm}^2/M_K^2)$ where the general two-loop function $\mathcal{I}(a, b)$ is given in [151] in terms of a three-dimensional integral which for $K_S \to \pi^+\pi^- \to \gamma\gamma \to \ell^+\ell^-$ is equal to $-2.821 + i\,1.216 \quad (\pm 0.0.001)$. $\mathrm{Br}(K_S^0 \to \gamma\gamma)^{\mathrm{EXP}} = (2.63 \pm 0.17) \times 10^{-6}$ [150] and while the experimental measurement has less than 7% uncertainty, a 30% uncertainty should be considered on the branching ratio due to possible higher-order chiral corrections to the dispersive long-distance $K_S \to \mu^+\mu^-$ amplitude which at the lowest order in the chiral expansion arises from two-loop diagrams of the type $K_S \to \pi^+\pi^- \to \gamma\gamma \to \mu^+\mu^-$ [141].

The general expression for $H(z)$ where $z = (p+p')^2/M_K^2$ with p and p' denoting the momenta of ℓ^- and ℓ^+ can be found in [154, 155]

$$F(z) = \begin{cases} 1 - \frac{4}{z}\arcsin^2\left(\frac{\sqrt{z}}{2}\right) & z \le 4 \\ 1 + \frac{1}{z}\left(\ln\frac{1-\sqrt{1-4/z}}{1+\sqrt{1-4/z}} + i\pi\right)^2 & z > 4 \end{cases}, \tag{786}$$

$$G(z) = \begin{cases} \sqrt{4/z - 1}\,\arcsin\left(\frac{\sqrt{z}}{2}\right) & z \le 4 \\ \frac{1}{2}\sqrt{1-4/z}\left(\ln\frac{1+\sqrt{1-4/z}}{1-\sqrt{1-4/z}} - i\pi\right) & z > 4 \end{cases}, \tag{787}$$

$$\begin{aligned} H(z) \quad = \quad & \frac{1}{2(1-z)^2}\left\{ zF\left(\frac{z}{r_\pi^2}\right) - F\left(\frac{1}{r_\pi^2}\right)\right. \\ & \left. -2z\left[G\left(\frac{z}{r_\pi^2}\right) - G\left(\frac{1}{r_\pi^2}\right)\right]\right\} \end{aligned} \tag{788}$$

with $r_\pi = m_\pi/M_K$.

E.11 Branching ratio of $B_u \to \tau\nu_\tau$

The purely leptonic decay $B_u \to \tau\nu_\tau$ occurs via W^+ and H^+ mediated annihilation processes. This decay is helicity suppressed in the SM, but there is no such suppression for the charged Higgs exchange at high $\tan\beta$, and the two contributions can therefore be of similar magnitudes. This decay is thus very sensitive to charged Higgs boson and provide important constraints.

The branching ratio of $B_u \to \tau\nu_\tau$ in Supersymmetry is given by [92]

$$\mathrm{BR}(B_u \to \tau\nu_\tau) = \frac{G_F^2 f_B^2 |V_{ub}|^2}{8\pi} \tau_B m_B m_\tau^2 \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 \left[1 - \left(\frac{m_B^2}{M_{H^+}^2}\right) \frac{\tan^2\beta}{1+\epsilon_0 \tan\beta}\right]^2 , \quad (789)$$

where ϵ_0 is given in Eq. (101), and τ_B is the $B^\pm$ meson lifetime which is given in Appendix G together with the other constants in this equation.

The following ratio is usually considered to express the new physics contributions:

$$R_{\tau\nu_\tau}^{\mathrm{MSSM}} = \frac{\mathrm{BR}(B_u \to \tau\nu_\tau)_{\mathrm{MSSM}}}{\mathrm{BR}(B_u \to \tau\nu_\tau)_{\mathrm{SM}}} = \left[1 - \left(\frac{m_B^2}{M_{H^+}^2}\right) \frac{\tan^2\beta}{1+\epsilon_0 \tan\beta}\right]^2 , \quad (790)$$

which is also implemented in `SuperIso`.

In the 2HDM, Eq. (789) takes the form

$$\mathrm{BR}(B_u \to \tau\nu_\tau) = \frac{G_F^2 f_B^2 |V_{ub}|^2}{8\pi} \tau_B m_B m_\tau^2 \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 \left[1 - \left(\frac{m_B^2}{M_{H^+}^2}\right) \lambda_{bb}\lambda_{\tau\tau}\right]^2 , \quad (791)$$

where the Yukawa couplings $\lambda_{bb}, \lambda_{\tau\tau}$ can be found in Table 4 for the four types of 2HDM Yukawa sectors.

E.12 Branching ratio of $B \to D\tau\nu_\tau$

The semileptonic decay $B \to D\tau\nu_\tau$ is similar to $B_u \to \tau\nu_\tau$. The SM helicity suppression here occurs only near the kinematic endpoint. The branching ratio of $B \to D\tau\nu_\tau$ on the other hand is about 50 times larger that the branching ratio of $B_u \to \tau\nu_\tau$ in the SM.

In Supersymmetry, the partial rate of the transition $B \to D\ell\nu_\ell$ (where $\ell = e, \mu$ or τ) can be written in function of w as [93]

$$\begin{aligned}\frac{d\Gamma(B \to D\ell\nu_\ell)}{dw} &= \frac{G_F^2 |V_{cb}|^2 m_B^5}{192\pi^3} \rho_V(w) \quad (792)\\ &\times \left[1 - \frac{m_\ell^2}{m_B^2} \left|1 - t(w) \frac{m_b}{(m_b - m_c) M_{H^+}^2} \frac{\tan^2\beta}{1+\epsilon_0 \tan\beta}\right|^2 \rho_S(w)\right] ,\end{aligned}$$

where w is a kinematic variable defined as:

$$w = \frac{1 + (m_D/m_B)^2 - (p_B - p_D)^2/m_B^2}{2 m_D/m_B} , \quad (793)$$

with p_D and p_B the meson four-momenta, and $t(w) = m_B^2 + m_D^2 - 2w\, m_D\, m_B$. Again, ϵ_0 is given in Eq. (101), and

$$t(w) = m_B^2 + m_D^2 - 2w m_D m_B \; . \tag{794}$$

In general 2HDM, Eq. (792) is replaced by

$$\begin{aligned} \frac{d\Gamma(B \to D\ell\nu_\ell)}{dw} &= \frac{G_F^2 |V_{cb}|^2 m_B^5}{192\pi^3} \rho_V(w) \qquad (795) \\ &\quad \times \left[1 - \frac{m_\ell^2}{m_B^2} \left| 1 - t(w) \frac{m_b \lambda_{bb} - m_c \lambda_{cc}}{(m_b - m_c) M_{H^+}^2} \lambda_{\ell\ell} \right|^2 \rho_S(w) \right] , \end{aligned}$$

where the Yukawa couplings $\lambda_{bb}, \lambda_{\ell\ell}$ can be found in Table 4 for the four types of 2HDM Yukawa sectors.

The vector and scalar Dalitz density contributions read [93]

$$\rho_V(w) = 4 \left(1 + \frac{m_D}{m_B} \right)^2 \left(\frac{m_D}{m_B} \right)^3 (w^2 - 1)^{\frac{3}{2}} \left(1 - \frac{m_\ell^2}{t(w)} \right)^2 \left(1 + \frac{m_\ell^2}{2t(w)} \right) G(w)^2 \; , \tag{796}$$

$$\rho_S(w) = \frac{3}{2} \frac{m_B^2}{t(w)} \left(1 + \frac{m_\ell^2}{2t(w)} \right)^{-1} \frac{1 + w}{1 - w} \Delta(w)^2 \; , \tag{797}$$

where $G(w)$ and $\Delta(w)$ are hadronic form factors. $G(w)$ can be parametrized as

$$G(w) = G(1) \times [1 - 8\rho^2 z(w) + (51\rho^2 - 10) z(w)^2 - (252\rho^2 - 84) z(w)^3] \; , \tag{798}$$

with

$$z(w) = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}} \; , \tag{799}$$

and $\Delta(w)$ is [94]

$$\Delta(w) = 0.46 \pm 0.02 \; . \tag{800}$$

The parameters $G(1)$ and ρ^2 are given in Appendix G. Integrating Eq. (792) over w leads as a result to the value of the branching ratio.

The following ratio

$$\xi_{D\ell\nu} = \frac{\mathrm{BR}(B \to D^0 \tau \nu_\tau)}{\mathrm{BR}(B \to D^0 e \nu_e)} \tag{801}$$

is also considered in order to reduce some of the theoretical uncertainties. It can be calculated using Eq. (792).

E.13 Branching ratio of $K \to \mu\nu_\mu$

The leptonic kaon decay $K \to \mu\nu_\mu$ is also very similar to $B_u \to \tau\nu_\tau$, and is mediated via W^+ and H^+ annihilation processes. The charged Higgs contribution is however reduced as H^+ couples to lighter quarks in this case.

We consider the following ratio in `SuperIso` in order to reduce the theoretical uncertainties from f_K [95], which reads in Supersymmetry

$$\frac{\mathrm{BR}(K \to \mu\nu_\mu)}{\mathrm{BR}(\pi \to \mu\nu_\mu)} = \frac{\tau_K}{\tau_\pi}\left|\frac{V_{us}}{V_{ud}}\right|^2 \frac{f_K^2}{f_\pi^2}\frac{m_K}{m_\pi}\left(\frac{1-m_\ell^2/m_K^2}{1-m_\ell^2/m_\pi^2}\right)^2 \times\left[1-\frac{m_{K^+}^2}{M_{H^+}^2}\left(1-\frac{m_d}{m_s}\right)\frac{\tan^2\beta}{1+\epsilon_0\tan\beta}\right]^2(1+\delta_{\mathrm{em}}) \;, \tag{802}$$

where $\delta_{\mathrm{em}} = 0.0070 \pm 0.0035$ is a long distance electromagnetic correction factor, the ratio f_K/f_π is given in Appendix G, and ϵ_0 for the second generation of quarks reads:

$$\epsilon_0 = -\frac{2\,\alpha_s\,\mu}{3\,\pi\,m_{\tilde{g}}} H_2\left(\frac{m_{\tilde{q}_L}^2}{m_{\tilde{g}}^2}, \frac{m_{\tilde{d}_R}^2}{m_{\tilde{g}}^2}\right) \;, \tag{803}$$

where $H_2(x,y)$ is given in Eq. (103).

The additional quantity $R_{\mu 23}$ [95] is also implemented in `SuperIso`

$$R_{\mu 23} = \left|\frac{V_{us}(K_{\ell 2})}{V_{us}(K_{\ell 3})}\times\frac{V_{ud}(0^+ \to 0^+)}{V_{ud}(\pi_{\ell 2})}\right| = \left|1-\frac{m_{K^+}^2}{M_{H^+}^2}\left(1-\frac{m_d}{m_s}\right)\frac{\tan^2\beta}{1+\epsilon_0\tan\beta}\right| \;, \tag{804}$$

where ℓi refers to leptonic decays with i particles in the final state, and $0^+ \to 0^+$ corresponds to nuclear beta decay.

In general 2HDM, Eq. (804) reads

$$R_{\mu 23} = \left|1-\frac{m_{K^+}^2}{M_{H^+}^2}\left(1-\frac{m_d}{m_s}\right)\lambda_{ss}\lambda_{\mu\mu}\right| \;, \tag{805}$$

where the Yukawa couplings $\lambda_{ss}, \lambda_{\mu\mu}$ can be found in Table 4 for the four types of 2HDM Yukawa sectors.

E.14 Branching ratio of $D_s \to \ell\nu_\ell$

The purely leptonic decays $D_s \to \ell\nu_\ell$ are very similar to $K \to \mu\nu_\mu$ and proceed via annihilation of the heavy meson into W^+ and H^+. The charged Higgs boson contribution can only suppress the branching ratio and is therefore slightly disfavoured. In Supersymmetry the branching fraction is given by (where $\ell = e, \mu$ or τ) [96, 97]:

$$\mathrm{BR}(D_s \to \ell\nu_\ell) = \frac{G_F^2}{8\pi}|V_{cs}|^2 f_{D_s}^2 m_\ell^2 m_{D_s}\tau_{D_s}\left(1-\frac{m_\ell^2}{m_{D_s}^2}\right)^2 \times\left[1+\left(\frac{1}{m_c+m_s}\right)\left(\frac{m_{D_s}}{M_{H^+}}\right)^2\left(m_c-\frac{m_s\tan^2\beta}{1+\epsilon_0\tan\beta}\right)\right]^2 \;, \tag{806}$$

where τ_{D_s} and f_{D_s} are the $D_s^\pm$ meson lifetime and decay constant respectively, which are given in Appendix G together with the other constants in this equation, and

$$\epsilon_0 = -\frac{2\,\alpha_s\,\mu}{3\,\pi\,m_{\tilde{g}}} H_2\left(\frac{m^2_{\tilde{q}_L}}{m^2_{\tilde{g}}}, \frac{m^2_{\tilde{u}_R}}{m^2_{\tilde{g}}}\right) \;, \tag{807}$$

with $H_2(x,y)$ given in Eq. (103).

In general 2HDM, Eqs. (807) becomes

$$\begin{aligned} \mathrm{BR}(D_s \to \ell\nu_\ell) &= \frac{G_F^2}{8\pi} |V_{cs}|^2 f_{D_s}^2 m_\ell^2 m_{D_s} \tau_{D_s} \left(1 - \frac{m_\ell^2}{m_{D_s}^2}\right)^2 \\ &\times \left[1 - \left(\frac{m_{D_s}}{M_{H^+}}\right)^2 \frac{m_s\lambda_{ss} - m_c\lambda_{cc}}{(m_c+m_s)}\lambda_{\ell\ell}\right]^2 \;, \end{aligned} \tag{808}$$

where the Yukawa couplings $\lambda_{cc}, \lambda_{ss}, \lambda_{\ell\ell}$ can be found in Table 4 for the four types of 2HDM Yukawa sectors.

E.15 Branching ratio of $D \to \mu\nu_\mu$

The decay $D \to \mu\nu_\mu$ is also measured experimentally. In Supersymmetry the branching fraction is given by:

$$\begin{aligned} \mathrm{BR}(D \to \mu\nu_\mu) &= \frac{G_F^2}{8\pi} |V_{cd}|^2 f_D^2 m_\ell^2 m_D \tau_D \left(1 - \frac{m_\mu^2}{m_D^2}\right)^2 \\ &\times \left[1 + \left(\frac{1}{m_c + m_d}\right)\left(\frac{m_D}{M_{H^+}}\right)^2 \left(m_c - \frac{m_d \tan^2\beta}{1 + \epsilon_0 \tan\beta}\right)\right]^2 \;, \end{aligned} \tag{809}$$

where τ_D and f_D are the $D^\pm$ meson lifetime and decay constant respectively, which are given in Appendix G together with the other constants in this equation, and ϵ_0 is given in Eq. (807) and $H_2(x,y)$ in Eq. (103). Contrary to $D_s \to \ell\nu_\ell$, the $\tan\beta$ terms are suppressed in this decay.

In general 2HDM, Eqs. (810) becomes

$$\begin{aligned} \mathrm{BR}(D \to \mu\nu_\mu) &= \frac{G_F^2}{8\pi} |V_{cd}|^2 f_D^2 m_\ell^2 m_D \tau_D \left(1 - \frac{m_\ell^2}{m_D^2}\right)^2 \\ &\times \left[1 - \left(\frac{m_D}{M_{H^+}}\right)^2 \frac{m_d\lambda_{dd} - m_c\lambda_{cc}}{(m_c+m_d)}\lambda_{\mu\mu}\right]^2 \;, \end{aligned} \tag{810}$$

where the Yukawa couplings $\lambda_{cc}, \lambda_{dd}, \lambda_{\mu\mu}$ can be found in Table 4 for the four types of 2HDM Yukawa sectors.

Appendix F Muon anomalous magnetic moment

The magnetic moment of the muon can be written as

$$M = \frac{e\hbar}{2m_\mu}(1 + a_\mu) \ , \tag{811}$$

where $e\hbar/2m_\mu$ is the Dirac moment and the small higher order correction to the tree level is called the anomalous magnetic moment $a_\mu = (g_\mu - 2)/2$.

F.1 Supersymmetry

Supersymmetry can contribute to this anomaly through chargino-sneutrino and neutralino-smuon loops. The one-loop SUSY contributions to a_μ are [98]

$$\delta a_\mu^{\chi^0} = \frac{m_\mu}{16\pi^2} \sum_{i=1}^{n_{\chi^0}} \sum_{m=1}^{2} \left\{ -\frac{m_\mu}{12 m_{\tilde{\mu}_m}^2} (|n_{im}^L|^2 + |n_{im}^R|^2) F_1^N(x_{im}) + \frac{m_{\chi_i^0}}{3 m_{\tilde{\mu}_m}^2} \mathrm{Re}[n_{im}^L n_{im}^R] F_2^N(x_{im}) \right\} \ , \tag{812}$$

and

$$\delta a_\mu^{\chi^\pm} = \frac{m_\mu}{16\pi^2} \sum_{k=1}^{2} \left\{ \frac{m_\mu}{12 m_{\tilde{\nu}_\mu}^2} \left(|c_k^L|^2 + |c_k^R|^2 \right) F_1^C(x_k) + \frac{2 m_{\chi_k^\pm}}{3 m_{\tilde{\nu}_\mu}^2} \mathrm{Re}[c_k^L c_k^R] \, F_2^C(x_k) \right\} \ , \tag{813}$$

where n_{χ^0} is 4 in the MSSM and 5 in the NMSSM. i, m and k are neutralino, smuon and chargino mass eigenstate labels respectively, and

$$n_{im}^R = \sqrt{2} g' N_{i1} X_{m2} + y_\mu N_{i3} X_{m1} \ , \tag{814}$$

$$n_{im}^L = \frac{1}{\sqrt{2}} \left(g N_{i2} + g' N_{i1} \right) X_{m1}^* - y_\mu N_{i3} X_{m2}^* \ , \tag{815}$$

$$c_k^R = y_\mu U_{k2} \ , \tag{816}$$

$$c_k^L = -g V_{k1} \ , \tag{817}$$

where $y_\mu = g m_\mu / \sqrt{2} M_W \cos\beta$ is the muon Yukawa coupling, and the X is the smuon mixing matrix. The functions F_i^N and F_i^C depend respectively on $x_{im} = m_{\chi_i^0}^2 / m_{\tilde{\mu}_m}^2$ and $x_k = m_{\chi_k^\pm}^2 / m_{\tilde{\nu}_\mu}^2$ as

$$F_1^N(x) = \frac{2}{(1-x)^4} \Big(1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x \Big) \ , \tag{818}$$

$$F_2^N(x) = \frac{3}{(1-x)^3} \Big(1 - x^2 + 2x \ln x \Big) \ , \tag{819}$$

$$F_1^C(x) = \frac{2}{(1-x)^4} \Big(2 + 3x - 6x^2 + x^3 + 6x \ln x \Big) \ , \tag{820}$$

$$F_2^C(x) = -\frac{3}{2(1-x)^3} \Big(3 - 4x + x^2 + 2 \ln x \Big) \ . \tag{821}$$

The muon anomalous magnetic moment can also receive at one loop contributions from the Higgs bosons, which can be large in the NMSSM [99]:

$$\delta a_\mu^{H^0} = \frac{G_\mu m_\mu^2}{4\sqrt{2}\pi^2} \sum_{i=1}^{3} \frac{(U_{i2}^H)^2}{\cos^2\beta} \int_0^1 \frac{x^2(2-x)\,dx}{x^2 + \left(\frac{M_{h_i}}{m_\mu}\right)^2 (1-x)} \,, \tag{822}$$

$$\delta a_\mu^{A^0} = -\frac{G_\mu m_\mu^2}{4\sqrt{2}\pi^2} \sum_{i=1}^{2} (U_{i1}^A)^2 \tan^2\beta \int_0^1 \frac{x^3\,dx}{x^2 + \left(\frac{M_{a_i}}{m_\mu}\right)^2 (1-x)} \,, \tag{823}$$

$$\delta a_\mu^{H^+} = \frac{G_\mu m_\mu^2}{4\sqrt{2}\pi^2} \tan^2\beta \int_0^1 \frac{x(x-1)\,dx}{x - 1 + \left(\frac{M_{H^\pm}}{m_\mu}\right)^2} \,, \tag{824}$$

where $G_\mu = g/4\sqrt{2}M_W^2$, and U^H and U^A are respectively the CP-even and CP-odd Higgs mixing matrices given in Eqs. (114) and (115), and M_{h_i} and M_{a_i} refer respectively to the masses of the three CP-even and the two CP-odd Higgs bosons.
In addition to these contributions, we also consider the leading logarithm QED correction from two-loop evaluation [100]:

$$a_{\mu,\,2\,\text{loop}}^{\text{SUSY}} = a_{\mu,\,1\,\text{loop}}^{\text{SUSY}} \left(1 - \frac{4\alpha}{\pi} \ln \frac{M_{\text{SUSY}}}{m_\mu}\right) \,, \tag{825}$$

where M_{SUSY} is a typical superpartner mass scale.

The dominant two loop contributions from the photonic Barr-Zee diagrams with physical Higgs bosons are as follows [101]

$$a_\mu^{(\chi\gamma H)} = \frac{\alpha^2 m_\mu^2}{8\pi^2 M_W^2 s_W^2} \sum_{k=1,2} \left\{ \sum_{a_i} \text{Re}[\lambda_\mu^{a_i} \lambda_{\chi_k^\pm}^{a_i}]\ f_{PS}(m_{\chi_k^\pm}^2/M_{a_i}^2) \right. \tag{826}$$

$$\left. + \sum_{h_i} \text{Re}[\lambda_\mu^{h_i} \lambda_{\chi_k^\pm}^{h_i}]\ f_S(m_{\chi_k^\pm}^2/M_{h_i}^2) \right\} ,$$

$$a_\mu^{(\tilde{f}\gamma H)} = \frac{\alpha^2 m_\mu^2}{8\pi^2 M_W^2 s_W^2} \sum_{\tilde{f}=\tilde{t},\tilde{b},\tilde{\tau}} (N_c Q^2)_{\tilde{f}} \sum_{j=1,2} \sum_{h_i} \text{Re}[\lambda_\mu^{h_i} \lambda_{\tilde{f}_j}^{h_i}]\ f_{\tilde{f}}(m_{\tilde{f}_j}^2/M_{h_i}^2) \,, \tag{827}$$

where h_i and a_i stand respectively for (h^0, H^0) and A^0 in the MSSM, and (h^0, H^0, H_3^0) and (A_1^0, A_2^0) in the NMSSM. N_c is the colour number and Q the electric charge.

The couplings of the Higgs to muon, charginos and sfermions in the MSSM are given by:

$$\lambda_{\mu}^{[h^0,H^0,A^0]} = \left[-\frac{\sin\alpha}{\cos\beta}, \frac{\cos\alpha}{\cos\beta}, \tan\beta\right] , \tag{828}$$

$$\lambda_{\chi_k^\pm}^{[h^0,H^0,A^0]} = \frac{\sqrt{2}M_W}{m_{\chi_k^\pm}}\Big\{U_{k1}V_{k2}\big[\cos\alpha, \sin\alpha, -\cos\beta\big] \tag{829}$$

$$+U_{k2}V_{k1}\big[-\sin\alpha, \cos\alpha, -\sin\beta\big]\Big\} ,$$

$$\lambda_{\tilde{t}_i}^{[h^0,H^0]} = \frac{2m_t}{m_{\tilde{t}_i}^2 \sin\beta}\Big\{+\mu^*\big[\sin\alpha, -\cos\alpha\big] + A_t\big[\cos\alpha, \sin\alpha\big]\Big\} \, (D_{i1}^{\tilde{t}})^* D_{i2}^{\tilde{t}} , \tag{830}$$

$$\lambda_{\tilde{b}_i}^{[h^0,H^0]} = \frac{2m_b}{m_{\tilde{b}_i}^2 \cos\beta}\Big\{-\mu^*\big[\cos\alpha, \sin\alpha\big] + A_b\big[-\sin\alpha, \cos\alpha\big]\Big\}(D_{i1}^{\tilde{b}})^* D_{i2}^{\tilde{b}} , \tag{831}$$

$$\lambda_{\tilde{\tau}_i}^{[h^0,H^0]} = \frac{2m_\tau}{m_{\tilde{\tau}_i}^2 \cos\beta}\Big\{-\mu^*\big[\cos\alpha, \sin\alpha\big] + A_\tau\big[-\sin\alpha, \cos\alpha\big]\Big\} \, (D_{i1}^{\tilde{\tau}})^* D_{i2}^{\tilde{\tau}} , \tag{832}$$

where U and V are the chargino mixing matrices, and $D_{\tilde{f}}$ is the sfermion $\tilde{f}$ mixing matrix.

In the NMSSM, these couplings can be generalized as [99]:

$$\lambda_{\mu}^{h_i} = \frac{U_{i2}^H}{\cos\beta} , \tag{833}$$

$$\lambda_{\mu}^{a_i} = U_{i2}^A \tan\beta , \tag{834}$$

$$\lambda_{\chi_k^\pm}^{h_i} = \frac{\sqrt{2}M_W}{g\, m_{\chi_k^\pm}}\Big[\lambda U_{k2}V_{k2}U_{i3}^H + g\left(U_{k1}V_{k2}U_{i1}^H + U_{k2}V_{k1}U_{i2}^H\right)\Big] , \tag{835}$$

$$\lambda_{\chi_k^\pm}^{a_i} = \frac{\sqrt{2}M_W}{g\, m_{\chi_k^\pm}}\Big[\lambda U_{k2}V_{k2}U_{i2}^A - g\left(U_{k1}V_{k2}\cos\beta + U_{k2}V_{k1}\sin\beta\right)U_{i1}^A\Big] , \tag{836}$$

$$\lambda_{\tilde{t}_k}^{h_i} = \frac{2\sqrt{2}M_W}{g m_{\tilde{t}_i}^2}\Bigg\{h_t\big[A_t U_{i1}^H - \lambda(x U_{i2}^H + v_d U_{i3}^H)\big]\mathrm{Re}[(D_{k1}^{\tilde{t}})^* D_{k2}^{\tilde{t}}] \tag{837}$$

$$+\Big[h_t^2 v_u U_{i1}^H - \frac{g'^2}{3}\left(v_u U_{i1}^H - v_d U_{i2}^H\right)\Big]\left|D_{k2}^{\tilde{t}}\right|^2$$

$$+\Big[h_t^2 v_u U_{i1}^H - \frac{3g^2 - g'^2}{12}(v_u U_{i1}^H - v_d U_{i2}^H)\Big]\left|D_{k1}^{\tilde{t}}\right|^2\Bigg\} ,$$

(838)

$$\lambda^{h_i}_{\tilde{b}_k} = \frac{2\sqrt{2}M_W}{g m^2_{\tilde{b}_i}} \Bigg\{ h_b \big[A_b U^H_{i2} - \lambda(x U^H_{i1} + v_u U^H_{i3})\big] \mathrm{Re}[(D^{\tilde{b}}_{k1})^* D^{\tilde{b}}_{k2}] \tag{839}$$

$$+ \Big[h_b^2 v_d U^H_{i2} + \frac{g'^2}{6}\left(v_u U^H_{i1} - v_d U^H_{i2}\right)\Big] \left|D^{\tilde{b}}_{k2}\right|^2$$

$$+ \Big[h_b^2 v_d U^H_{i2} + \frac{3g^2 + g'^2}{12}\left(v_u U^H_{i1} - v_d U^H_{i2}\right)\Big] \left|D^{\tilde{b}}_{k1}\right|^2 \Bigg\} ,$$

$$\lambda^{h_i}_{\tilde{\tau}_k} = \frac{2\sqrt{2}M_W}{g m^2_{\tilde{\tau}_i}} \Bigg\{ h_\tau \left[A_\tau U^H_{i2} - \lambda\left(x U^H_{i1} + v_u U^H_{i3}\right)\right] \mathrm{Re}[(D^{\tilde{\tau}}_{k1})^* D^{\tilde{\tau}}_{k2}] \tag{840}$$

$$+ \left[h_\tau^2 v_d U^H_{i2} + \frac{g'^2}{2}\left(v_u U^H_{i1} - v_d U^H_{i2}\right)\right] \left|D^{\tilde{\tau}}_{k2}\right|^2$$

$$+ \left[h_\tau^2 v_d U^H_{i2} + \frac{g^2 - g'^2}{4}\left(v_u U^H_{i1} - v_d U^H_{i2}\right)\right] \left|D^{\tilde{\tau}}_{k1}\right|^2 \Bigg\} ,$$

where v_u, v_d and x are the VEV of H_u, H_d and S such as

$$v_u^2 = \frac{\sin^2\beta}{\sqrt{2}G_F} \qquad , \qquad \tan\beta = \frac{v_u}{v_d} . \tag{841}$$

The loop integral function f_{PS} is given by:

$$f_{PS}(x) = x\int_0^1 \mathrm{d}z \frac{1}{z(1-z)-x} \ln\frac{z(1-z)}{x} = \frac{2x}{y}\Big[\mathrm{Li}_2\Big(1-\frac{1-y}{2x}\Big) - \mathrm{Li}_2\Big(1-\frac{1+y}{2x}\Big)\Big] , \tag{842}$$

with $y = \sqrt{1-4x}$. The other loop functions are related to f_{PS} as

$$f_S(x) = (2x-1) f_{PS}(x) - 2x(2+\log x) , \tag{843}$$

$$f_{\tilde{f}}(x) = \frac{x}{2}\Big[2 + \log x - f_{PS}(x)\Big] . \tag{844}$$

The contribution from the bosonic electroweak two loop diagrams can be written as [99,102]:

$$\delta a^{bos}_\mu = \frac{5\, G_F\, m^2_\mu\, \alpha}{24\sqrt{2}\,\pi^3}\left(c_L \ln\frac{m^2_\mu}{M^2_W} + c_0\right) , \tag{845}$$

where

$$c_L = \frac{1}{30}\left[98 + 9c^h_L + 23\left(1-4s^2_W\right)^2\right] . \tag{846}$$

c^h_L in the MSSM is given by

$$c^h_L = \frac{\cos 2\beta M^2_Z}{\cos\beta}\left[\frac{\cos\alpha\cos(\alpha+\beta)}{M^2_{H^0}} + \frac{\sin\alpha\sin(\alpha+\beta)}{M^2_{h^0}}\right] , \tag{847}$$

and in the NMSSM is extended to:

$$c_L^h = \cos 2\beta M_Z^2 \left[\sum_{i=1}^{3} \frac{U_{i2}^H (U_{i2}^H - \tan\beta U_{i1}^H)}{M_{h_i}^2} \right] , \tag{848}$$

from which the SM bosonic electroweak two loop contributions have to be deduced:

$$\delta a_\mu^{SM} = \frac{5\, G_F\, m_\mu^2\, \alpha}{24\sqrt{2}\, \pi^3} \left(c_L^{SM} \ln \frac{m_\mu^2}{M_W^2} + c_0^{SM} \right) , \tag{849}$$

with

$$c_L^{SM} = \frac{1}{30} \left[107 + 23 \left(1 - 4 s_W^2\right)^2 \right] , \tag{850}$$

and c_0 and c_0^{SM} are neglected.

F.2 2HDM

In the 2HDM the dominant contribution can be obtained by generalizing the results of [103]:

$$\begin{aligned} \delta a_\mu^H \;=\; & \sum_f \frac{\alpha\, m_\mu m_f}{8\pi^3} (N_c Q^2)_f \Bigg\{ -\frac{2\, T_3^f}{M_{A^0}^2} \rho^f \rho^\mu g\left(x_{fA_0}\right) \qquad (851) \\ & - \frac{1}{M_{h^0}^2} \Big[\kappa^f \sin(\beta-\alpha) + \rho^f \cos(\beta-\alpha) \Big] \Big[\kappa^\mu \sin(\beta-\alpha) + \rho^\mu \cos(\beta-\alpha) \Big] f\left(x_{fh_0}\right) \\ & - \frac{1}{M_{H^0}^2} \Big[\kappa^f \cos(\beta-\alpha) - \rho^f \sin(\beta-\alpha) \Big] \Big[\kappa^\mu \cos(\beta-\alpha) - \rho^\mu \sin(\beta-\alpha) \Big] f\left(x_{fH_0}\right) \Bigg\} \end{aligned}$$

where $\kappa^f = \sqrt{2} m_f / v$, $\rho^f = \lambda_{ff} \kappa^f$, $x_{fX} = m_f^2 / m_X^2$, and T_3^f is the third component of the weak isospin, $-1/2$ for down-type fermions and $1/2$ for up-type fermions. Finally, the f and g functions are given by

$$f(x) = \int_0^1 \mathrm{d}y \frac{1 - 2y(1-y)}{y(1-y) - x} \ln \frac{y(1-y)}{x} , \tag{852}$$

and

$$g(x) = \int_0^1 \mathrm{d}y \frac{1}{y(1-y) - x} \ln \frac{y(1-y)}{x} . \tag{853}$$

Appendix G Useful parameters

The masses of quarks and mesons, as well as some other useful parameters such as lifetimes, CKM matrix elements and decay constants, are given in Table 17.

Appendix H Suggested limits

In Table 18, we present our suggested limits for each observable, which can be used to constrain SUSY parameters.
We would like to stress however that some of the inputs in this table suffer from large uncertainties from the determination of CKM matrix elements and/or hadronic parameters. The constraints obtained using these observables should therefore not be over-interpreted (see [21] for more details).

The limits on the masses of Higgs and SUSY particles from direct searches at colliders are given in Table 20. Some of the limits are subject to auxiliary conditions (see [15]) which are also taken into account in the program. These values are encoded in `src/excluded_masses.c` and can be updated by the user if necessary.

Appendix I LHA file format for 2HDM

`SuperIso` needs a Les Houches Accord (LHA) inspired input file for the calculations in 2HDM. This file, in addition to the usual MODSEL, SMINPUTS, GAUGE, MASS and ALPHA blocks of SLHA format, needs three additional blocks to specify the Yukawa coupling matrices for up-type and down-type quarks and for leptons. Also, the 2HDM model must be specified in the MODSEL block with the entry 0 followed by a positive integer. Such a file can be generated by `2HDMC`. An example is given in the following:

```
Block MODSEL # Select Model
    0   10     #  10 = THDM
Block SMINPUTS  # Standard Model inputs
    1        1.27910000e+02   # 1/alpha_em(MZ) SM MSbar
    2        1.16637000e-05   # G Fermi
    3        1.17600000e-01   # alpha_s(MZ) SM MSbar
    4        9.11876000e+01   # MZ
    5        4.24680224e+00   # mb(mb)
    6        1.71200000e+02   # mt (pole)
    7        1.77700000e+00   # mtau(pole)
Block GAUGE  # SM Gauge couplings
    1        3.58051564e-01   # g'
    2        6.48408288e-01   # g
    3        1.47780518e+00   # g_3
Block MASS      #  Mass spectrum (kinematic masses)
#  PDG      Mass
    25        9.11123204e+01   # Mh1, lightest CP-even Higgs
    35        5.00013710e+02   # Mh2, heaviest CP-even Higgs
```

Meson masses in MeV [15]								
m_π	m_K	m_{K^*}	m_{D^0}	m_D	m_{D_s}	m_B	m_{B_s}	m_{B_d}
139.57	493.677	895.81	1864.84	1869.61	1968.30	5279.26	5366.77	5279.58

Meson lifetimes in ps [15]						
τ_π	τ_K	τ_B	τ_{B_s}	τ_{B_d}	τ_D	τ_{D_s}
26033	12380	1.638	1.512	1.519	1.040	0.500

Quark masses in GeV and $\boldsymbol{\alpha_s}$ [15]				
$\overline{m}_b(\overline{m}_b)$	$\overline{m}_c(\overline{m}_c)$	m_s	m_t^{pole}	$\alpha_s(M_Z)$
4.18 ± 0.03	1.275 ± 0.025	0.095 ± 0.005	$173.34 \pm 0.27 \pm 0.71$	0.1184 ± 0.0008

Hadronic parameters in MeV ($\mu = 1$ GeV)						
f_K/f_π [104]	f_{K^*} [105]	$f_{K^*}^\perp$ [105]	f_B [50]	λ_B [86]	f_{B_s} [50]	f_{D_s} [106]
1.193 ± 0.006	220 ± 5	185 ± 9	194 ± 10	460 ± 110	234 ± 10	248 ± 2.5

Meson mass and coupling parameters ($\mu = 1$ GeV) [61]					
ζ_3^A	ζ_3^V	ζ_3^T	$\omega_{1,0}^A$	$\tilde{\delta}_+$	$\tilde{\delta}_-$
0.032	0.013	0.024	-2.1	0.16	-0.16

Meson related parameters ($\mu = 1$ GeV)			
$a_1^\perp(K^*)$ [83]	$a_2^\perp(K^*)$ [83]	$a_1^\parallel(K^*)$ [83]	$a_2^\parallel(K^*)$ [83]
0.10 ± 0.07	0.13 ± 0.08	0.10 ± 0.07	0.09 ± 0.05
$G(1)$ [94]	ρ^2 [93]	$\Delta(w)$ [94]	$T_1^{B\to K^*}$ [80]
1.026 ± 0.017	1.17 ± 0.18	0.46 ± 0.02	0.268 ± 0.032

CKM matrix elements [15]				
$\lvert V_{ud}\rvert$	$\lvert V_{us}\rvert$ [15]	$\lvert V_{ub}\rvert$	$\lvert V_{cd}\rvert$	$\lvert V_{cs}\rvert$
0.97427±0.00015	0.22537 ± 0.0007	(3.55±0.02) 10^{-3}	0.2252 ± 0.0007	0.97343±0.00016

$\lvert V_{cb}\rvert$	$\lvert V_{td}\rvert$	$\lvert V_{ts}\rvert$	$\lvert V_{tb}\rvert$
$(4.13 \pm 0.11) \times 10^{-2}$	$(8.86 \pm 0.26) \times 10^{-3}$	$(4.05 \pm 0.11) \times 10^{-2}$	0.999139 ± 0.000045

$\boldsymbol{b \to s\gamma}$ and $\boldsymbol{b \to s\ell^+\ell^-}$ related parameters				
μ_G^2 [54]	ρ_D^3 [54]	ρ_{LS}^3 [54]	λ_2 (GeV^2) [15,53]	BR$(B \to X_c e\bar{\nu})_{\text{exp}}$ [54]
0.336 ± 0.064	0.153 ± 0.45	-0.145 ± 0.098	0.12	0.1065 ± 0.0016

Table 17: Input parameters.

Observable	**Combined experimental value**	**95% C.L. Bound**
$\text{BR}(B \to X_s\gamma)$	$(3.43 \pm 0.21 \pm 0.07) \times 10^{-4}$ [109]	$2.63 \times 10^{-4} \leq \text{BR}(b \to s\gamma) \leq 4.23 \times 10^{-4}$
$\Delta_0(B \to K^*\gamma)$	$(5.2 \pm 2.6) \times 10^{-2}$ (a)	$-1.7 \times 10^{-2} < \Delta_0 < 8.9 \times 10^{-2}$
$\text{BR}(B_u \to \tau\nu_\tau)$ $R_{\tau\nu_\tau}$	$(1.64 \pm 0.34) \times 10^{-4}$ [109] 1.63 ± 0.54 (b)	$0.71 \times 10^{-4} < \text{BR}(B_u \to \tau\nu_\tau) < 2.57 \times 10^{-4}$ $0.56 < R_{\tau\nu_\tau} < 2.70$
$\text{BR}(B \to D^0\tau\nu_\tau)$ $\xi_{D\ell\nu}$	$(8.6 \pm 2.4 \pm 1.1 \pm 0.6) \times 10^{-3}$ [112] $0.416 \pm 0.117 \pm 0.052$ [112]	$2.9 \times 10^{-3} < \text{BR}(B \to D^0\tau\nu_\tau) < 14.2 \times 10^{-3}$ $0.151 < \xi_{D\ell\nu} < 0.681$
$\text{BR}(B_s \to \mu^+\mu^-)$	$(2.9 \pm 0.7) \times 10^{-9}$ [113]	$1.3 \times 10^{-9} < \text{BR}(B_s \to \mu^+\mu^-) < 4.5 \times 10^{-9}$
$\text{BR}(B_d \to \mu^+\mu^-)$	$(3.6^{+1.6}_{-1.4}) \times 10^{-10}$ [113]	$\text{BR}(B_d \to \mu^+\mu^-) < 1.1 \times 10^{-9}$
$\dfrac{\text{BR}(K \to \mu\nu)}{\text{BR}(\pi \to \mu\nu)}$ $R_{\mu 23}$	0.6358 ± 0.0011 (c) 0.999 ± 0.007 [104]	$0.6257 < \dfrac{\text{BR}(K \to \mu\nu)}{\text{BR}(\pi \to \mu\nu)} < 0.6459$ $0.985 < R_{\mu 23} < 1.013$ (d)
$\text{BR}(D_s \to \tau\nu_\tau)$ $\text{BR}(D_s \to \mu\nu_\mu)$ $\text{BR}(D \to \mu\nu_\mu)$	$(5.38 \pm 0.32) \times 10^{-2}$ [97] $5.81 \pm 0.43 \times 10^{-3}$ [97] $(3.82 \pm 0.33) \times 10^{-4}$ [15]	$4.7 \times 10^{-2} < \text{BR}(D_s \to \tau\nu_\tau) < 6.1 \times 10^{-2}$ $4.9 \times 10^{-3} < \text{BR}(D_s \to \mu\nu_\mu) < 6.7 \times 10^{-3}$ $3.0 \times 10^{-4} < \text{BR}(D \to \mu\nu_\mu) < 4.6 \times 10^{-4}$
δa_μ	$(2.55 \pm 0.80) \times 10^{-9}$ [114]	$-2.4 \times 10^{-10} < \delta a_\mu < 5.0 \times 10^{-9}$

Table 18: Suggested limits for the observables implemented in `SuperIso v3.4`. The 95% C.L. bounds presented in this table include both the experimental and the theoretical uncertainties [*to be updated*].

(a) Value obtained combining the Babar measurement [110] with the results of [15, 111].
(b) Value deduced from [109].
(c) Value obtained combining the results of [15, 95].
(d) See [21] for a discussion on the uncertainties.

Observable	Experiment [115–117]	SM prediction
$\mathrm{BR}(B \to X_s \ell^+ \ell^-)_{q^2 \in [1,6] \mathrm{GeV}^2}$	$(1.56 \pm 0.39) \times 10^{-6}$	$(1.73 \pm 0.16) \times 10^{-6}$
$\mathrm{BR}(B \to X_s \ell^+ \ell^-)_{q^2 > 14.4 \mathrm{GeV}^2}$	$(4.79 \pm 1.04) \times 10^{-7}$	$(2.20 \pm 0.44) \times 10^{-7}$
$\langle dBR/dq^2 (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [0.1,2] \mathrm{GeV}^2}$	$(0.60 \pm 0.06 \pm 0.05 \pm 0.04 \pm 0.05) \times 10^{-7}$	$(0.70 \pm 0.81) \times 10^{-7}$
$\langle F_L (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [0.1,2] \mathrm{GeV}^2}$	$0.37 \pm 0.10 \pm 0.04$	0.32 ± 0.20
$\langle P_1 (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [0.1,2] \mathrm{GeV}^2}$	$-0.19 \pm 0.40 \pm 0.02$	-0.01 ± 0.04
$\langle P_2 (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [0.1,2] \mathrm{GeV}^2}$	$0.03 \pm 0.15 \pm 0.01$	0.17 ± 0.02
$\langle P_4' (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [0.1,2] \mathrm{GeV}^2}$	$0.00 \pm 0.52 \pm 0.06$	-0.37 ± 0.03
$\langle P_5' (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [0.1,2] \mathrm{GeV}^2}$	$0.45 \pm 0.22 \pm 0.09$	0.52 ± 0.04
$\langle P_6' (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [0.1,2] \mathrm{GeV}^2}$	$0.24 \pm 0.22 \pm 0.05$	-0.05 ± 0.04
$\langle P_8' (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [0.1,2] \mathrm{GeV}^2}$	$-0.12 \pm 0.56 \pm 0.04$	0.02 ± 0.04
$\langle dBR/dq^2 (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [2,4.3] \mathrm{GeV}^2}$	$(0.30 \pm 0.03 \pm 0.03 \pm 0.02 \pm 0.02) \times 10^{-7}$	$(0.35 \pm 0.29) \times 10^{-7}$
$\langle F_L (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [2,4.3] \mathrm{GeV}^2}$	$0.74 \pm 0.10 \pm 0.03$	0.76 ± 0.20
$\langle P_1 (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [2,4.3] \mathrm{GeV}^2}$	$-0.29 \pm 0.65 \pm 0.03$	-0.05 ± 0.05
$\langle P_2 (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [2,4.3] \mathrm{GeV}^2}$	$0.50 \pm 0.08 \pm 0.02$	0.25 ± 0.09
$\langle P_4' (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [2,4.3] \mathrm{GeV}^2}$	$0.74 \pm 0.58 \pm 0.16$	0.54 ± 0.07
$\langle P_5' (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [2,4.3] \mathrm{GeV}^2}$	$0.29 \pm 0.39 \pm 0.07$	-0.33 ± 0.11
$\langle P_6' (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [2,4.3] \mathrm{GeV}^2}$	$-0.15 \pm 0.38 \pm 0.05$	-0.06 ± 0.06
$\langle P_8' (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [2,4.3] \mathrm{GeV}^2}$	$-0.3 \pm 0.58 \pm 0.14$	0.04 ± 0.05
$\langle dBR/dq^2 (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [4.3,8.68] \mathrm{GeV}^2}$	$(0.49 \pm 0.04 \pm 0.04 \pm 0.03 \pm 0.04) \times 10^{-7}$	$(0.48 \pm 0.53) \times 10^{-7}$
$\langle F_L (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [4.3,8.68] \mathrm{GeV}^2}$	$0.57 \pm 0.07 \pm 0.03$	0.63 ± 0.14
$\langle P_1 (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [4.3,8.68] \mathrm{GeV}^2}$	$0.36 \pm 0.31 \pm 0.03$	-0.11 ± 0.06
$\langle P_2 (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [4.3,8.68] \mathrm{GeV}^2}$	$-0.25 \pm 0.08 \pm 0.02$	-0.36 ± 0.05
$\langle P_4' (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [4.3,8.68] \mathrm{GeV}^2}$	$1.18 \pm 0.30 \pm 0.10$	0.99 ± 0.03
$\langle P_5' (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [4.3,8.68] \mathrm{GeV}^2}$	$-0.19 \pm 0.16 \pm 0.03$	-0.83 ± 0.05
$\langle P_6' (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [4.3,8.68] \mathrm{GeV}^2}$	$0.04 \pm 0.15 \pm 0.05$	-0.02 ± 0.06
$\langle P_8' (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [4.3,8.68] \mathrm{GeV}^2}$	$0.58 \pm 0.38 \pm 0.06$	0.02 ± 0.06
$\langle dBR/dq^2 (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [14.18,16] \mathrm{GeV}^2}$	$(0.56 \pm 0.06 \pm 0.04 \pm 0.04 \pm 0.05) \times 10^{-7}$	$(0.67 \pm 1.17) \times 10^{-7}$
$\langle F_L (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [14.18,16] \mathrm{GeV}^2}$	$0.33 \pm 0.08 \pm 0.03$	0.39 ± 0.24
$\langle P_1 (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [14.18,16] \mathrm{GeV}^2}$	$0.07 \pm 0.28 \pm 0.02$	-0.32 ± 0.70
$\langle P_2 (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [14.18,16] \mathrm{GeV}^2}$	$-0.50 \pm 0.03 \pm 0.01$	-0.47 ± 0.14
$\langle P_4' (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [14.18,16] \mathrm{GeV}^2}$	$-0.18 \pm 0.70 \pm 0.08$	1.15 ± 0.33
$\langle P_5' (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [14.18,16] \mathrm{GeV}^2}$	$-0.79 \pm 0.20 \pm 0.18$	-0.82 ± 0.36
$\langle P_6' (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [14.18,16] \mathrm{GeV}^2}$	$0.18 \pm 0.25 \pm 0.03$	0.00 ± 0.00
$\langle P_8' (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [14.18,16] \mathrm{GeV}^2}$	$-0.40 \pm 0.60 \pm 0.06$	0.00 ± 0.01
$\langle dBR/dq^2 (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [16,19] \mathrm{GeV}^2}$	$(0.41 \pm 0.04 \pm 0.04 \pm 0.03 \pm 0.03) \times 10^{-7}$	$(0.43 \pm 0.78) \times 10^{-7}$
$\langle F_L (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [16,19] \mathrm{GeV}^2}$	$0.38 \pm 0.09 \pm 0.03$	0.36 ± 0.13
$\langle P_1 (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [16,19] \mathrm{GeV}^2}$	$-0.71 \pm 0.35 \pm 0.06$	-0.55 ± 0.59
$\langle P_2 (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [16,19] \mathrm{GeV}^2}$	$-0.32 \pm 0.08 \pm 0.01$	-0.41 ± 0.15
$\langle P_4' (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [16,19] \mathrm{GeV}^2}$	$0.70 \pm 0.52 \pm 0.06$	1.24 ± 0.25
$\langle P_5' (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [16,19] \mathrm{GeV}^2}$	$-0.60 \pm 0.19 \pm 0.09$	-0.66 ± 0.37
$\langle P_6' (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [16,19] \mathrm{GeV}^2}$	$-0.31 \pm 0.38 \pm 0.10$	0.00 ± 0.00
$\langle P_8' (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [16,19] \mathrm{GeV}^2}$	$0.12 \pm 0.54 \pm 0.04$	0.00 ± 0.04
$\langle dBR/dq^2 (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [1,6] \mathrm{GeV}^2}$	$(0.34 \pm 0.03 \pm 0.04 \pm 0.02 \pm 0.03) \times 10^{-7}$	$(0.38 \pm 0.33) \times 10^{-7}$
$\langle F_L (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [1,6] \mathrm{GeV}^2}$	$0.65 \pm 0.08 \pm 0.03$	0.70 ± 0.21
$\langle P_1 (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [1,6] \mathrm{GeV}^2}$	$0.15 \pm 0.41 \pm 0.03$	-0.06 ± 0.04
$\langle P_2 (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [1,6] \mathrm{GeV}^2}$	$0.33 \pm 0.12 \pm 0.02$	0.10 ± 0.08
$\langle P_4' (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [1,6] \mathrm{GeV}^2}$	$0.58 \pm 0.36 \pm 0.06$	0.53 ± 0.07
$\langle P_5' (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [1,6] \mathrm{GeV}^2}$	$0.21 \pm 0.21 \pm 0.03$	-0.34 ± 0.10
$\langle P_6' (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [1,6] \mathrm{GeV}^2}$	$0.18 \pm 0.21 \pm 0.03$	-0.05 ± 0.05
$\langle P_8' (B \to K^* \mu^+ \mu^-) \rangle_{q^2 \in [1,6] \mathrm{GeV}^2}$	$0.46 \pm 0.38 \pm 0.04$	0.03 ± 0.04

Table 19: Experimental results and theoretical predictions for the $b \to s\ell\ell$ observables [*to be updated*].

Particle	Limits (GeV)	Conditions
h^0	111	
H^+	79.3	
A^0	93.4	
$\tilde{\chi}_1^0$	46	
$\tilde{\chi}_2^0$	62.4	$\tan\beta < 40$
$\tilde{\chi}_3^0$	99.9	$\tan\beta < 40$
$\tilde{\chi}_4^0$	116	$\tan\beta < 40$
$\tilde{\chi}_1^\pm$	94	$\tan\beta < 40$, $m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0} > 5$ GeV
$\tilde{e}_R$	73	
$\tilde{e}_L$	107	
$\tilde{\mu}_{L,R}$	94	$\tan\beta < 40$, $m_{\tilde{\mu}_R} - m_{\tilde{\chi}_1^0} > 10$ GeV
$\tilde{\tau}_1$	81.9	$m_{\tilde{\tau}_1} - m_{\tilde{\chi}_1^0} > 15$ GeV
$\tilde{\nu}_\ell\ _{(\ell=e,\mu,\tau)}$	94	$\tan\beta < 40$, $m_{\tilde{\ell}_R} - m_{\tilde{\chi}_1^0} > 10$ GeV
$\tilde{u}_R$	100	$m_{\tilde{u}_R} - m_{\tilde{\chi}_1^0} > 10$ GeV
$\tilde{u}_L$	100	$m_{\tilde{u}_L} - m_{\tilde{\chi}_1^0} > 10$ GeV
$\tilde{t}_1$	95.7	$m_{\tilde{t}_1} - m_{\tilde{\chi}_1^0} > 10$ GeV
$\tilde{d}_R$	100	$m_{\tilde{d}_R} - m_{\tilde{\chi}_1^0} > 10$ GeV
$\tilde{d}_L$	100	$m_{\tilde{d}_L} - m_{\tilde{\chi}_1^0} > 10$ GeV
$\tilde{b}_1$	100	$m_{\tilde{b}_1} - m_{\tilde{\chi}_1^0} > 5$ GeV
$\tilde{g}$	195	

Table 20: Limit on the masses of Higgs and MSSM particles from direct searches at LEP and Tevatron [15]. The limit on the h^0 mass includes a 3 GeV intrinsic uncertainty.

```
    36        4.99999960e+02   # Mh3, CP-odd Higgs
    37        5.06332023e+02   # Mhc
Block ALPHA     # Effective Higgs mixing parameter
              3.12022092e+00   # alpha
Block UCOUPL
    1     1     0.00000000e+00   # LU_{1,1}
    1     2     0.00000000e+00   # LU_{1,2}
```

```
        1     3      0.00000000e+00   # LU_{1,3}
        2     1      0.00000000e+00   # LU_{2,1}
        2     2      2.00000000e-02   # LU_{2,2}
        2     3      0.00000000e+00   # LU_{2,3}
        3     1      0.00000000e+00   # LU_{3,1}
        3     2      0.00000000e+00   # LU_{3,2}
        3     3      2.00000000e-02   # LU_{3,3}
    Block DCOUPL
        1     1      0.00000000e+00   # LD_{1,1}
        1     2      0.00000000e+00   # LD_{1,2}
        1     3      0.00000000e+00   # LD_{1,3}
        2     1      0.00000000e+00   # LD_{2,1}
        2     2     -5.00000000e+01   # LD_{2,2}
        2     3      0.00000000e+00   # LD_{2,3}
        3     1      0.00000000e+00   # LD_{3,1}
        3     2      0.00000000e+00   # LD_{3,2}
        3     3     -5.00000000e+01   # LD_{3,3}

    Block LCOUPL
        1     1     -5.00000000e+01   # LL_{1,1}
        1     2      0.00000000e+00   # LL_{1,2}
        1     3      0.00000000e+00   # LL_{1,3}
        2     1      0.00000000e+00   # LL_{2,1}
        2     2     -5.00000000e+01   # LL_{2,2}
        2     3      0.00000000e+00   # LL_{2,3}
        3     1      0.00000000e+00   # LL_{3,1}
        3     2      0.00000000e+00   # LL_{3,2}
        3     3     -5.00000000e+01   # LL_{3,3}
```

The UCOUPL, DCOUPL and LCOUPL blocks contain respectively the Yukawa couplings λ_{UU}, λ_{DD} and λ_{LL}. If some entries in these blocks are missing, they will be set to 0. The Yukawa couplings for the types I–IV are specified in Table 4.

Appendix J Sample FLHA output file

`SuperIso` generates Flavour Les Houches Accord (FLHA) output files. A sample file is presented below. Some lines in the blocks `FOBS` and `FOBSMS` are broken for better readability.

```
# SuperIso output in Flavour Les Houches Accord format
Block FCINFO  # Program information
     1     SUPERISO         # flavour calculator
     2     3.3              # version number
Block MODSEL  # Model selection
     1     1   # Minimal supergravity (mSUGRA,CMSSM) model
```

```
Block SMINPUTS  # Standard Model inputs
     1     1.27839951e+02   # alpha_em^(-1)
     2     1.16570000e-05   # G_Fermi
     3     1.17200002e-01   # alpha_s(M_Z)
     4     9.11699982e+01   # m_{Z}(pole)
     5     4.19999981e+00   # m_{b}(m_{b})
     6     1.72399994e+02   # m_{top}(pole)
     7     1.77699995e+00   # m_{tau}(pole)
Block FMASS  # Mass spectrum in GeV
#PDG_code  mass        scheme scale particle
     5     4.68765531e+00   3   0   # b (1S)
   211     1.39600000e-01   0   0   # pi+
   313     8.91700000e-01   0   0   # K*
   321     4.93700000e-01   0   0   # K+
   421     1.86960000e+00   0   0   # D0
   431     1.96847000e+00   0   0   # D_s+
   521     5.27917000e+00   0   0   # B+
   531     5.36630000e+00   0   0   # B_s
Block FLIFE  # Lifetime in sec
#PDG_code  lifetime        particle
   211     2.60330000e-08   # pi+
   321     1.23800000e-08   # K+
   431     5.00000000e-13   # D_s+
   521     1.63800000e-12   # B+
   531     1.42500000e-12   # B_s
Block FCONST  # Decay constant in GeV
#PDG_code number decay_constant particle
   431     1   2.48000000e-01     0      0   # D_s+
   521     1   1.94000000e-01     0      0   # B+
   531     1   2.34000000e-01     0      0   # B_s
Block FCONSTRATIO  # Ratio of decay constants
#PDG_code1 code2  nb1 nb2 ratio            comment
   321     211    1   1   1.19300000e+00     0      0   # f_K/f_pi
Block FOBS  # Flavour observables
# ParentPDG type  value       q   NDA   ID1   ID2  ID3 ... comment
    5    1   3.01680109e-04   0     2     3    22        # BR(b->s gamma)
  521    4   7.94262137e-02   0     2   313    22        # Delta0(B->K* gamma)
  531    1   3.47501488e-09   0     2    13   -13        # BR(B_s->mu+ mu-)
  521    1   7.97936023e-05   0     2   -15    16        # BR(B_u->tau nu)
  521    2   9.96640789e-01   0     2   -15    16        # R(B_u->tau nu)
  431    1   5.09631717e-02   0     2   -15    16        # BR(D_s->tau nu)
  431    1   5.22975346e-03   0     2   -13    14        # BR(D_s->mu nu)
  521    1   6.74263197e-03   0     3   421   -15    16  # BR(B+->D0 tau nu)

  521   11   2.97215970e-01   0     3   421   -15    16  # BR(B+->D0 tau nu)
                                                            /BR(B+-> D0 e nu)
```

```
  321   11   6.34226493e-01   0     2   -13    14        # BR(K->mu nu)/BR(pi->mu nu)
  321   12   9.99985352e-01   0     2   -13    14        # R_mu23
Block FOBSSM  # SM predictions for flavour observables
# ParentPDG type  value       q   NDA   ID1   ID2  ID3 ... comment
    5    1   3.04981464e-04   0     2     3    22        # BR(b->s gamma)
  521    4   7.95890809e-02   0     2   313    22        # Delta0(B->K* gamma)
  531    1   3.49460689e-09   0     2    13   -13        # BR(B_s->mu+ mu-)
  521    1   8.00625493e-05   0     2   -15    16        # BR(B_u->tau nu)
  521    2   1.00000000e+00   0     2   -15    16        # R(B_u->tau nu)
  431    1   5.09649311e-02   0     2   -15    16        # BR(D_s->tau nu)
  431    1   5.22993400e-03   0     2   -13    14        # BR(D_s->mu nu)
  521    1   6.74722565e-03   0     3   421   -15    16  # BR(B+->D0 tau nu)
  521   11   2.97418460e-01   0     3   421   -15    16  # BR(B+->D0 tau nu)
                                                            /BR(B+-> D0 e nu)
  321   11   6.34245073e-01   0     2   -13    14        # BR(K->mu nu)/BR(pi->mu nu)
  321   12   1.00000000e+00   0     2   -13    14        # R_l23
```


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The literature on flavour physics observables and indirect searches for new physics is very rich, therefore it is impossible to refer to the complete list here. Since the present manuscript is meant to serve as a manual for the program, only the articles used directly in the program are acknowledged for clarity. We refer to [118] for a fairly complete review of available tools in SUSY.

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